Ideal Gas: properties

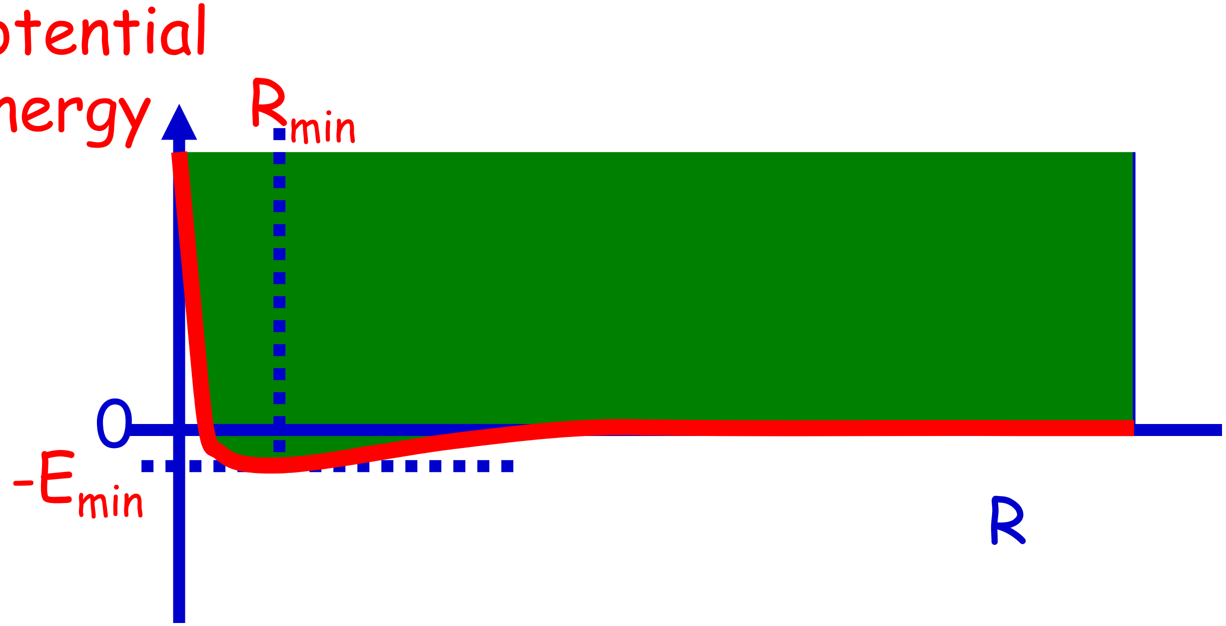
Collection of atoms/molecules that

- Exert no force upon each other

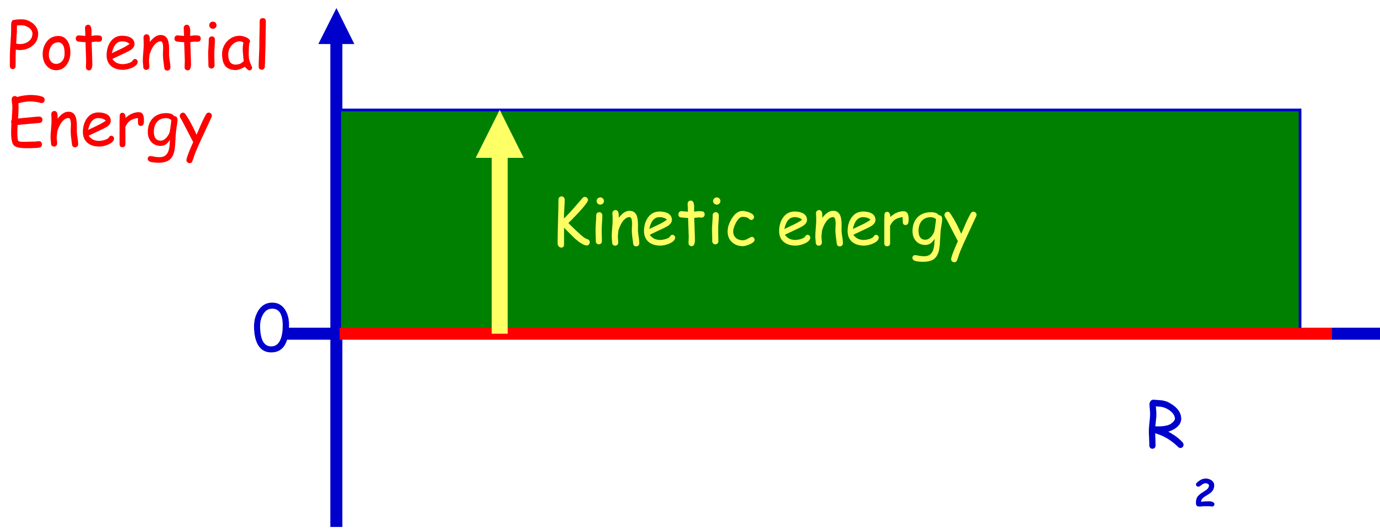
  The energy of a system of two atoms/molecules cannot be reduced by bringing them close to each other

