Isaac Newton (1642-1727)

"In the beginning of 1665 I found the…rule for reducing any degree of binomial to a series. The same year in May I found the method of tangents and in November the method of fluxions and in the next year in January had the Theory of Colours and in the following May I had the entrance into the inverse method of fluxions and in the same year I began to think of gravity extending to the orbit of the moon…and…compared the force required to keep the Moon in its orbit with the force of Gravity on the surface of the Earth."

"Nature and Nature's Laws lay hid in night:
God said, Let Newton be! And all was light."
Alexander Pope (1688-1744)

Newton's Laws

- First Law: If the net force exerted on an object is zero the object continues in its original state of motion; if it was at rest, it remains at rest. If it was moving with a certain velocity, it will keep on moving with the same velocity.
- Second Law: The acceleration of an object is proportional to the net force acting on it, and inversely proportional to its mass: $F = m a$
- If two objects interact, the force exerted by the first object on the second is equal but opposite in direction to the force exerted by the second object on the first: $F_{12} = -F_{21}$

Inertia

The tendency of an object to resist a change in its velocity is called inertia.

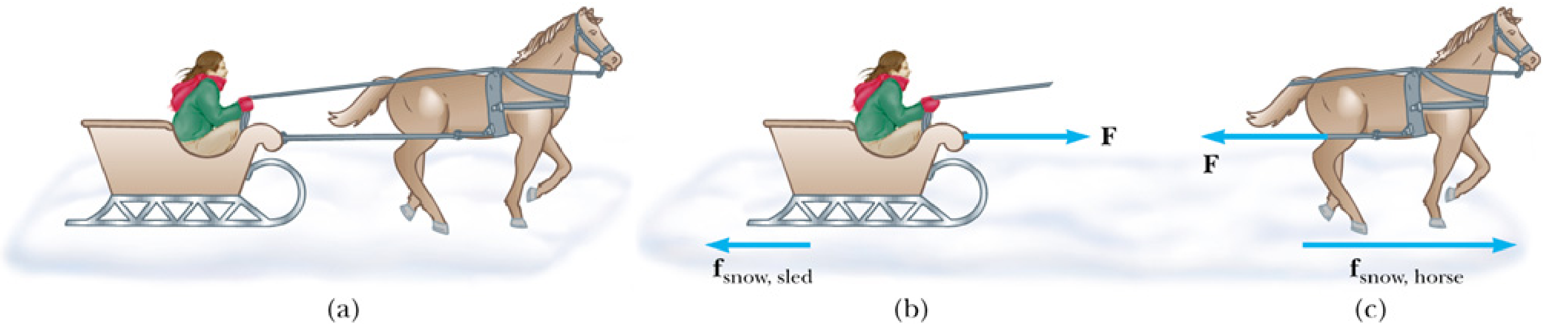
The measure of inertia is mass.

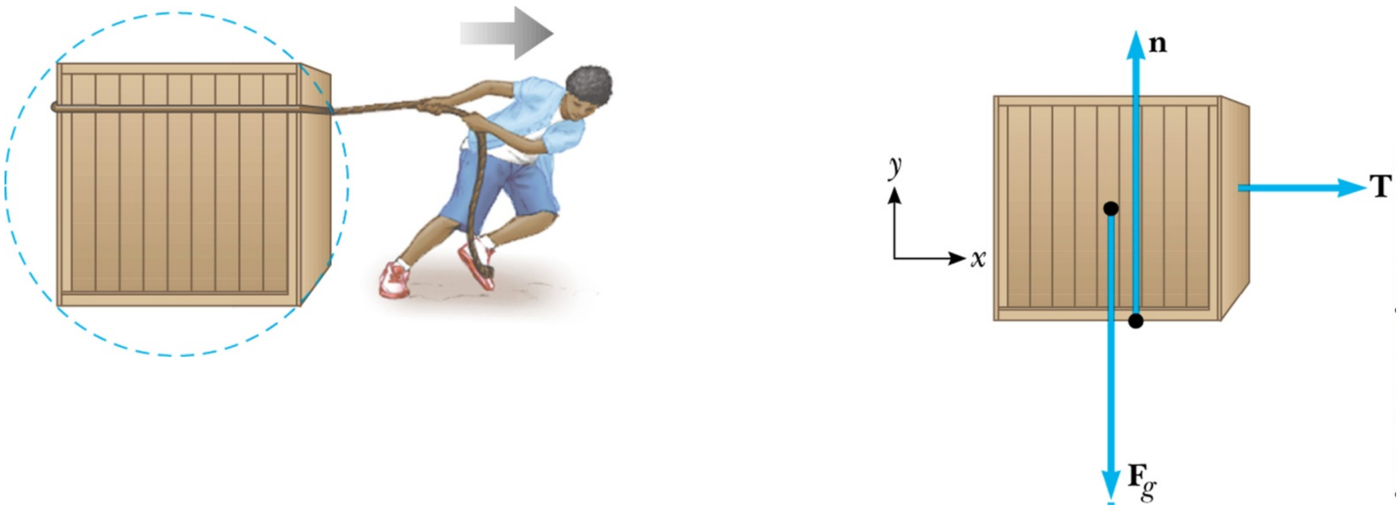
- SI units measure mass as multiples of the standard kilogram (kg = 1000g) stored at the International Bureau of Weights and Measures in Sèvres, France.

Newton's First Law tells us about motion if

$\mathbf{F} = 0$. What if $\mathbf{F} \neq 0$?

Identifying the forces in a system





Examples

1. An object that is moving and that continues to move with constant velocity without any force acting on it.
    - A hockey puck sliding (almost without friction) across the ice
2. An object at rest that remains at rest.
3. What about pushing a chair?
    - If the floor pushes just as hard (friction) the net force (vector sum) is zero.
4. What happens when you turn a corner quickly in your car?
    - The car would continue straight ahead unless the friction from the road pushes inwards to guide the car around the circle (centripetal force).

Force

- Forces are quantified in units of *Newton* (N).

$$1 \text{ N} = 1 \text{ kg m/s}^2$$

$$F = m\,a$$

For comparison:

$$1 \text{ lb.} = 4.448 \text{ N}$$

Notice pounds and kilograms do not directly convert.

The British unit of mass is the slug (don't ask).

The force of gravity (near Earth's surface) acting on a 1 kg mass is 2.2 lb.:

$$(1.0 \text{ kg})\,(g) = 2.2 \text{ lb.}$$

Do not confuse g=gram with *g*=9.8m/s2=acceleration due to gravity.

- *A* force is a vector: it has direction.

Example

An object of mass 5 kg undergoes an acceleration of 8 m/s^2 $\hat{y}$ = 8 m/s^2 in + y direction

What is the force on that object?

F = m**a**

= (5 kg)(8 m/s^2) $\hat{y}$ = 40 kg·m/s^2 $\hat{y}$

$\hat{y}$ = vector on unit length (no dimensions) in +y direction.

The force is in the same direction as the acceleration.

Problem

A catcher stops a 92 mi/h pitch in his glove, bringing it to rest (with uniform acceleration) in 0.15 m. If the force exerted by the catcher is 803 N, what is the mass of the ball?

Chapter 3: $v^2 = v_0^2 + 2a(x-x_0)$

$a = (v^2 - v_0^2)/ [2 (x-x_0)]$

$$v_0 = 92\frac{mi}{hr} = 41.14m/s$$

$$v = 0, \qquad x = 0, \qquad x_0 = 0.15m$$

$$a = \frac{0-(41.14m/s)^2}{2(0.15m-0)} = -5{,}641\frac{m}{s^2}$$

Chapter 4: F = ma $\qquad$ m = |F/a| = (803 N) / (5641 m/s^2) =0.142kg

Weight

The weight of any object on the Earth is the gravitational force exerted on it by the Earth:

$$\mathbf{W} = m\mathbf{g}$$

Note:

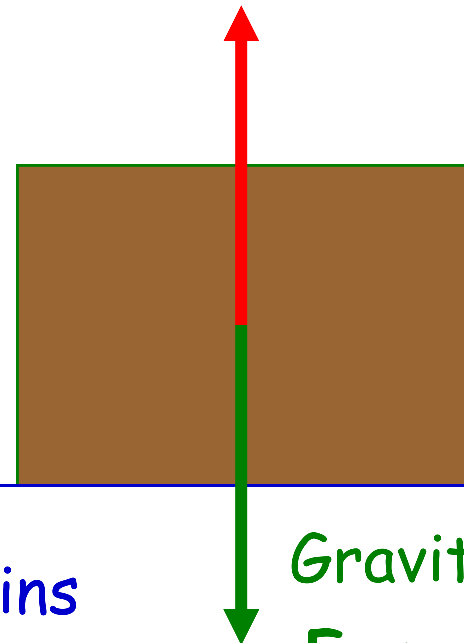
Weight is a force (and therefore a vector).

Weight is not equivalent to mass.

Can a person's weight be zero?

When we say we want to "lose weight", what do we really mean?

Two Forces that luckily act upon us nearly all the time.



