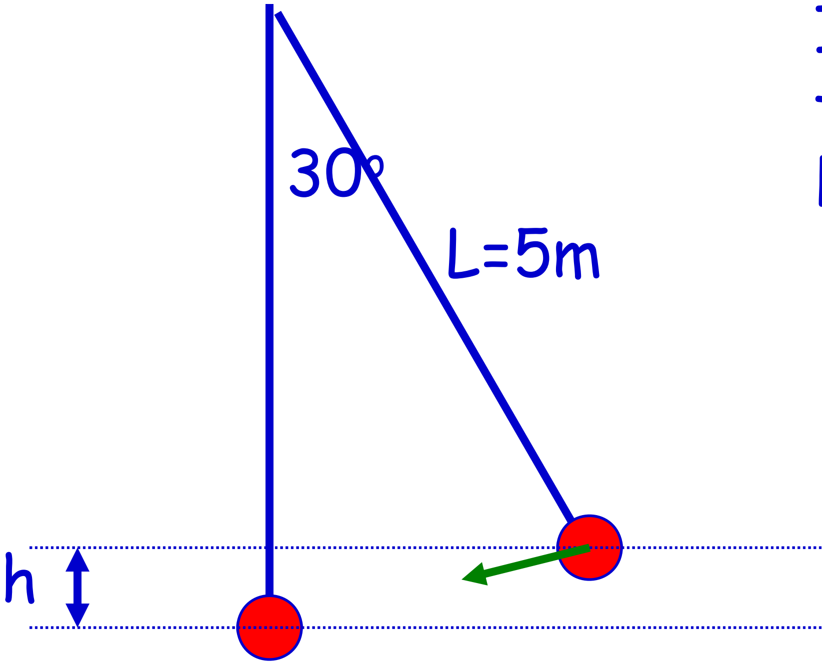
A swing



If relieved from rest, what is the velocity of the ball at the lowest point?

(PE+KE)=constant

$PE_{release}$=mgh (h=5-5cos(30°))

=6.57m J

$KE_{release}$=0

PE_{bottom}=0

$KE_{bottom}=\frac{1}{2}mv^2$

$\frac{1}{2}mv^2$=6.57m so v=3.6 m/s

A running person

While running, a person dissipates about 0.60 J of mechanical energy per step per kg of body mass. If a 60 kg person develops a power of 70 Watt during a race, how fast is she running (1 step=1.5 m long)
What is the force the person exerts on the road?

$W=F\Delta x$ $P=W/\Delta t=Fv$

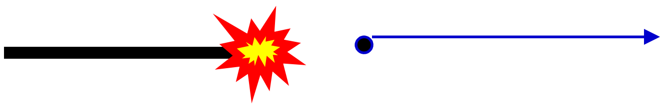
Work per step: 0.60 J/kg * 60 kg=36 J

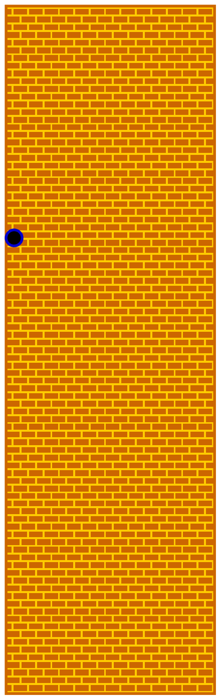
Work during race: 36*(racelength(L)/steplength)=24L

Power= $W/\Delta t=24L/\Delta t=24v_{average}=70$ so $v_{average}=2.9$ m/s

F=P/v so F=24 N

Momentum





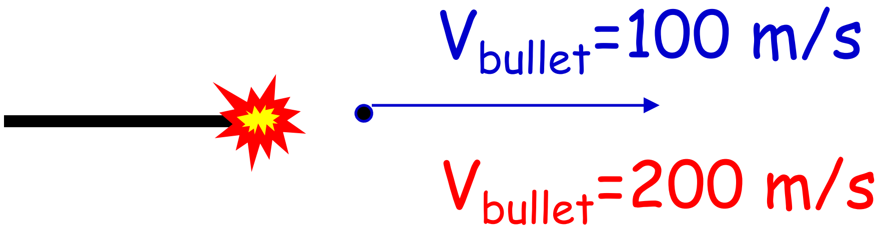
When a bullet hits the wall, its velocity is very much reduced. The wall does not move, although the force on the ball is the same as the force on the wall (Newton's 3rd law: $F_{wall\text{-}bullet}=-F_{bullet\text{-}wall}$).

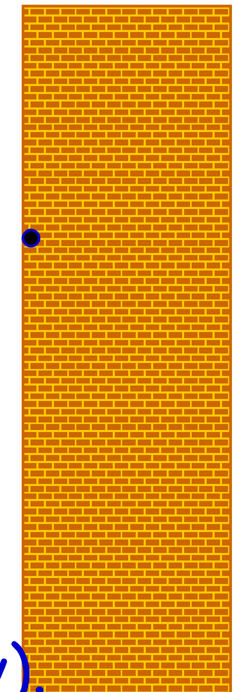
$$F_{wall\text{-}bullet}=m_{bullet}a_{bullet}$$
$$F_{bullet\text{-}wall}=m_{wall}a_{wall}$$

→ $m_{bullet} \ll m_{wall}$

→ $|a_{bullet}| \gg a_{wall}$

Is it only the mass???





When the bullet gets stopped in the wall, it deccelerates from its initial velocity to 0. So, its acceleration is $v_{bullet}/\Delta t$, with Δt some small time (independent of v).

Second law: $F_{wall\text{-}bullet}=m_{bullet}a_{bullet}=m_{bullet}v_{bullet}/\Delta t$

The force also depends on the velocity of the bullet!

More general…and formal.