- Take no volume

  The volume taken by the atoms/molecules is negligible compared to the volume they are sitting in



Ideal gas: we are neglecting the potential energy between
The atoms/molecules



Number of particles: mol

1 mol of particles: 6.02×10^{23} particles

Avogadro's number $N_A = 6.02 \times 10^{23}$ particles per mol

It doesn't matter what kind of particles:
1 mol is always N_A particles

What is the weight of 1 mol of atoms?

Number of protons

Name

Z

X

A

molar mass

1 H 1.01																	2 He 4.00
3 Li 6.94	4 Be 9.01											5 B 10.8	6 C 12.0	7 N 14.0	8 O 16.0	9 F 19.0	10 Ne 20.2
11 Na 23.0	12 Mg 24.3											13 Al 27.0	14 Si 28.1	15 P 31.0	16 S 32.1	17 Cl 35.5	18 Ar 40.0
19 K 39.1	20 Ca 40.1	21 Sc 45.0	22 Ti 47.9	23 V 50.9	24 Cr 52.0	25 Mn 54.9	26 Fe 55.8	27 Co 58.9	28 Ni 58.7	29 Cu 63.5	30 Zn 65.4	31 Ga 69.7	32 Ge 72.6	33 As 74.9	34 Se 79.0	35 Br 79.9	36 Kr 83.8
37 Rb 85.5	38 Sr 87.6	39 Y 88.9	40 Zr 91.2	41 Nb 92.9	42 Mo 95.9	43 Tc 98	44 Ru 101	45 Rh 103	46 Pd 106	47 Ag 108	48 Cd 112	49 In 115	50 Sn 119	51 Sb 122	52 Te 128	53 I 127	54 Xe 131
55 Cs 133	56 Ba 137	57 La 139	72 Hf 178	73 Ta 181	74 W 184	75 Re 186	76 Os 190	77 Ir 192	78 Pt 195	79 Au 197	80 Hg 201	81 Tl 204	82 Pb 207	83 Bi 209	84 Po 210	85 At 210	86 Rn 222
87 Fr 223	88 Ra 226	89 Ac 227	104 Rf 227	105 Db 262	106 Sg 263	107 Bh 264	108 Hs 265	109 Mt 268	110 Uun 269	111 Uuu 269	112 Uub 277	113	114 Uuq 289	115	116 Uun 289	117	118 Uuo 293

58 Ce 140	59 Pr 141	60 Nd 144	61 Pm 147	62 Sm 150	63 Eu 152	64 Gd 157	65 Tb 159	66 Dy 163	67 Ho 165	68 Er 167	69 Tm 169	70 Yb 173	71 Lu 175
90 Th 232	91 Pa 231	92 U 238	93 Np 237	94 Pu 244	95 Am 243	96 Cm 247	97 Bk 247	98 Cf 251	99 Es 254	100 Fm 257	101 Md 258	102 No 255	103 Lr 256

color code= light metals - brittle metals - ductile metals - low melting metals - non-metals - noble gases - lanthanides - actinides

Weight of 1 mol of atoms

1 mol of atoms: *A* gram (*A*: mass number)

Example: 1 mol of Carbon = 12 g
1 mol of Zinc = 65.4 g

What about molecules?
H_2O 1 mol of water molecules:
2x 1 g (due to Hydrogen)
1x 16 g (due to Oxygen)
Total: 18 g

Example

A cube of silicon (molar mass 28.1 g) is 250 g.