Normal Force: elastic force acting perpendicular to the surface the object is resting on.
Name: **n**

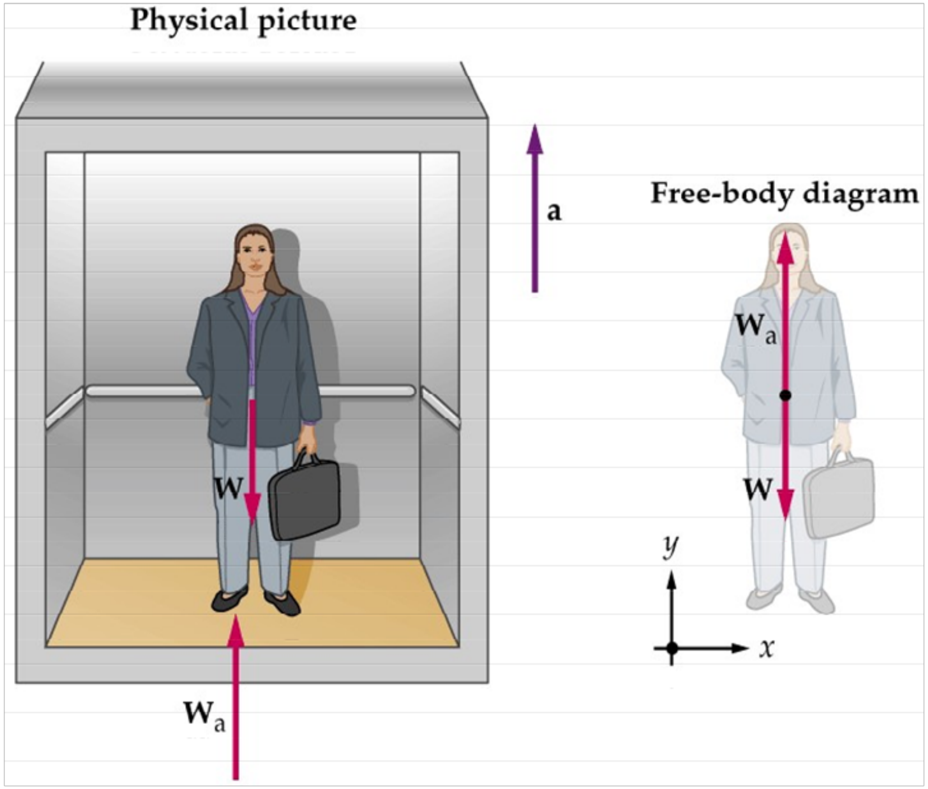
Gravitational Force

$\mathbf{F}_g$=mg (referred to as weight)
g=9.81 m/s^2

1. No net force: remains at rest.
2. $F_g = m\,g = n$
3. $F_{mass\text{-}ground} = -F_{ground\text{-}mass}$

Apparent Weight

Our sensation of weight comes from the force of the floor pushing up on us. We can feel light or heavy if the floor is accelerating down or up. The upward force of the floor on our feet is known as apparent weight W_a.



It is your apparent weight that is measured on a scale.

Translational Equilibrium

When the sum of the forces on an object is zero, it is in translational equilibrium. It must have zero acceleration.

Example: A 5-kg mass is raised at a constant speed of 6 m/s using a rope. What is the tension in the rope?

$T=mg$

The 5kg mass is raised with a constant acceleration of 1.0 m/s2. What is the tension in the rope?

$T=m(g+a)$

Problem

As part of a physics experiment, you stand on a bathroom scale in an elevator. Though your normal weight is 610 N, the scale at the moment reads 730 N.

(a) Is the acceleration of the elevator upward, downward, or zero?

(b) Calculate the magnitude of the elevator's acceleration.

The Vector Nature of Forces

In the formula **F** = m**a**, **F** is the total (net) force acting on the object. We must consider the vector sum of all forces acting on an object. We can also consider each dimension separately:

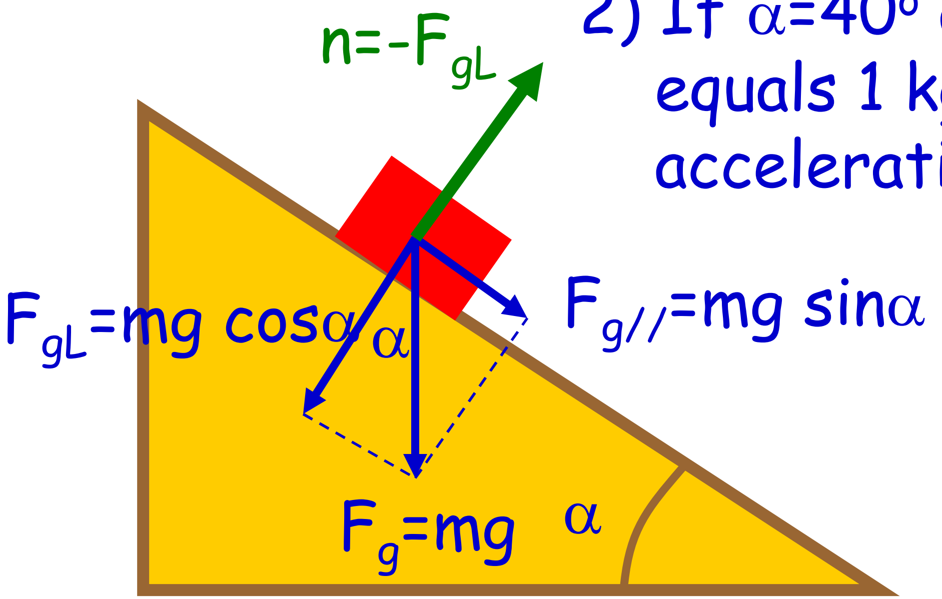
$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$

Example

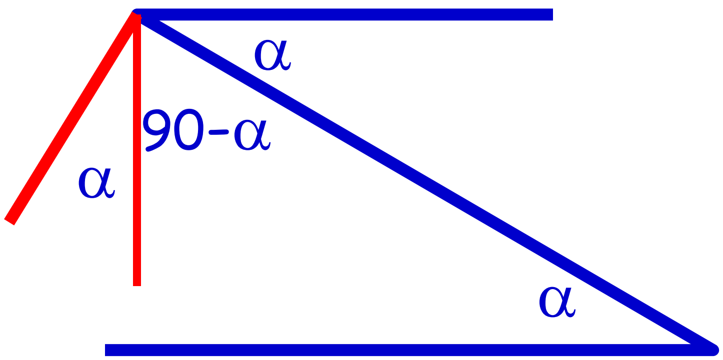
1) Draw all forces acting on the red object
2) If $\alpha=40^o$ and the mass of the red object equals 1 kg, what is the resulting acceleration (no friction).



Balance forces in directions where you expect no acceleration; whatever is left causes the object to accelerate!

$m\ a = m\ g\ \sin\alpha = 6.3$ N
$a = 6.3\ m/s^2$

Some handy things to remember.



angles

Choose your coordinate system in a clever way:
Define one axis along the direction where you expect an object to start moving, the other axis perpendicular to it (these are not necessarily the horizontal and vertical direction.

Gravity, mass and weights.

Weight=mass times gravitational acceleration

$F_g(N) = M(kg)\ g(m/s^2)$

Newton's law of universal gravitation:

$F_{gravitation} = Gm_1m_2/r^2$

$G = 6.67 \cdot 10^{-11}\ N\ m^2/kg^2$

For objects on the surface of the earth:

$m_1 = m_{earth}$ = fixed

r = "radius" of earth = fixed

The earth is a point object relative to m_2

Measuring mass and weight.

Given that g_{earth}=9.81 m/s^2, g_{sun}=274 m/s^2, g_{moon}=1.67 m/s^2, what is the mass of a person on the sun and moon if his/her mass on earth is 70 kg? And what is his/her weight on each of the three surfaces?