$F=ma$	Newton's 2nd law
$F=m\Delta v/\Delta t$	$a=\Delta v/\Delta t$
$F=m(v_{final}-v_{inital})/\Delta t$	
Define $p=mv$	p: momentum (kg m/s)
$F=(p_{final}-p_{initial})/\Delta t$	
$F=\Delta p/\Delta t$	

The net force acting on an object equals the change in momentum (Δp) in a certain time period (Δt).

Since velocity is a vector, momentum is also a vector, pointing in the same direction as v.

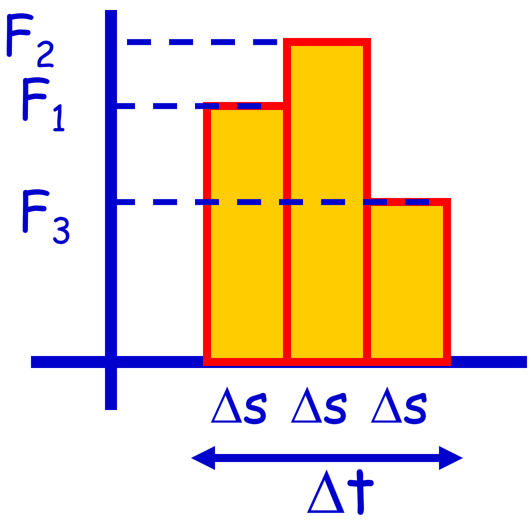
Impulse

$F=\Delta p/\Delta t$ Force=change in (mv) per time period (Δt).

$\Delta p=F\Delta t$ The change in momentum equals the force acting on the object times as long as you apply the force.

Definition: Δp=Impulse

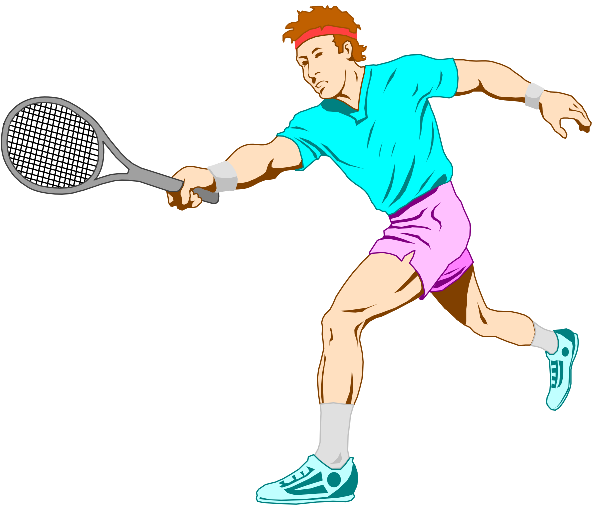
What if the force is not constant within the time period Δt?



$$\Delta p=F\Delta t=(F_1\Delta s+F_2\Delta s+F_3\Delta s)=$$
$$= \Delta t(F_1\Delta s+F_2\Delta s+F_3\Delta s)/\Delta t$$
$$= \Delta t F_{average}$$

$$\Delta p=F_{average}\,\Delta t$$

Some examples



A tennis player receives a shot approaching him (horizontally) with 50 m/s and returns the ball in the opposite direction with 40 m/s. The mass of the ball is 0.060 kg.

A) What is the impulse delivered by the ball to the racket?

B) What is the work done by the racket on the ball?

A) Impulse=change in momentum (Δp).

$$\Delta p = m(v_{final} - v_{initial}) = 0.060(-40-50) = -5.4 \text{ kg m/s}$$

B) $W = KE_{final} - KE_{initial} = \frac{1}{2}mv_{final}^2 - \frac{1}{2}mv_{inital}^2$ (no PE!)

$$= \frac{1}{2}0.060([-40]^2 - [50]^2) = -27 \text{ J}$$

Child safety

A friend claims that it is safe to go on a car trip with your child without a child seat since he can hold onto your 12 kg child even if the car makes a frontal collision (lasting 0.05 s and causing the vehicle to stop completely) at v=50 km/h (about 30 miles/h). Is he to be trusted?

$F=\Delta p/\Delta t$ force=impulse per time period

$=m(v_f-v_i)/\Delta t$

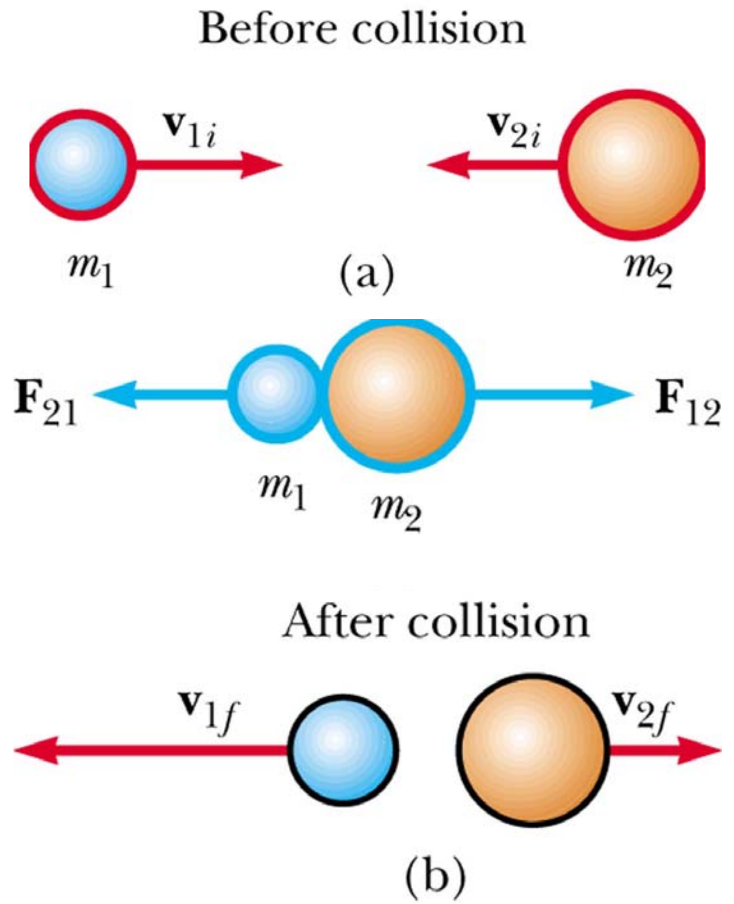
$v_f=0$ and $v_i=50$ km/h=13.9 m/s m=12 kg $\Delta t=0.05$ s

F=12(13.9)/0.05=3336 N

This force corresponds to lifting a mass of 340 kg or about 680 pounds! DON'T TRUST THIS GUY!