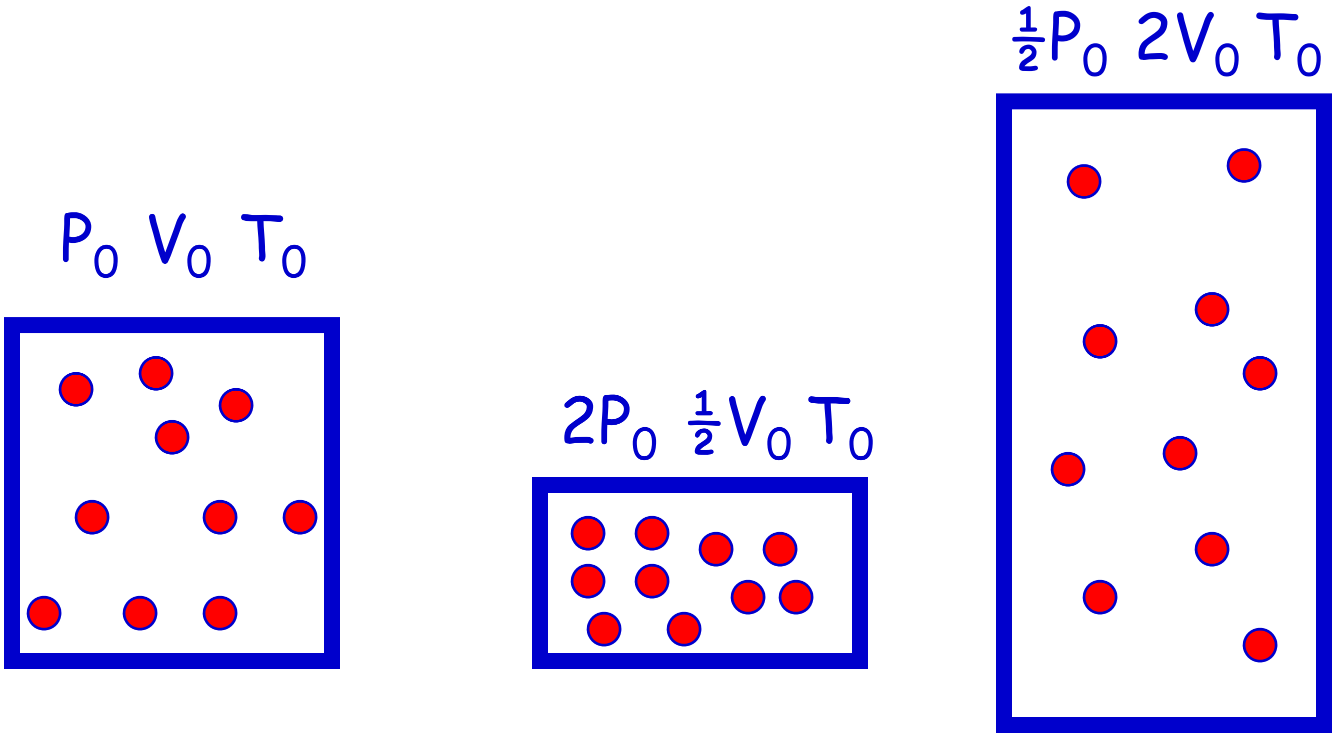
A) How much silicon atoms are in the cube?