- The mass is the same on each of the surfaces

- On Earth: w=686.7 N
- On the Moon: w=116.7 N
- On the Sun: 19180 N

Newton's Laws

- First Law: If the net force exerted on an object is zero the object continues in its original state of motion; if it was at rest, it remains at rest. If it was moving with a certain velocity, it will keep on moving with the same velocity.
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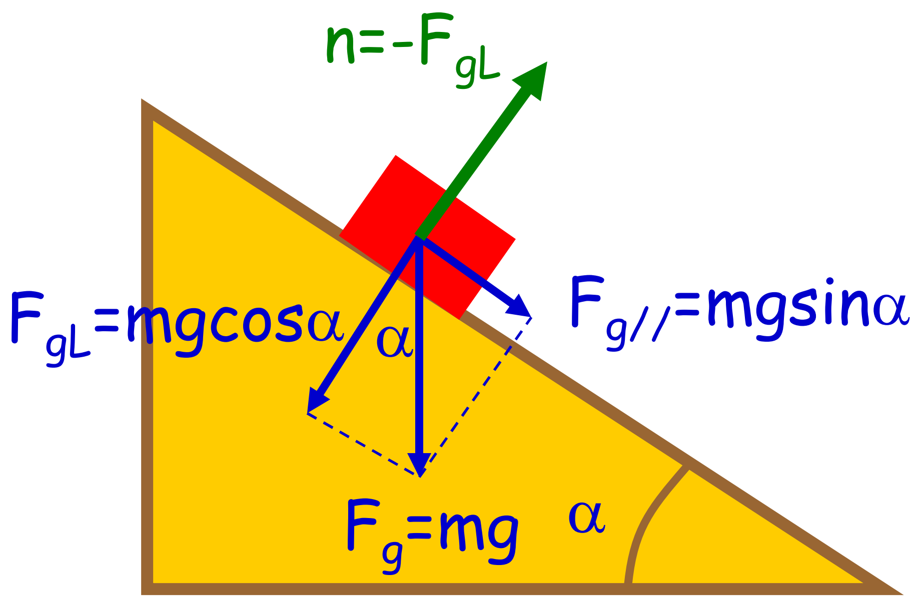
Forces seen in the previous lecture

- Gravity: Force between massive objects
- Normal force: Elasticity force from supporting surface



Gravity, mass and weight.

Weight=mass times gravitational acceleration

$$F_g(N) = m(kg)\, g(m/s^2)$$

Newton's law of universal gravitation:

$$F_{gravitation} = Gm_1m_2/r^2$$

$$G = 6.67 \cdot 10^{-11}\ N\ m^2/kg^2$$

For objects on the surface of the earth:

$$m_1 = m_{earth}$$

$$r = r_{earth}$$

The earth acts as point object from a distance r_{earth} from m_2

$$m_2 = m, \quad F_g = m\ .\ (G\ m_{earth}/r_{earth}{}^2) = m\ g$$

$$g = G\ m_{earth}/r_{earth}{}^2 = 9.8\ m/s^2$$

Measuring mass and weight.

Given that g_{earth}=9.81 m/s^2, g_{sun}=274 m/s^2, g_{moon}=1.67 m/s^2, what is the mass of a person on the sun and moon if his mass on earth is 70 kg? And what is his weight on each of the three surfaces?

- The mass is the same on each of the surfaces

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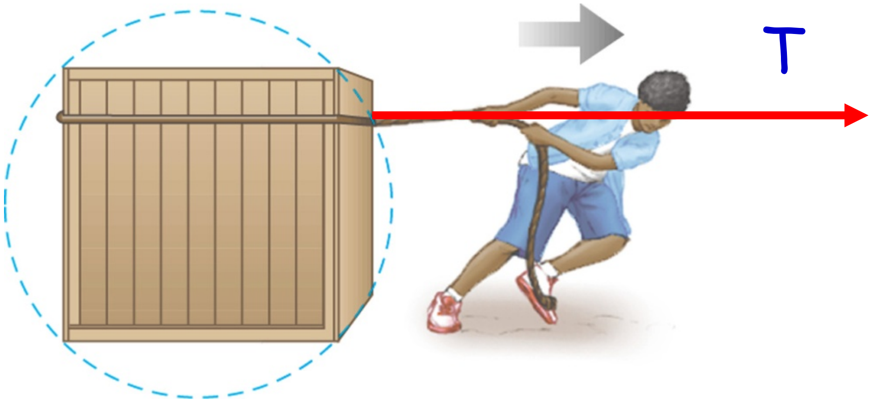
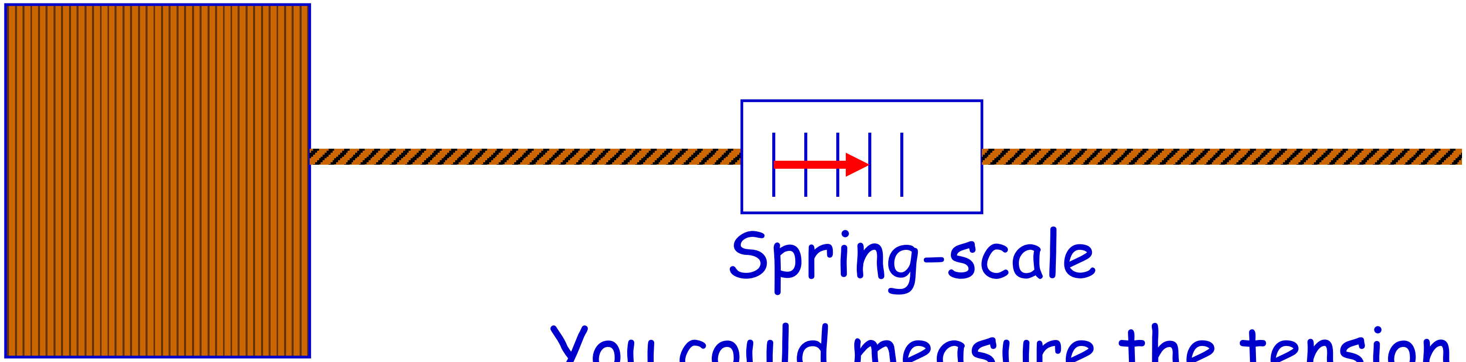
Tension



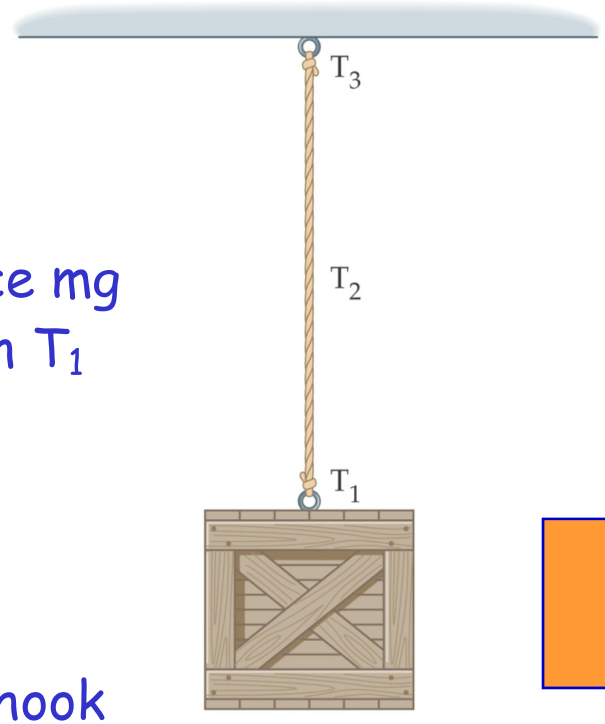
The magnitude of the force T acting on the crate, is the same as the tension in the rope.

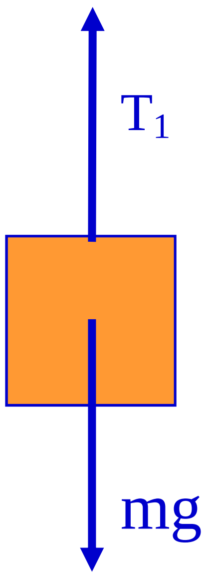


You could measure the tension by inserting a spring-scale...

Tension

- Free Body Diagram for the Box.
  - The Earth pulls down with force mg
  - The string pulls up with tension T_1
  - If $a = 0$, then $T_1 - mg = 0$.
- By Action-Reaction,
  - Tension $T_2 = T_1$
  - $T_3 = T_2 = T_1$
- The rope pulls down on the upper hook with tension $T = T_1$
- If a string pulls at one end with tension T, it pulls (in the opposite direction) at the other end with the same tension T.





Strings
(application of Action-Reaction)