Conservation of Momentum

$F_{21}\Delta t = m_1 v_{1f} - m_1 v_{1i}$
$F_{12}\Delta t = m_2 v_{2f} - m_2 v_{2i}$



Newton's 3rd law:
$F_{12} = -F_{21}$

$(m_1 v_{1f} - m_1 v_{1i}) = -(m_2 v_{2f} - m_2 v_{2i})$

Rewrite:

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$
$p_{1i} + p_{2i} = p_{1f} + p_{2f}$

CLOSED SYSTEM!

CONSERVATION OF MOMENTUM

Moving in space

An astronaut (100 kg) is drifting away from the spaceship with v=0.2 m/s. To get back he throws a wrench (2 kg) in the direction away from the ship. With what velocity does he need to throw the wrench to move with v=0.1 m/s towards the ship?

Initial momentum: $m_{ai}v_{ai}+m_{wi}v_{wi}$ =100*0.2+2*0.2=20.4 kg m/s

After throw: $m_{af}v_{af}+m_{wf}v_{wf}$=100*(-0.1)+2*$v_{wf}$ kg m/s

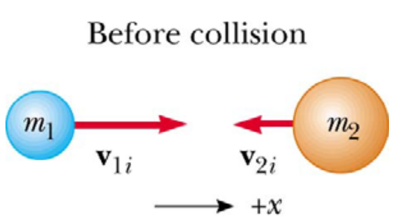
Conservation of momentum: $m_{ai}v_{ai}+m_{wi}v_{wi}= m_{af}v_{af}+m_{wf}v_{wf}$

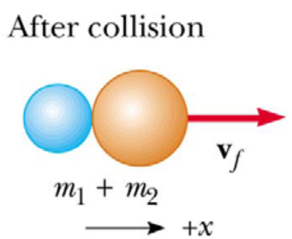
20.4=-10+2*v_{wf} $\qquad$ v_{wf}=15.7 m/s

Types of collisions

Inelastic collisions

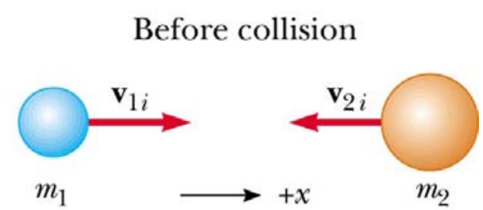
- Momentum is conserved
- Some energy is lost in the collision: KE not conserved
- Perfectly inelastic: the objects stick together.

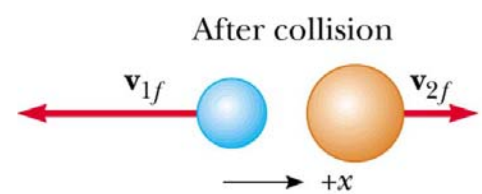




Elastic collisions

- Momentum is conserved
- No energy is lost in the collision: KE conserved





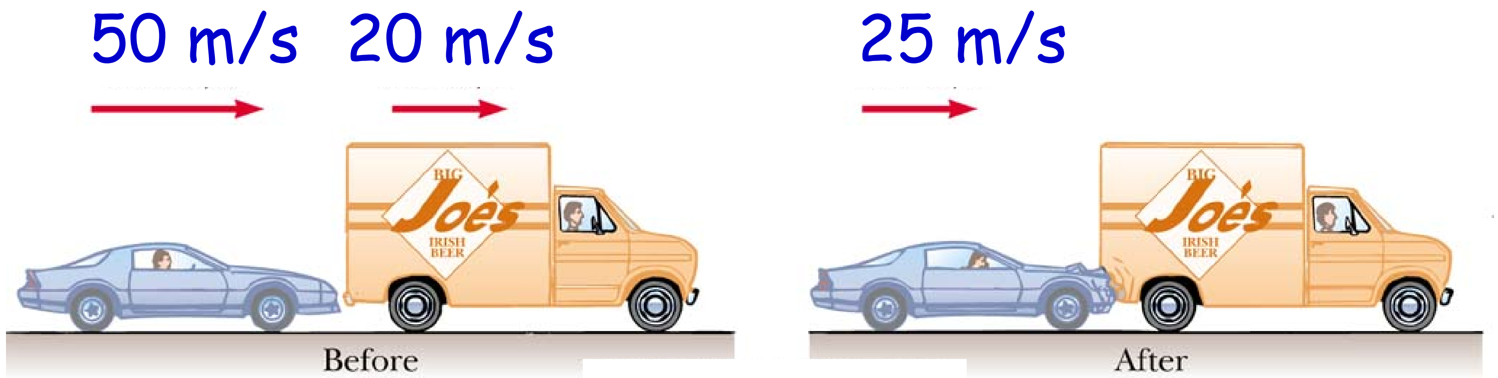
Perfectly inelastic collisions

Conservation of P: $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

After the collision m_1 and m_2 form one new object with mass $M=m_1+m_2$

$$m_1v_{1i}+m_2v_{2i}=v_f(m_1+m_2)$$
$$v_f=(m_1v_{1i}+m_2v_{2i})/\ (m_1+m_2)$$

Perfect inelastic collision: an example



A car collides into the back of a truck and their bumpers get stuck. What is the ratio of the mass of the truck and the car? ($m_{truck}=c^*m_{car}$) What is the fraction of KE lost?