B) What would be the mass for the same number of gold atoms (molar mass 197 g)

A) Total number of mol: 250/28.1 = 8.90 mol
8.9 mol x 6.02×10^{23} particles = 5.4×10^{24} atoms

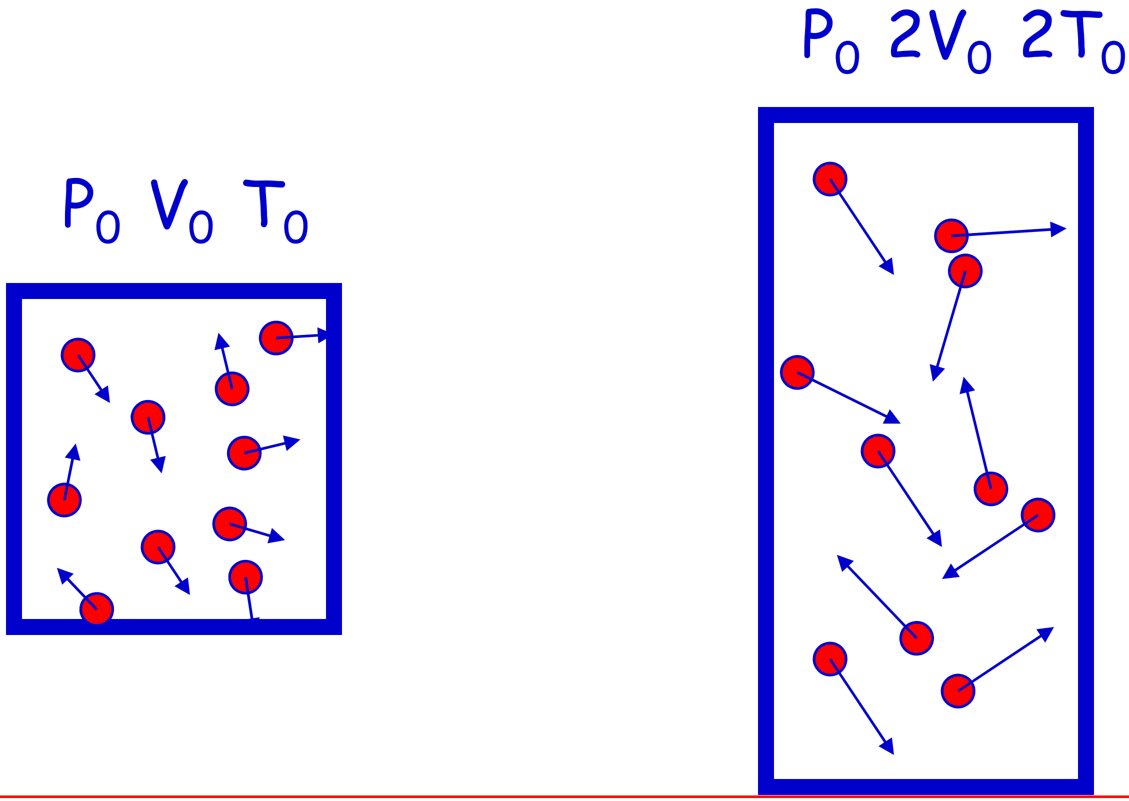
B) 8.90 mol x 197 g = 1.75×10^{3} g

Boyle's Law



At constant temperature: $P \sim 1/V$

Charles' law



If you want to maintain a constant pressure, the temperature must be increased linearly with the volume

$V \sim T$

Gay-Lussac's law

$P_0\ V_0\ T_0$ $\qquad$ $2P_0\ V_0\ 2T_0$

If, at constant volume, the temperature is increased, the pressure will increase by the same factor

$$P \sim T$$

Boyle & Charles & Gay-Lussac
IDEAL GAS LAW

PV/T = nR

n: number of mols in the gas
R: universal gas constant 8.31 J/mol·K

If no molecules are extracted from or added to a system:

$$\frac{PV}{T} = \text{constant} \qquad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Example

An ideal gas occupies a volume of 1.0 cm^3 at 20 ^0C at 1 atm.
A) How many molecules are in the volume?
B) If the pressure is reduced to 1.0×10^{-11} Pa, while the temperature drops to 0 ^0C, how many molecules remained in the volume?