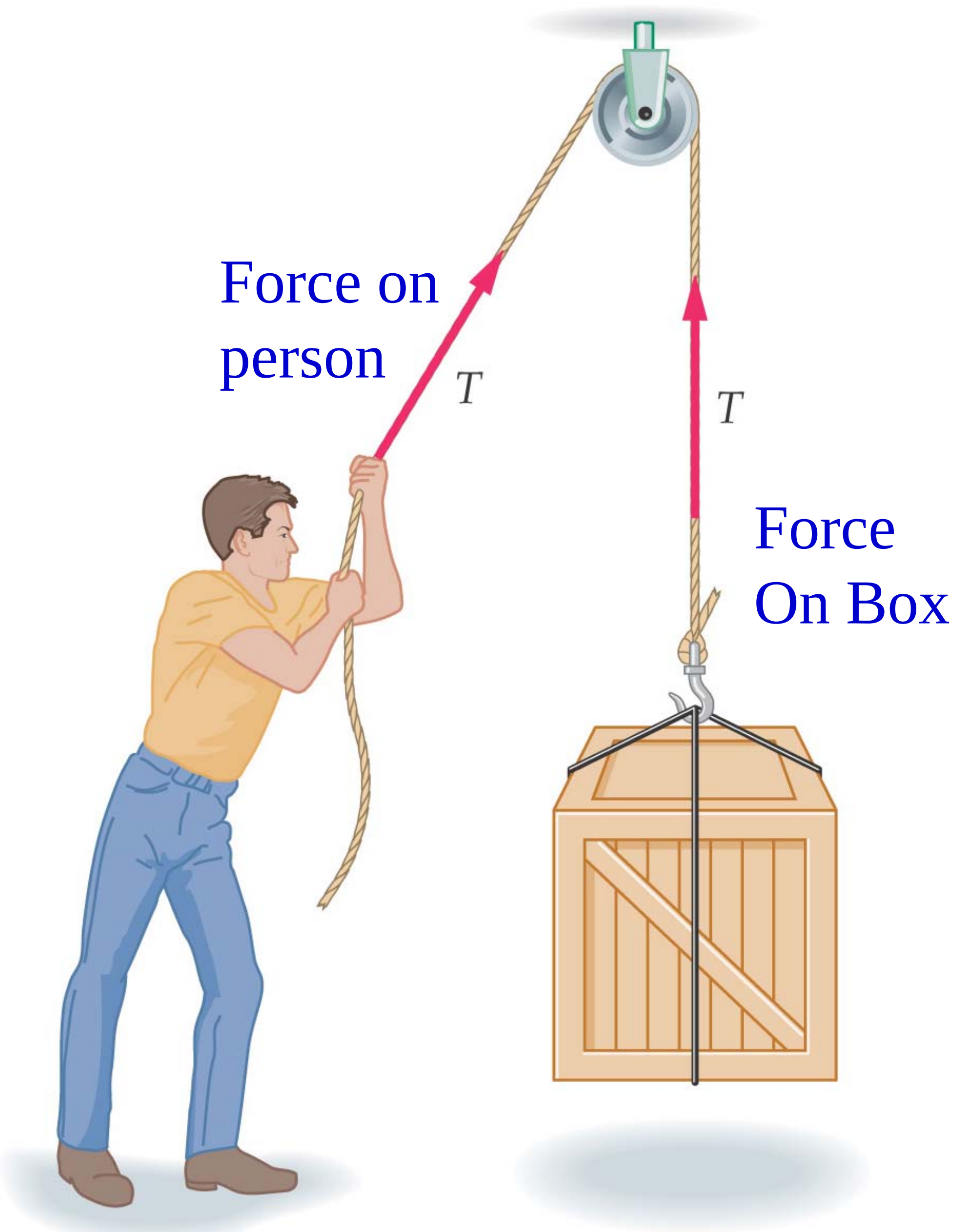
- A String is floppy, it cannot push either straight or sideways. It can only pull in tension.
- A String is a chain of tiny masses:

- Consider F=ma for the middle segment.

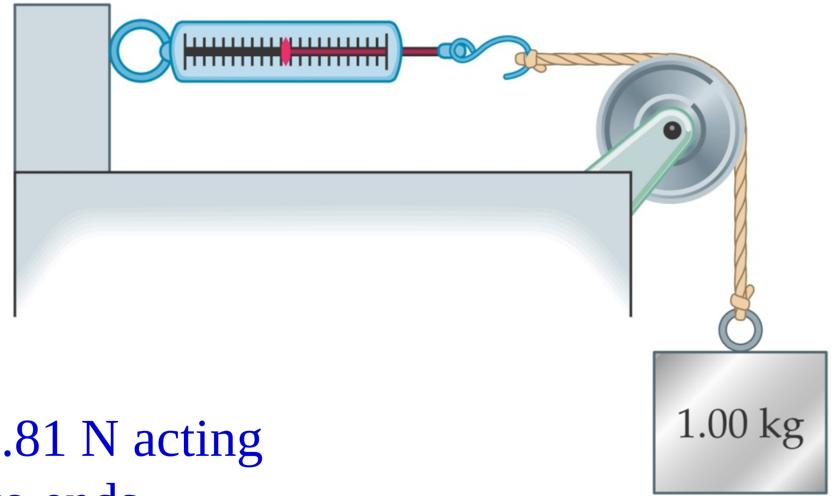
  - T_1 and T_2 are the forces on the mass segment from the adjacent segments to left and right.
  - If the mass m of each segment is small enough, then ma=0 even if $a \neq 0$. Therefore $F_{net} = 0$, and $T_1 = T_2$.
  - By action-reaction, the segment pulls back on its neighbors with equal and opposite forces.

Ideal Pulley

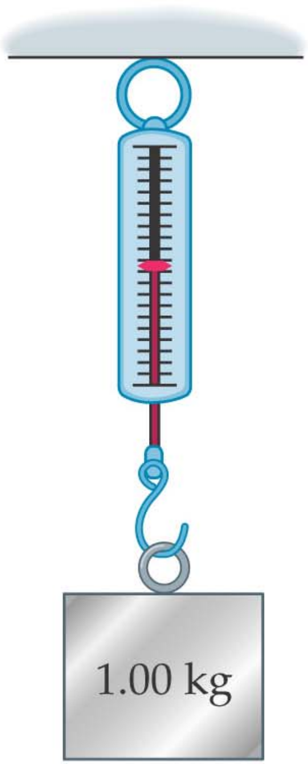
- Pulley spins freely without friction, neglect (rotational) inertia (mass) of pulley.
- Pulley changes direction, not magnitude, of tension.

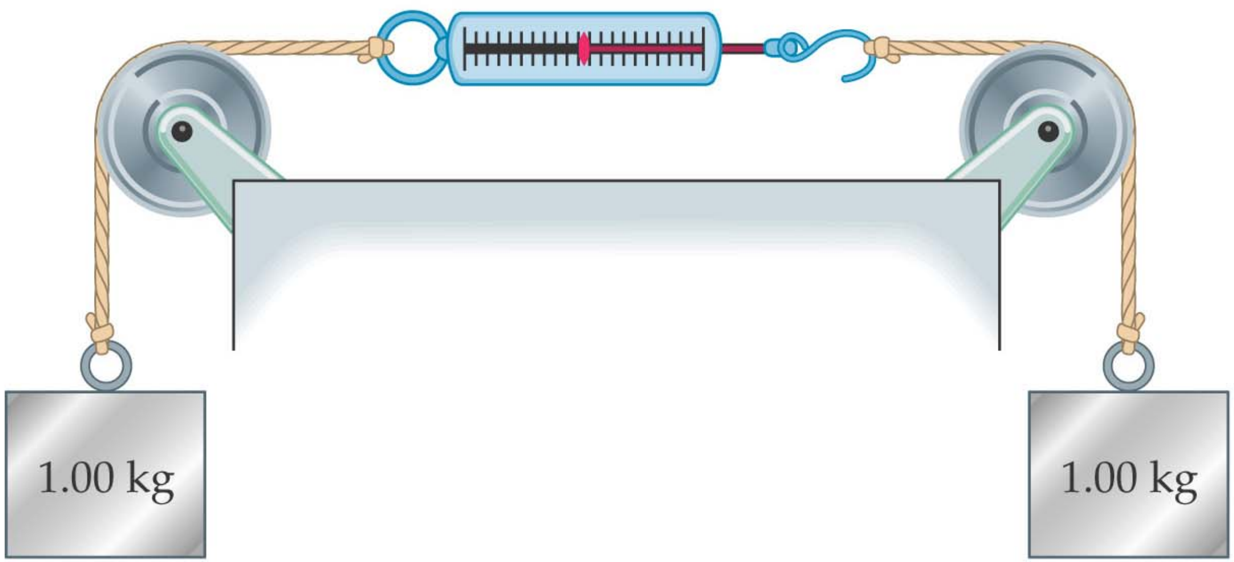


Each Spring-Scale reads 9.81 N !!!



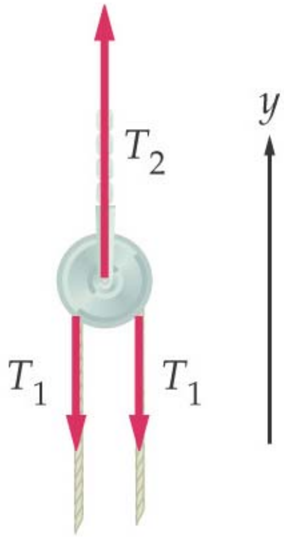
Each Scale has two forces of 9.81 N acting in opposite directions on its two ends.