$m_1v_{1i}+m_2v_{2i}=v_f(m_1+m_2)$ $\qquad$ $50m_c+20c^*m_c=25(m_c+c^*m_c)$

so $c=25m_c/5m_c=5$

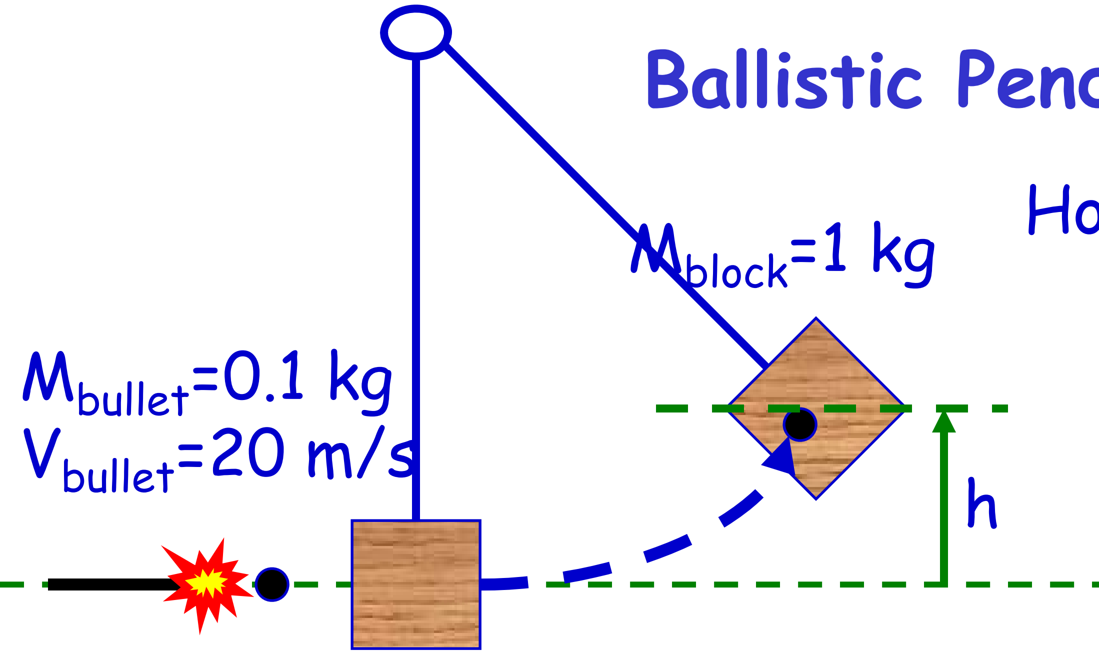
Before collision: $KE_i=\frac{1}{2}m_c50^2+\frac{1}{2}5m_c20^2$

After collision: $KE_f=\frac{1}{2}6m_c25^2$

Ratio: $KE_f/KE_i= (6^*25^2)/(50^2+5^*20^2)=0.83$

17% of the KE is lost (damage to cars!)

Ballistic Pendulum



How high will the block go?

There are 2 stages:

- The collision
- The swing of the block

The collision The bullet gets stuck in the block (perfect inelastic collision). Use conservation of momentum.

$m_1v_{1i}+m_2v_{2i}=v_f(m_1+m_2)$ so: $0.1*20+1*0=v_f(0.1+1)$ $v_f=1.8$ m/s

The swing of the block Use conservation of mechanical energy.

$(mgh+\frac{1}{2}mv^2)_{start\ of\ swing}=(mgh+\frac{1}{2}mv^2)_{at\ highest\ point}$

$0+\frac{1}{2}1.1(1.8)^2=1.1*9.81*h$ so h=0.17 m

Why can't we use conservation of ME right from the start??

Elastic collisions

Conservation of momentum: $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

Conservation of KE: $\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2=\frac{1}{2}m_1v_{1f}^2+\frac{1}{2}m_2v_{2f}^2$

Rewrite conservation of KE:

a) $m_1(v_{1i}-v_{1f})(v_{1i}+v_{1f}) = m_2(v_{2f}-v_{2i})(v_{2f}+v_{2i})$

Rewrite conservation of P :

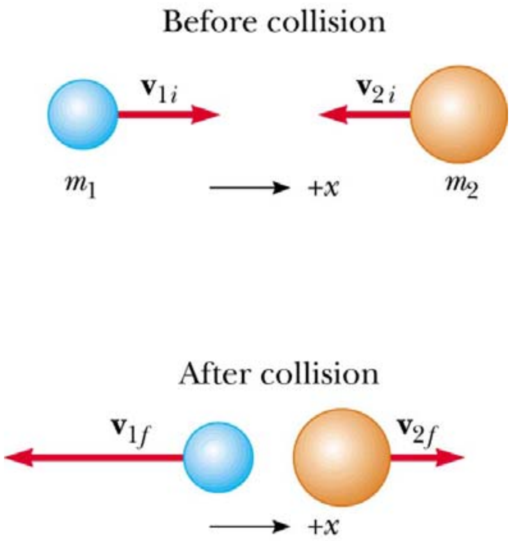
b) $m_1(v_{1i}-v_{1f}) = m_2(v_{2f}-v_{2i})$

Divide a) by b): $(v_{1i}+v_{1f})=(v_{2f}+v_{2i})$

rewrite: $(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

Use in problems

Elastic collision of equal masses



Given $m_2=m_1$.
What is the velocity of m_1 and m_2 after the collision in terms of the initial velocity of m_2 if m_1 is originally at rest?