A) PV/T=nR, so n=PV/(TR) R=8.31 J/mol.K
T=20 ^0C=293 K P=1 atm=1.013×10^5 Pa V=1.0 cm^3=1×10^{-6} m^3
n=4.2×10^{-5} mol n=4.2×10^{-5}*N_A=2.5×10^{19} molecules

B) T=0 ^0C=273 K P=1.0×10^{-11} Pa V=1×10^{-6} m^3
n=4.4×10^{-21} mol n=2.6×10^3 particles (almost vacuum)

And another!

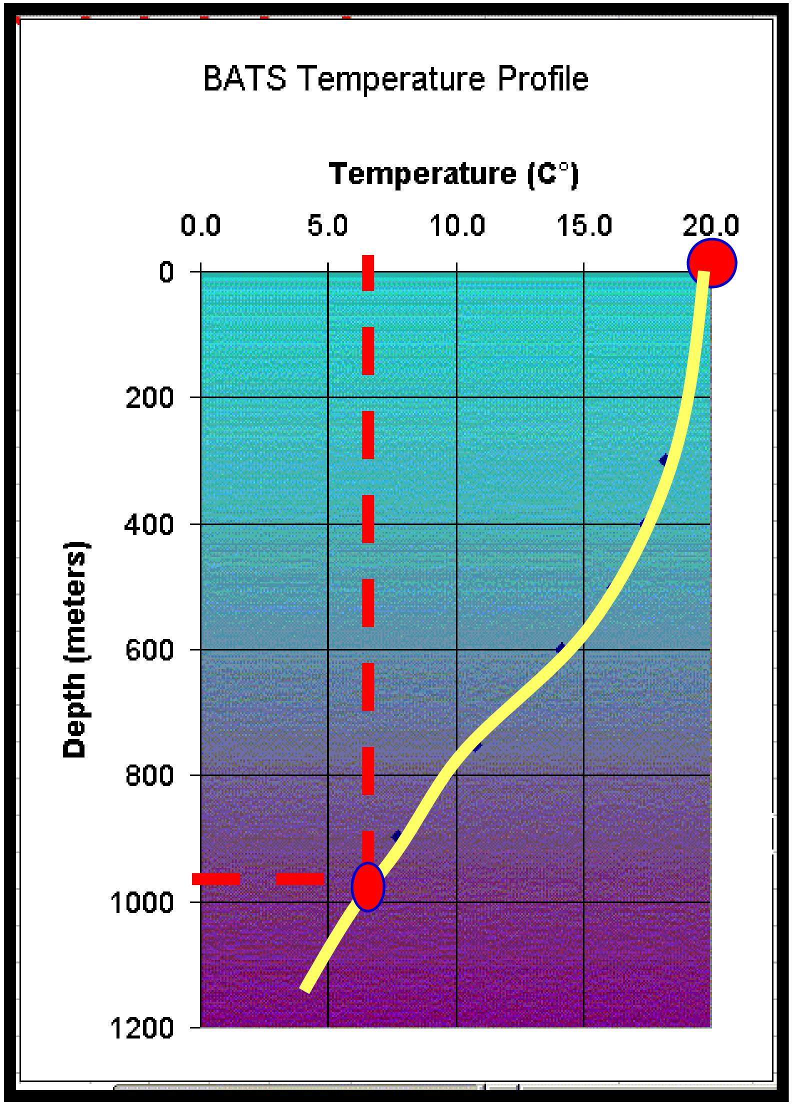
An air bubble has a volume of 1.50 cm^3 at 950 m depth (T=7 °C). What is its volume when it reaches the surface (ρ_{water}=1.0x10^3 kg/m^3)?

$P_{950m} = P_0 + \rho_{water} g h$
$= 1.013\times10^5 + 1.0\times10^3 \times 950 \times 9.81$
$= 9.42\times10^6$ Pa

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{9.42\cdot10^6 \times 1.50\cdot10^{-6}}{280} = \frac{1.013\cdot10^5 \times V_{surface}}{293}$$

$V_{surface}$=1.46x10^{-4} m^3=146 cm^3



Correlations

A volume with dimensions L x W x H is kept under pressure P at temperature T. A) If the temperature is raised by a factor of 2, and the height of the volume made 5 times smaller, by what factor does the pressure change?