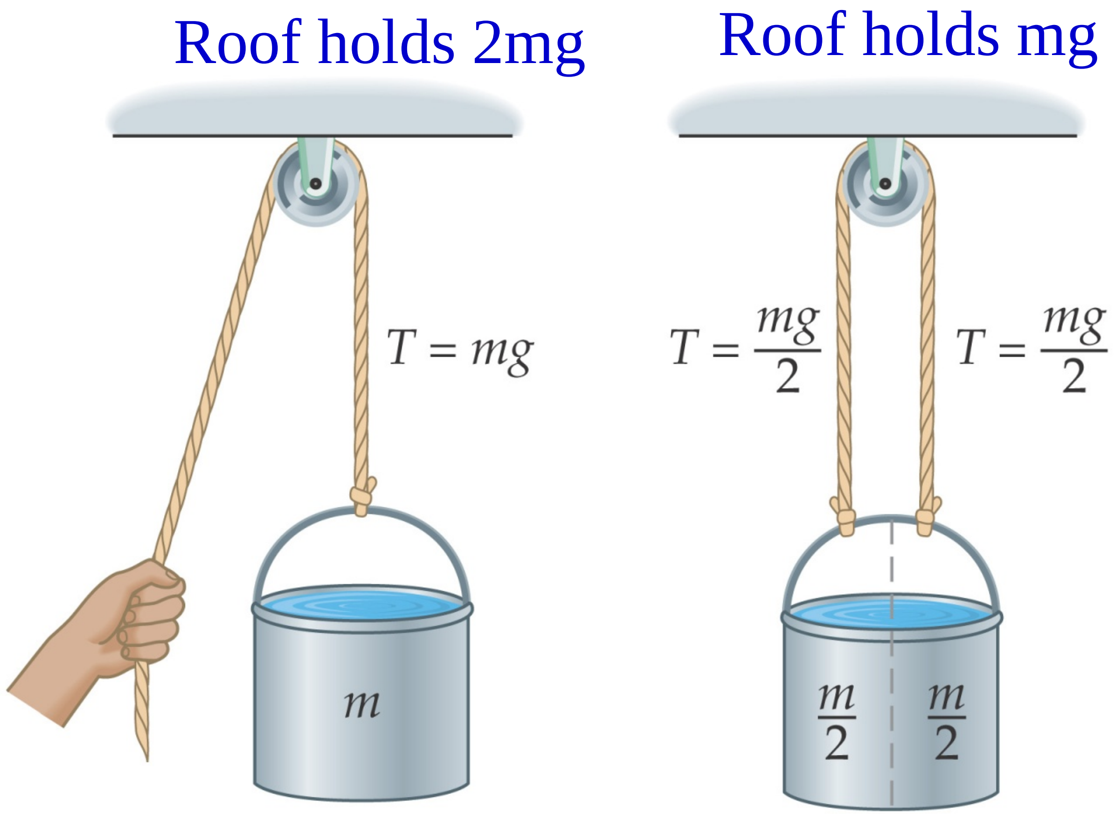


Tension, and Forces on Pulleys

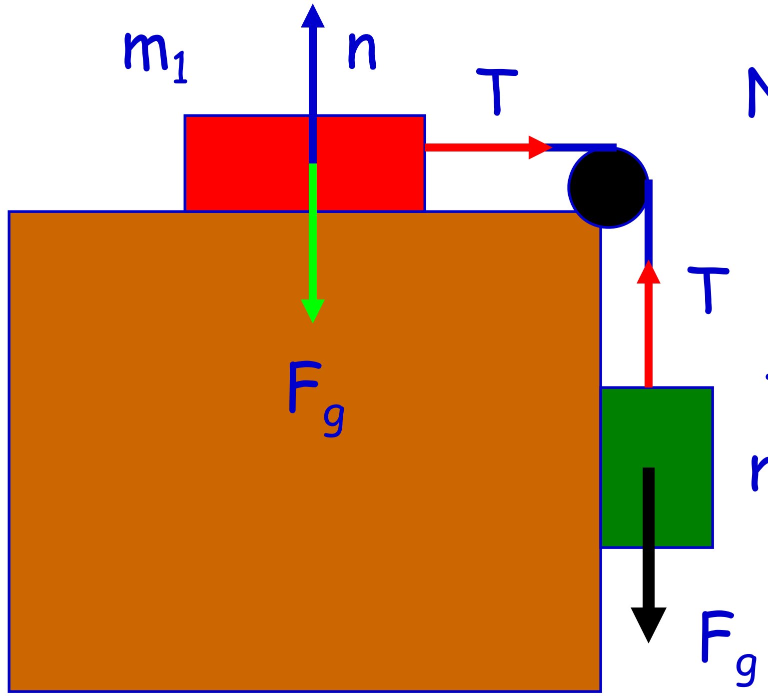
- Free body diagram of pulley.



Forces acting on the pulley



Newton's second law and tension



No friction.

What is the acceleration of the objects?

Object 1: $\Sigma F = m_1 a$, so $T = m_1 a$

Object 2: $\Sigma F = m_2 a$, so $F_g - T = m_2 a$

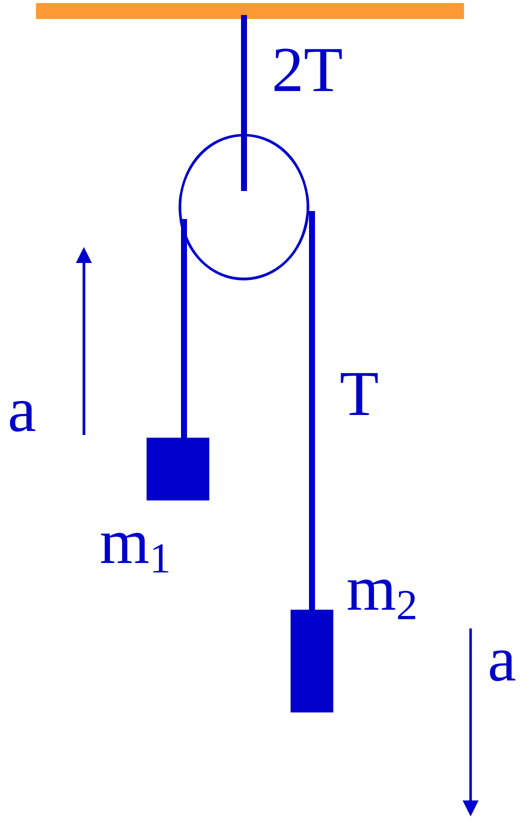
$m_2 g - T = m_2 a$

Combine 1&2 (Tension is the same): $a = m_2 g/(m_1+m_2)$

Atwood Machine

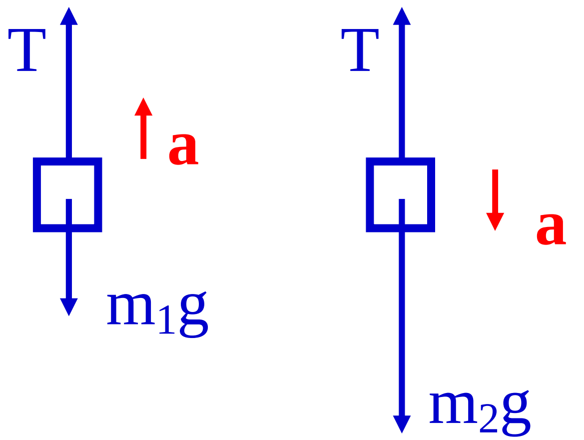
Find the acceleration a,
Tension, 2T, in support rope.

Expect: $a > 0$, if $m_2 > m_1$
Expect: $a << g$, if $m_2 \approx m_1$

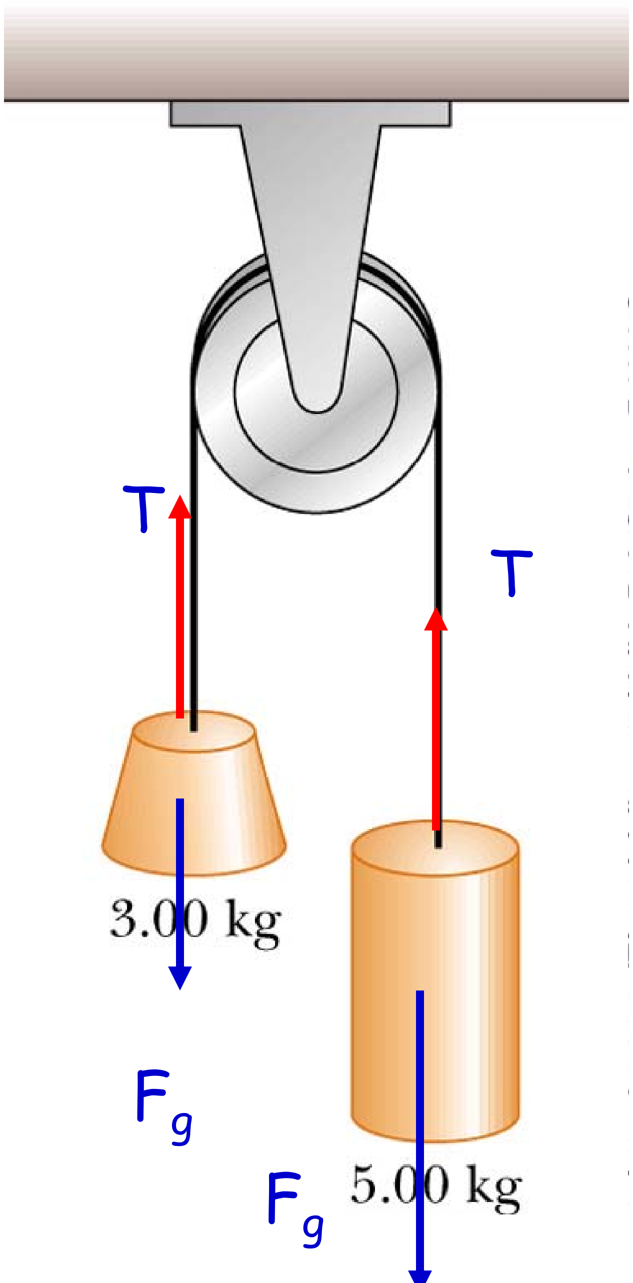


Atwood Machine, Free Body Diagrams

- $T - m_1 g = m_1 a$
- $T - m_2 g = - m_2 a$ (note sign)
- Get rid of T by subtracting:
  - $[T - m_1 g] - [T - m_2 g] = (m_1 + m_2) a$
  - $(m_2 - m_1) g = (m_1 + m_2) a$
  - $a = g\,(m_2 - m_1) / (m_1 + m_2) < g$
- Plug back in to get T
  - $T = m_1(g+a) = 2g\, m_1 m_2 / (m_1 + m_2)$
  - Note: if $m_1 = m_2$, then $a = 0$ and $T = m_1 g$



Problem



What is the tension in the string and what will be the acceleration of the two masses?