$$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$$

$$m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$$

$$v_{2i}=v_{1f}+v_{2f}$$

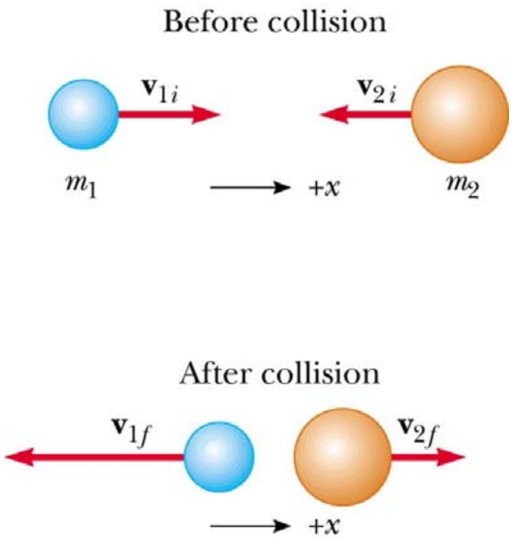
$$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$$

$$-v_{2i}= v_{2f}-v_{1f}$$

$$v_{2f}=0$$

$$v_{1f}=v_{2i}$$

Elastic collision of unequal masses



Given $m_2=3m_1$.
What is the velocity of m_1 and m_2 after the collision in terms of the initial velocity of the moving bullet if

a) m_1 is originally at rest

b) m_2 is originally at rest

A) $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

$3m_1v_{2i}=m_1v_{1f}+m_2v_{2f}$

$3v_{2i}=v_{1f}+3v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

$-v_{2i}= v_{2f}-v_{1f}$

$v_{2f}=v_{2i}/2$
$v_{1f}=3v_{2i}/2$

B) $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

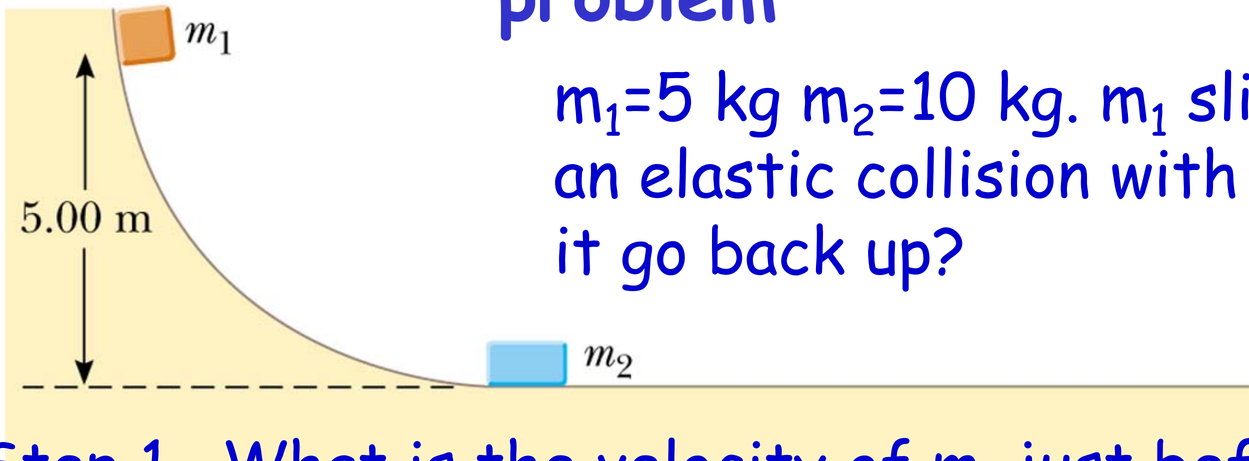
$m_1v_{1i}=m_1v_{1f}+m_2v_{2f}$

$v_{1i}=v_{1f}+3v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

$v_{1i}= v_{2f}-v_{1f}$

$v_{2f}=v_{1i}/2$
$v_{1f}=-v_{1i}/2$



problem

m_1=5 kg m_2=10 kg. m_1 slides down and makes an elastic collision with m_2. How high does it go back up?

Step 1. What is the velocity of m_1 just before it hits m_2?

Conservation of ME: $(m_1gh+0.5mv^2)_{start}=(m_1gh+0.5mv^2)_{bottom}$

$5*9.81*5+0=0+0.5*5*v^2$ so v_{1i}=9.9 m/s

Step 2. Collision: Elastic so conservation of momentum AND KE.

- $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f} \Rightarrow 5*9.9+0=5*v_{1f}+10v_{2f} \Rightarrow v_{2f}=4.95-0.5*v_{1f}$
- $(v_{1i}-v_{2i})=(v_{2f}-v_{1f}) \Rightarrow 9.9-0=v_{2f}-v_{1f}$ $\Rightarrow \underline{v_{2f}=9.9+v_{1f}}$ -

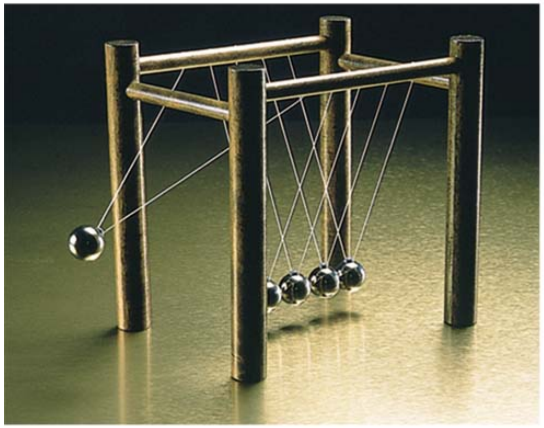
$v_{1f}=-3.3m/s \Leftarrow 0 \;=-4.95-1.5V_{1f}$

Step 3. m_1 moves back up; use conservation of ME again.