Use the fact PV/T is constant if no gas is added/leaked

$P_1V_1/T_1 = P_2V_2/T_2$

$P_1V_1/T_1 = P_2(V_1/5)/(2T_1)$

$P_2 = 5*2*P_1 = 10P_1$

A factor of 10.

Diving Bell

A cylindrical diving bell (diameter 3 m and 4 m tall, with an open bottom is submerged to a depth of 220 m in the sea. The surface temperature is 25 ^{0}C and at 220 m, T=5 ^{0}C. The density of sea water is 1025 kg/m^3. How high does the sea water rise in the bell when it is submerged?

Consider the air in the bell.

$P_{surf}=1.0\times10^5$ Pa $V_{surf}=\pi r^2 h=28.3$ m^3 $T_{surf}=25+273=298$ K

$P_{sub}=P_0+\rho_w g*depth=2.3\times10^6$ Pa $V_{sub}=?$ $T_{sub}=5+273=278$ K

Next, use PV/T=constant

$P_{surf}V_{surf}/T_{surf}=P_{sub}V_{sub}/T_{sub}$ plug in the numbers and find:

$V_{sub}=1.15$ m^3 (amount of volume taken by the air left)

$V_{taken\ by\ water}=28.3-1.15=27.15$ m^3= $\pi r^2 h$

$h=27.15/\pi r^2=3.8$ m rise of water level in bell.

A small matter of definition

Ideal gas law: $PV/T = nR$

$PV/T = (N/N_A)R$

$$n \text{ (number of mols)} = \frac{N \text{ (number of molecules)}}{N_A \text{ (number of molecules in 1 mol)}}$$

Rewrite ideal gas law: $PV/T = Nk_B$

where $k_B = R/N_A = 1.38 \times 10^{-23}$ J/K Boltzmann's constant

From macroscopic to microscopic descriptions: kinetic theory of gases

1) The number of molecules is large (statistical model)
2) Their average separation is large (take no volume)
3) Molecules follow Newton's laws
4) Any particular molecule can move in any direction with a large distribution of velocities
5) Molecules undergo elastic collision with each other
6) Molecules make elastic collisions with the walls
7) All molecules are of the same type

For derivations of the next equation, see the textbook

Pressure

Number of Molecules

Mass of 1 molecule

Averaged squared velocity

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\overline{v^2}\right)$$

Volume

Number of molecules per unit volume

Average translation kinetic energy

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right)$$ Microscopic

$$PV = Nk_BT$$ Macroscopic

$$T = \frac{2}{3k_B}\left(\frac{1}{2}m\overline{v^2}\right)$$ Temperature ~ average molecular kinetic energy

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$$ Average molecular kinetic energy

$$E_{kin} = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$ Total kinetic energy

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_bT}{m}} = \sqrt{\frac{3RT}{M}}$$

rms speed of a molecule
M=Molar mass (kg/mol)

example

What is the rms speed of air at 1 atm and room temperature? Assume it consist of molecular nitrogen only (N_2)?

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_bT}{m}} = \sqrt{\frac{3RT}{M}}$$

R=8.31 J/mol K T=293 K M=2*14x10^{-3} kg/mol

v_{rms}=511 m/s !!!!!

And another...

What is the total kinetic energy of the air molecules in the lecture room (assume only molecular nitrogen is present N_2)?

1) find the total number of molecules in the room

$PV/T = Nk_b$ $\quad P=1.015\times10^5$ Pa $\quad V=10*4*25=1000\ m^3$

$k_b=1.38\times10^{-23}$ J/K $\quad T=293$ K

$N=2.5\times10^{28}$ molecules (4.2×10^4 mol)

2) $E_{kin}=(3/2)Nk_BT=1.5\times10^8$ J

(same as driving a 1000 kg car at 547.7 m/s)