Draw the forces: what is positive & negative???

For 3.00 kg mass: $\Sigma F=ma$

$T-9.81\cdot 3.00=3.00\cdot a$

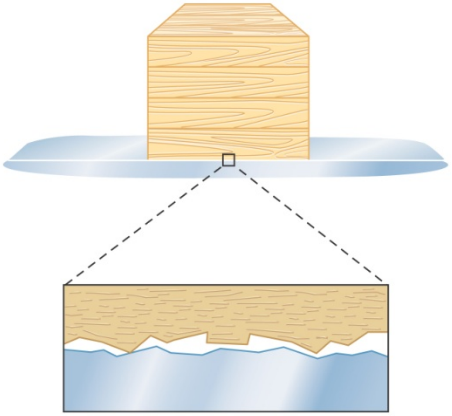
For 5.00 kg mass: $\Sigma F=ma$

$9.81\cdot 5.00-T=5.00\cdot a$

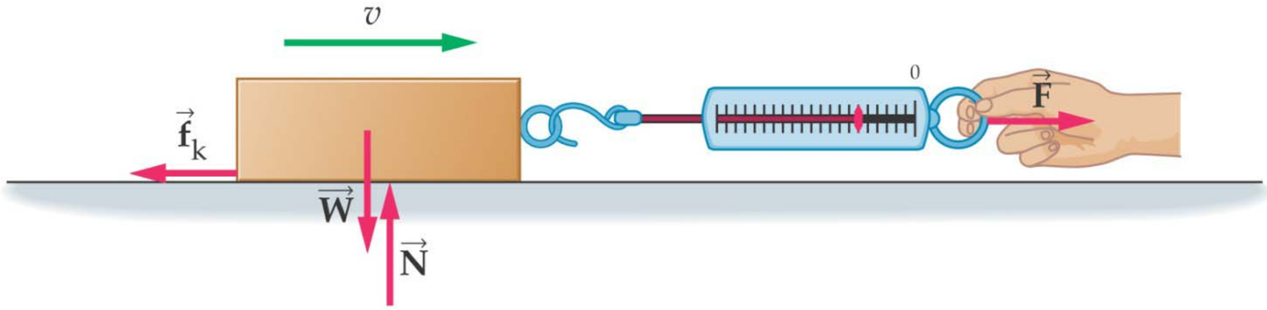
$T=36.8$ N

$a=2.45$ m/s^2

Friction



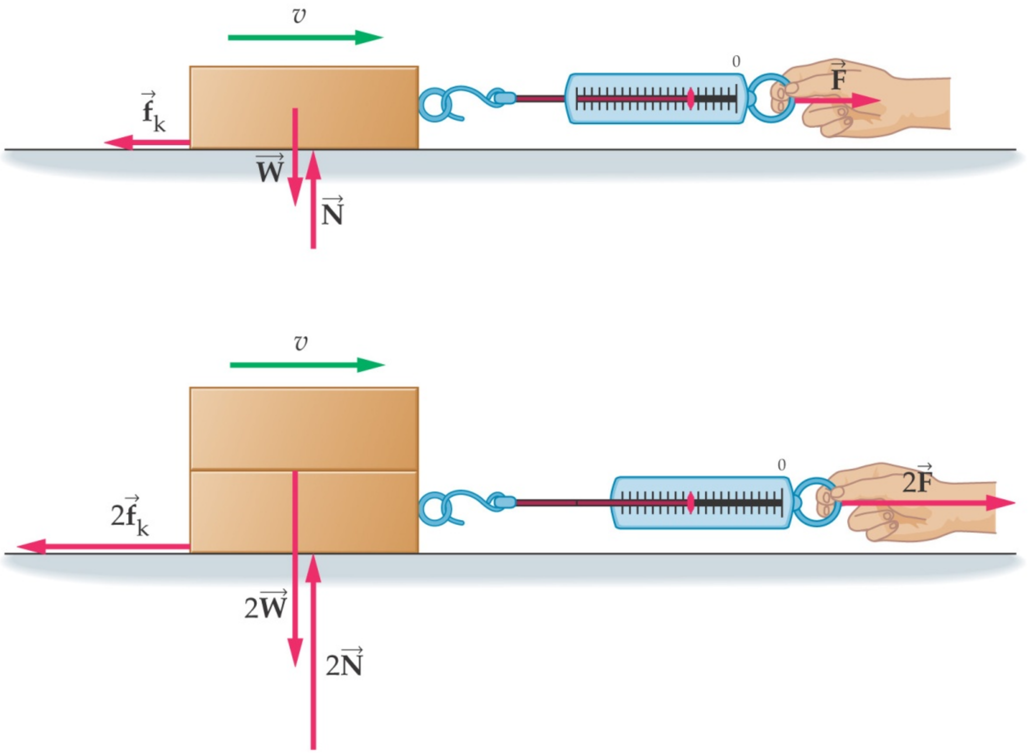
- As a block slides on the table, the force from the surface of the table acting on the bottom of the block has components both perpendicular to the surface, and parallel to the surface.
  - The component perpendicular to the surface we call the Normal force, N.
  - The component parallel to the surface is friction.



Approximate Model of Force of Kinetic (Sliding) Friction (F_k)

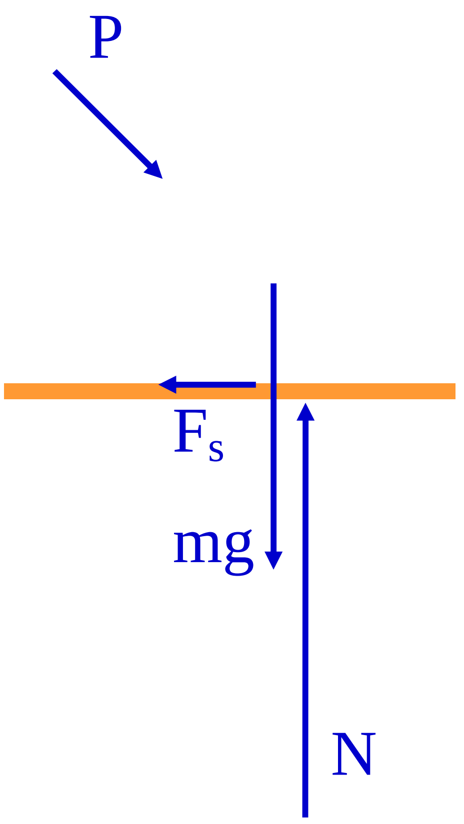
- Force of friction $\mathbf{F}_k$ points in direction opposite to velocity **v**.
- Force of friction $\mathbf{F}_k$ acting on object *m* is proportional to Normal force **N** also acting on *m* at the same surface: $|\mathbf{F}_k| = \mu_k |\mathbf{N}|$
  - N is sometimes but often not equal in magnitude to force of gravity
- $|\mathbf{F}_k|$ independent of area of contact and velocity **v**

- Coefficient μ_k
  - Depends on materials, surface conditions
  - Independent of **v** or **N**



Static Friction (v=0)

- In spite of the push force P, the block does not move (**v**=0, **a**=0)
- Static friction does not have a fixed magnitude or direction (but in the absence of glue, static friction must be parallel to surface).
- Static friction takes whatever value necessary to keep **a**=0 via $\mathbf{F}_{net} = 0$
  - But static friction has a maximum value $F_s \leq \mu_S N$
  - Due to a natural welding action, generally $\mu_S > \mu_k$.



Friction

Friction are the forces acting on an object due to interaction with the surroundings (air-friction, ground-friction etc).

Two variants:

- Static Friction: as long as an external force (F) trying to make an object move is smaller than $f_{s,max}$, the static friction f_s equals F but is pointing in the opposite direction: no movement!

  $\mathbf{f}_{s,max}=\mu_s \mathbf{n}$ μ_s=coefficient of static friction

- Kinetic Friction: After F has surpassed $f_{s,max}$, the object starts moving but there is still friction. However, the friction will be less than $f_{s,max}$!