$(m_1gh+0.5mv^2)_{bottom}=(m_1gh+0.5mv^2)_{final}$

$0 \;+0.5*5*(-3.3)^2=5*9.81*h+0$ h=0.55 m

Transporting momentum



For elastic collision of equal masses

$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

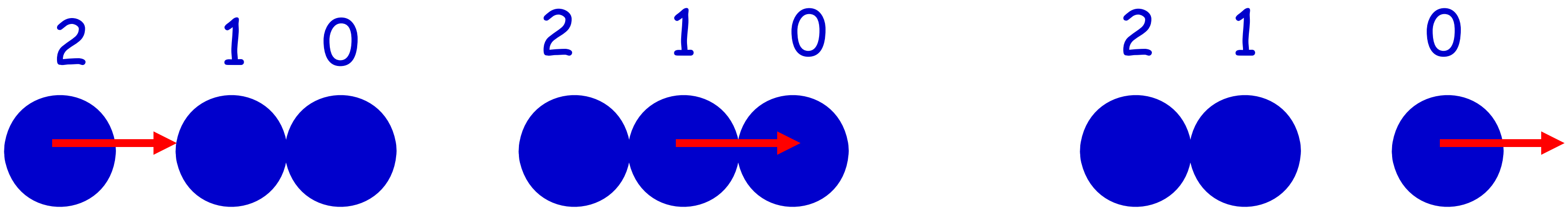
$m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

$v_{2i}=v_{1f}+v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

$-v_{2i}= v_{2f}-v_{1f}$

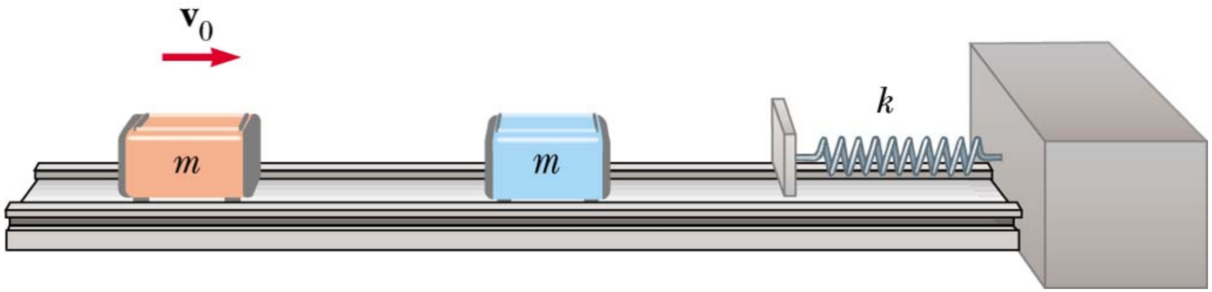
$v_{2f}=0$
$v_{1f}=v_{2i}$



$V_{2f}=0$
$V_{1f}=V_{2i}$

$V_{1f}=0$
$V_{0f}=V_{1i}$

Carts on a spring track



$k=50$ N/m
$v_0=5.0$ m/s
$m=0.25$ kg

What is the maximum compression of the spring if the carts collide a) elastically and b) inelastically?

A) Conservation of momentum and KE

$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f} \Rightarrow 0.25*5=0.25v_{1f}+0.25v_{2f} \Rightarrow v_{1f}=5-v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f}) \Rightarrow 5=v_{2f}-v_{1f}$ $v_{1f}=0$ $v_{2f}=5$ m/s

Conservation of energy: $\frac{1}{2}mv^2=\frac{1}{2}kx^2$ $0.5*0.25*5^2=0.5*50x^2$

$x=0.35$ m We could have skipped collision part!!

B) Conservation of momentum only

$m_1v_{1i}+m_2v_{2i}=(m_1+m_2)v_f \Rightarrow 0.25*5=0.5v_f \Rightarrow v_f=2.5$ m/s

Conservation of energy: $\frac{1}{2}mv^2=\frac{1}{2}kx^2$ $0.5*0.5*2.5^2=0.5*50x^2$

$x=0.25$ m Part of energy is lost!

Impact of a meteorite

Estimate what happens if a 1 km radius meteorite collides with earth: a) Is the orbit of earth around the sun changed?

b) how much energy is released?

Assume: meteorite has same density as earth, the collision is inelastic and the meteorites v is 10 km/s (relative to earth)

A) Earth's mass: 6E+24 kg radius: 6E+6 m

density=mass/volume=$M/(4\pi r^3/3)$=6.6E+3 kg/m^3

mass meteorite: $4(1000)^3/3\pi$*6.6E+3=2.8E+13 kg

Conservation of momentum: $m_e v_e + m_m v_m = (m_e + m_m) v_{me}$

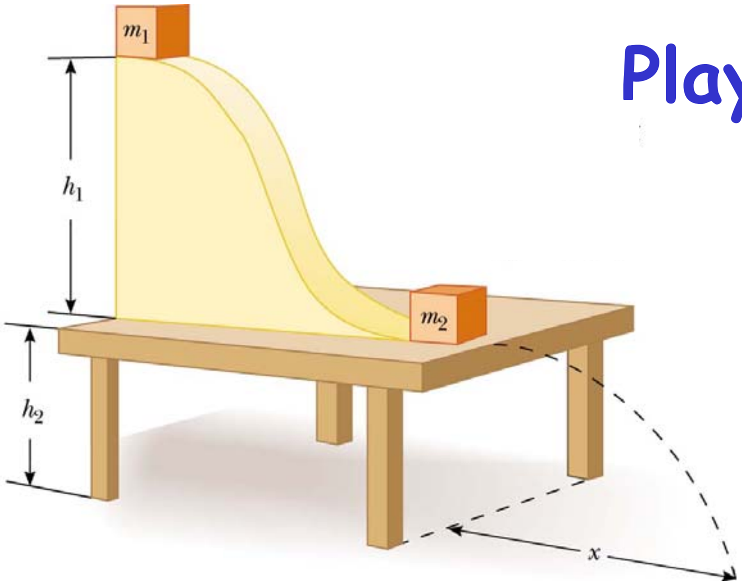
(2.8E+13)(1E+4)=(6E+24)v_{me} so v_{me}=4.7E-08 m/s (no change)

B) Energy=Kinetic energy loss: $(\frac{1}{2}m_e v_e^2 + \frac{1}{2}m_m v_m^2) - (\frac{1}{2}m_{m+e} v_{me}^2)$

0.5(2.8E+13)(1E+4)^2-0.5(6E+24)(4.7E-08)^2=1.4E+21 J

Largest nuclear bomb existing: 100 megaton TNT=4.2E+17 J

Energy release: 3.3E+3 nuclear bombs!!!!!

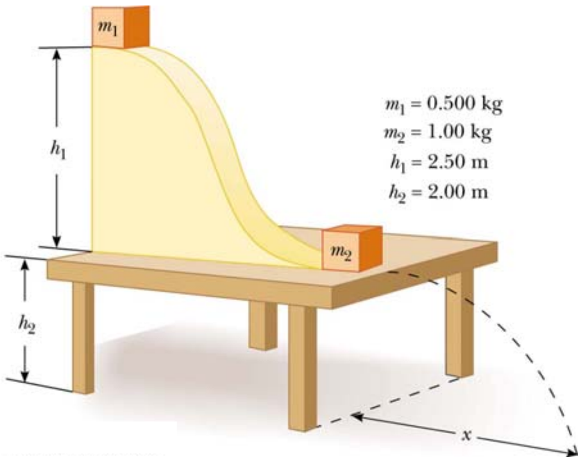


Playing with blocks

m_1=0.5 kg collision is elastic
m_2=1.0 kg
h_1=2.5 m
h_2=2.0 m

A) determine the velocity of the blocks after the collision
b) how far back up the track does m_1 travel?
C) how far away from the bottom of the table does m_2 land
d) how far away from the bottom of the table does m_1 land

Determine the velocity of the blocks after the collision



m_1=0.5 kg   collision is elastic
m_2=1.0 kg
h_1=2.5 m
h_2=2.0 m

Step 1: determine velocity of m_1 at the bottom of the slide

Conservation of ME $(mgh+\frac{1}{2}mv^2)_{top} = (mgh+\frac{1}{2}mv^2)_{bottom}$

$0.5*9.81*2.5+0=0+0.5*0.5*v^2$

so: v_1=7.0 m/s

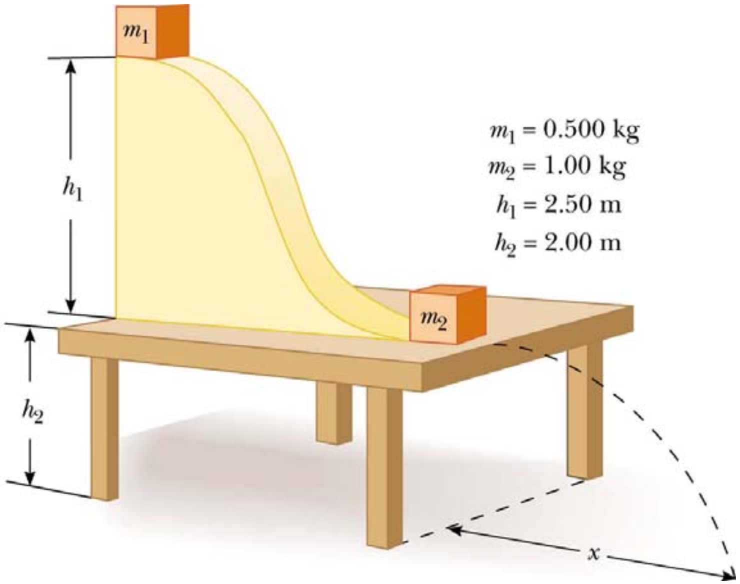
Step 2: Conservation of momentum and KE in elastic collision

$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$ so $0.5*7+0=0.5v_{1f}+v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$ so $7.0-0=v_{2f}-v_{1f}$

Combine these equations and find: v_{1f}=-2.3 m/s v_{2f}=4.7 m/s

How far back up does m_1 go after the collision?



m_1=0.5 kg   collision is elastic

m_2=1.0 kg

h_1=2.5 m

h_2=2.0 m

v_{1f}=-2.3 m/s v_{2f}=4.7 m/s

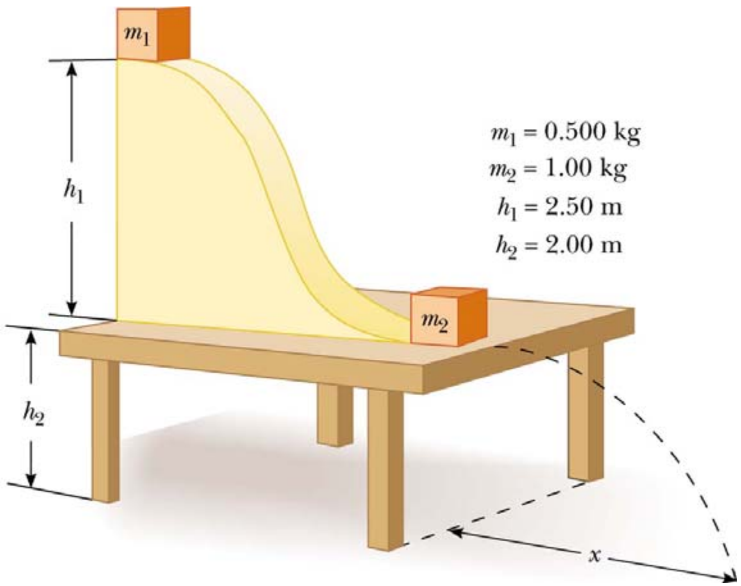
Use conservation of ME:

$$(mgh+\tfrac{1}{2}mv^2)_{bottom} = (mgh+\tfrac{1}{2}mv^2)_{back\ up\ slide}$$

$$0+0.5*0.5*(-2.3)^2=0.5*9.81*h+0$$

h=0.27 m

How far away from the table does m_2 land?