  $\mathbf{f}_k=\mu_k \mathbf{n}$ μ_k=coefficient of kinetic friction

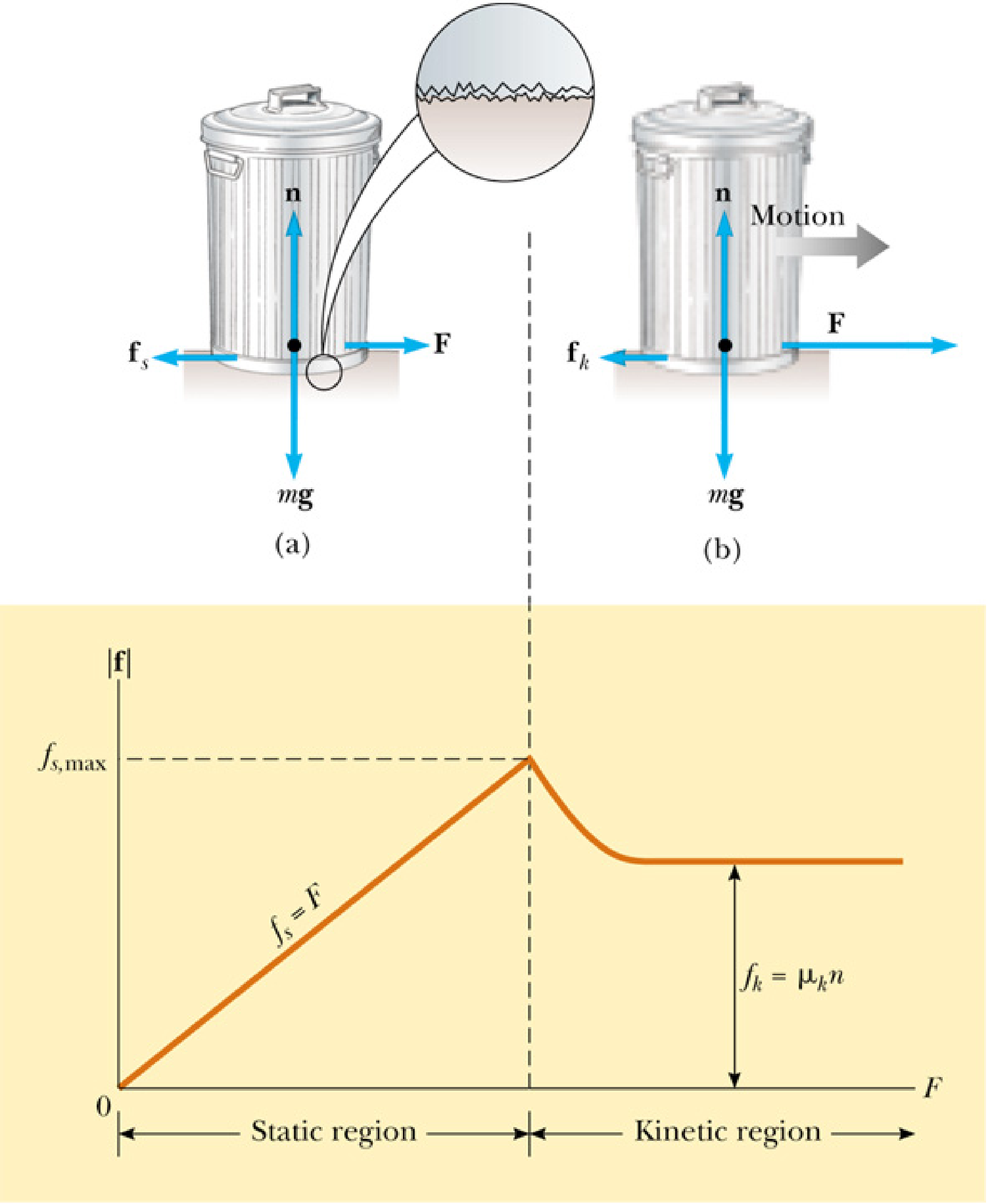


TABLE 4.2 **Coefficients of Friction**[a]

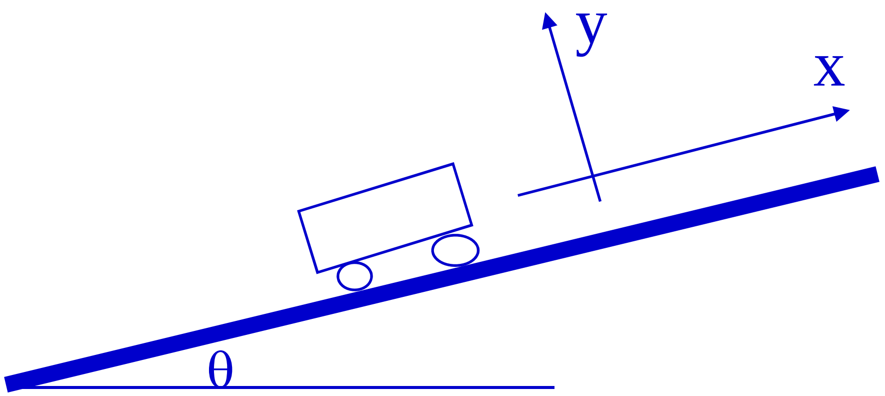
	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

[a] All values are approximate.

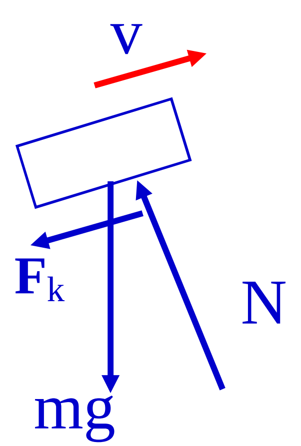
More Prolems: Motion on an Incline

Kinetic Friction

- Cart rolls down incline
- Friction in bearings is equivalent to kinetic friction of block sliding on incline.
- If the angle of the incline is θ=10° and the coefficient of kinetic friction is μ_k = 0.05,
- Find the acceleration of the cart as it rolls up the incline.
- Find the acceleration of the cart as it rolls down the incline.

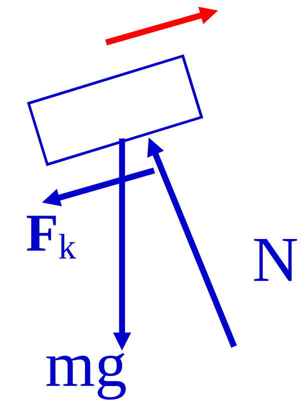


Free Body Diagram

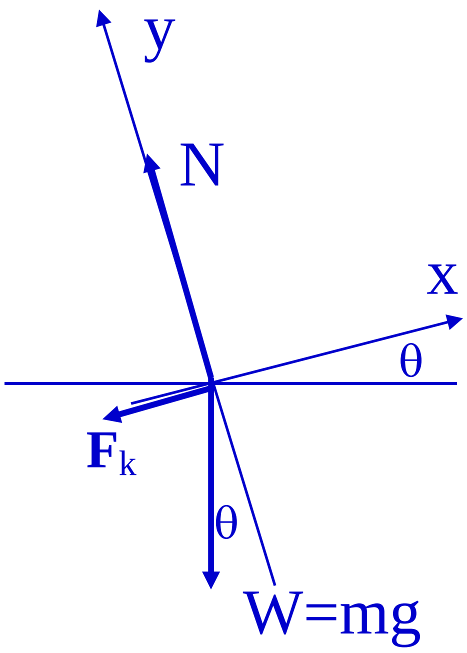


Motion on an Incline

Kinetic Friction, Sliding uphill



- Draw all force vectors with a common origin.
- Find all components of vectors
  - $N_x = 0$, $N_y = N$
  - $W_x = mg\cos(270-\theta) = -mg\sin\theta < 0$
  - $W_y = mg\sin(270-\theta) = -mg\cos\theta < 0$
  - $F_{k,x} = -\mu_k N$, $F_{k,y} = 0$
- Apply F=ma to each component:
  - Y: $N + mg\sin(270-\theta) = m\,a_y = 0$
  - . $N = -mg\sin(270-\theta) = mg\cos\theta$
  - X: $-\mu_k N - mg\sin\theta = m\,a_x$
- Combine & solve:
  - $-\mu_k mg\cos\theta - mg\sin\theta = m\,a_x$
  - $a_x = -[\mu_k g\cos\theta + g\sin\theta]$

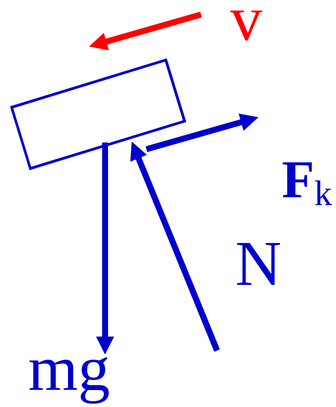


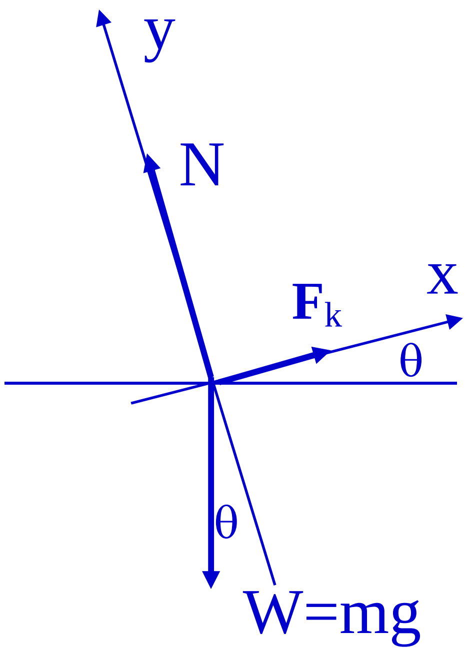
Magnitude bigger than without friction (as expected)

Motion on an Incline

Kinetic Friction, Sliding downhill

- Find all components of vectors
  - $N_x = 0$, $N_y = N$
  - $W_x = mg \cos(270 - \theta) = -mg \sin\theta < 0$
  - $W_y = mg \sin(270 - \theta) = -mg \cos\theta < 0$
  - $F_{k,x} = +\mu_k N$, $F_{k,y} = 0$
- Apply *F = ma* to each component:
  - Y: $N - mg \sin\theta = m\, a_y = 0$
  - . $N = mg \cos\theta$
  - X: $+\mu_k N - mg \sin\theta = m\, a_x$
- Combine & solve:
  - $+\mu_k mg \cos\theta - mg \sin\theta = m\, a_x$
  - $a_x = -[g \sin\theta - \mu_k g \cos\theta]$





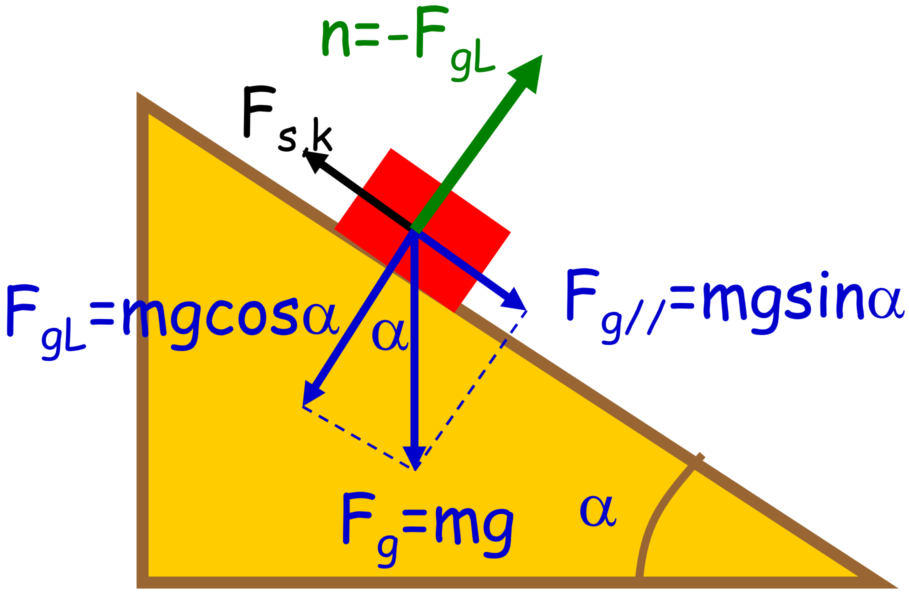
Motion down an incline
Kinetic Friction

- $a_x = -[g \sin\theta - \mu_k g \cos\theta] = -g[\sin\theta - \mu_k \cos\theta]$
  - Friction slows the downward acceleration for downward motion.
  - $\theta = 10°$, $\mu_k = 0.05$
  - $a_x = -g[\sin 10° - 0.05 \cos 10°] = -g[0.174 - 0.0492]$
  - $a_x = -g[0.124]$
- What if we make μ_k larger?
  - $\mu_k = \sin\theta / \cos\theta = \tan\theta$
    - $a_x = 0$. Constant downward velocity
  - $\mu_k > \sin\theta / \cos\theta$
    - $a_x > 0$ (as $v < 0$), motion slows to stop, even though started downhill.

Friction & Driving

- In ordinary driving, is the friction between tire and road static or kinetic?
- In a skid, is the friction between tire and road static or kinetic?
- How can anti-lock brakes stop a car in less distance than just slamming on the brakes?

Problem



A) If $\mu_s = 1.0$, what is the angle α for which the block just starts to slide?

B) The block starts moving. Given that $\mu_k = 0.5$, what is the acceleration of the block?

A) Parallel direction: $mg\sin\alpha - \mu_s n = 0$ ($\Sigma F = ma$)

Perpendicular direction: $mg\cos\alpha - n = 0$ so $n = mg\cos\alpha$

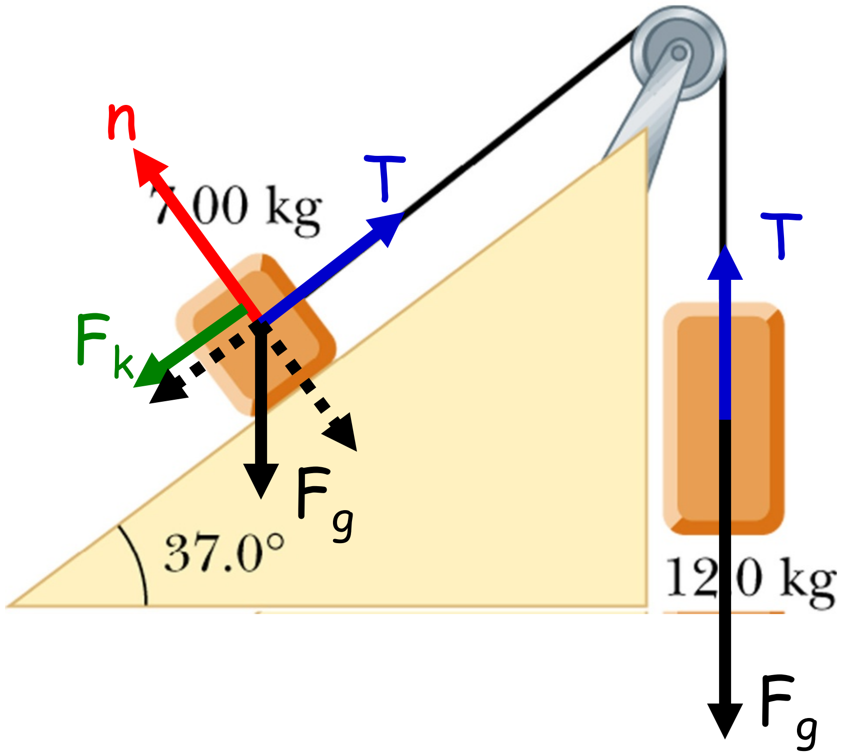
Combine: $mg\sin\alpha - \mu_s mg\cos\alpha = 0$

$$\mu_s = \sin\alpha/\cos\alpha = \tan\alpha = 1 \text{ so } \alpha = 45^o$$

B) Parallel direction: $mg\sin(45^o) - \mu_s mg\cos(45^o) = ma$ ($\Sigma F=ma$)

$g(\tfrac{1}{2}\sqrt{2} - \tfrac{1}{4}\sqrt{2}) = a$ so $a = g\tfrac{1}{4}\sqrt{2}$

All the forces come together...



If $a=3.30\ m/s^2$ (the 12kg block is moving downward), what is the value of μ_k?

For the 7 kg block parallel to the slope:
$T-mg\sin\alpha-\mu_k mg\cos\alpha=ma$
For the 12 kg block: $Mg-T=Ma$

Solve for μ_k $$\mu_k=\frac{-M(g-a)+mg\sin\alpha+ma}{-mg\cos\alpha}=0.25$$

General strategy

- If not given, make a drawing of the problem.
- Put all the relevant forces in the drawing, object by object.
    - Think about the axis
    - Think about the signs
- Decompose the forces in direction parallel to the motion and perpendicular to it.
- Write down Newton's first law for forces in the parallel direction and perpendicular direction.
- Solve for the unknowns.
- Check whether your answer makes sense.

APPENDIX: Manipulating Fractions

$$\frac{a}{b} = \frac{a}{b}1 = \frac{a}{b} \cdot \frac{c}{c} = \frac{a \cdot c}{b \cdot c}$$

Example : Simplify denominator :

$$\frac{a}{b/c} = \frac{c}{c} \cdot \frac{a}{b/c} = \frac{a \cdot c}{b \cdot (c/c)} = \frac{a \cdot c}{b}$$

Example : Introduce common factor :

$$\tan\theta = \frac{y}{x} = \frac{y}{x} \cdot \frac{(1/r)}{(1/r)} = \frac{y/r}{x/r} = \frac{\sin\theta}{\cos\theta}$$