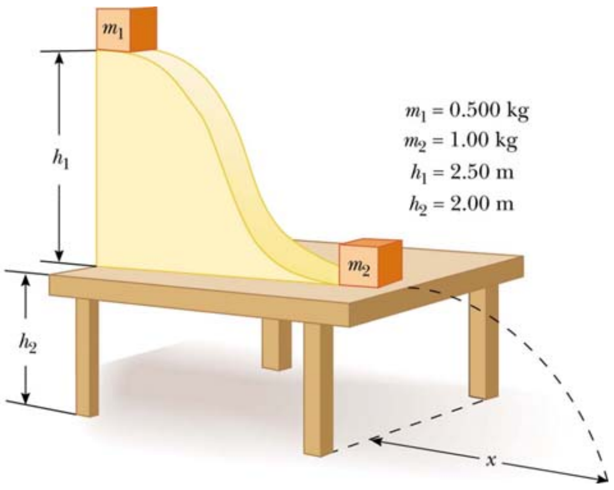


m_1=0.5 kg collision is elastic
m_2=1.0 kg
h_1=2.5 m
h_2=2.0 m
v_{1f}=-2.3 m/s v_{2f}=4.7 m/s
h_1=0.27 m (after collision back up)

This is a parabolic motion with initial horizontal velocity.

Horizontal	vertical
$x(t)=x(0)+v_x(0)t+\frac{1}{2}at^2$	$y(t)=y(0)+v_y(0)t-\frac{1}{2}gt^2$
$x(t)=4.7t$	$0=2.0-0.5*9.81*t^2$
$x(0.63)=2.96$ m ←	$t=0.63$ s

How far away from the table does m_1 land?



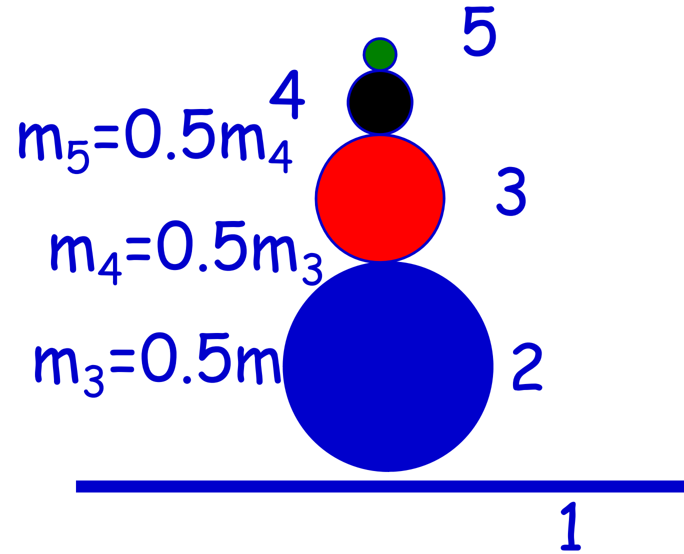
m_1=0.5 kg   collision is elastic
m_2=1.0 kg
h_1=2.5 m
h_2=2.0 m
v_{1f}=-2.3 m/s v_{2f}=4.7 m/s
h_1=0.27 m (after collision back up)
x_2=2.96 m

Use conservation of ME: m_1 has $-v_{1f}$=2.3 m/s when it returns back at the bottom of the slide.

This is a parabolic motion with initial horizontal velocity.

Horizontal	vertical
$x(t)=x(0)+v_x(0)t+\frac{1}{2}at^2$	$y(t)=y(0)+v_y(0)t-\frac{1}{2}gt^2$
$x(t)=2.3t$	$0=2.0-0.5*9.81*t^2$
$x_1(0.63)=1.45$ m ←	$t=0.63$ s

Ballistic balls



Consider only the lowest ball first.

$X(t)=1.5-0.5*9.8*t^2=0$ so $t=0.55$ s

$V(t)=-9.8t$ so $V(0.55)=-5.4$ m/s

Collision with earth:

- $m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$ (1: earth 2: ball)
- $v_{2i}=v_{2f}-v_{1f}$

$v_{2f}=(m_2-m_1)v_{2i}/(m_1+m_2)$ $m_1>>m_2$ so $v_{2f}=-v_{2i}=5.4$ m/s

Consider the collision of ball m(=n+1) with ball n

- $m_nv_{ni}+m_mv_{mi}=m_nv_{nf}+m_mv_{mf}$
- $(v_{ni}-v_{mi})=(v_{nf}-v_{mf})$

$v_{mf}=[2m_nv_{ni}+(m_m-m_n)v_{mi}]/[m_n+m_m]$ & $m_m=0.5m_n$

so $v_{mf}=[2m_nv_{ni}-0.5m_nv_{mi}]/[1.5m_n]=[1.33v_{ni}-0.33v_{mi}]$

- $V_{3f}=1.33*5.4-0.33(-5.4)=9.0$ m/s
- $V_{4f}=1.33*9.-0.33(-5.4)=13.7$ m/s
- $v_{5f}=1.33*13.7-0.33(-5.4)=20.$ m/s

Ballistic balls II

Highest point:

$v(t)=20.-9.8t=0$ so $t=2.0$ s

$x(t)=20t-0.5*9.8*2.0^2=20.$ m !!!