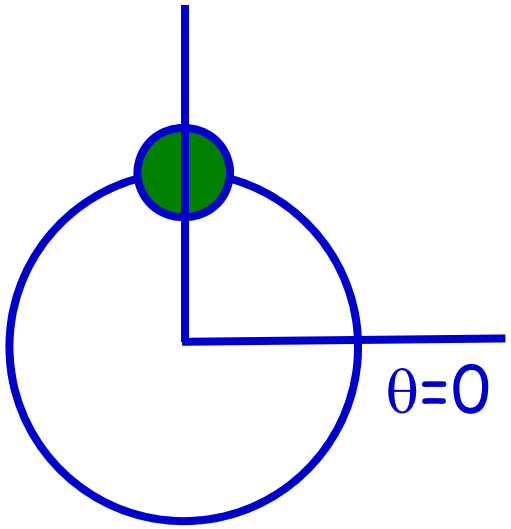
example

What is the angular velocity of earth around the sun? Give in rad/s, rev/s and rpm

Answer: in 1 year, the earth makes one full orbit around the sun.

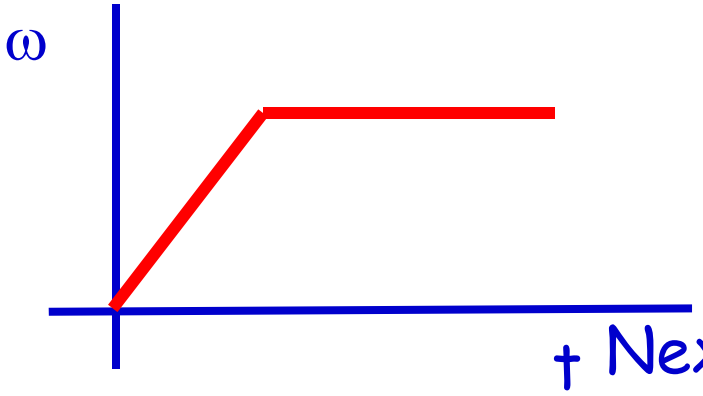
$$\varpi = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

rad/s	rev/s	rpm
= 2π rad/1 year	1 rev/1 year	1 rev/1 year
= 2π rad/(3.2E+7 s)	1 rev/(3.2E+7 s)	1 rev/(5.3E+5 min)
= 2.0E-07 rad/s	3.1E-8 rev/s	1.9E-6 rpm



example

A person is rotating a wheel. The handle is initially at $\theta=90^o$. For 5s the wheel gets an constant angular acceleration of 1 rad/s^2. After that The angular velocity is constant. Through what angle will the wheel have rotated after 10s.



First 5s: $\theta(5)= \theta(0)+\omega(0)t+\frac{1}{2}\alpha t^2$

$= \pi/2+ \ 0 \ + \frac{1}{2}1(5)^2$

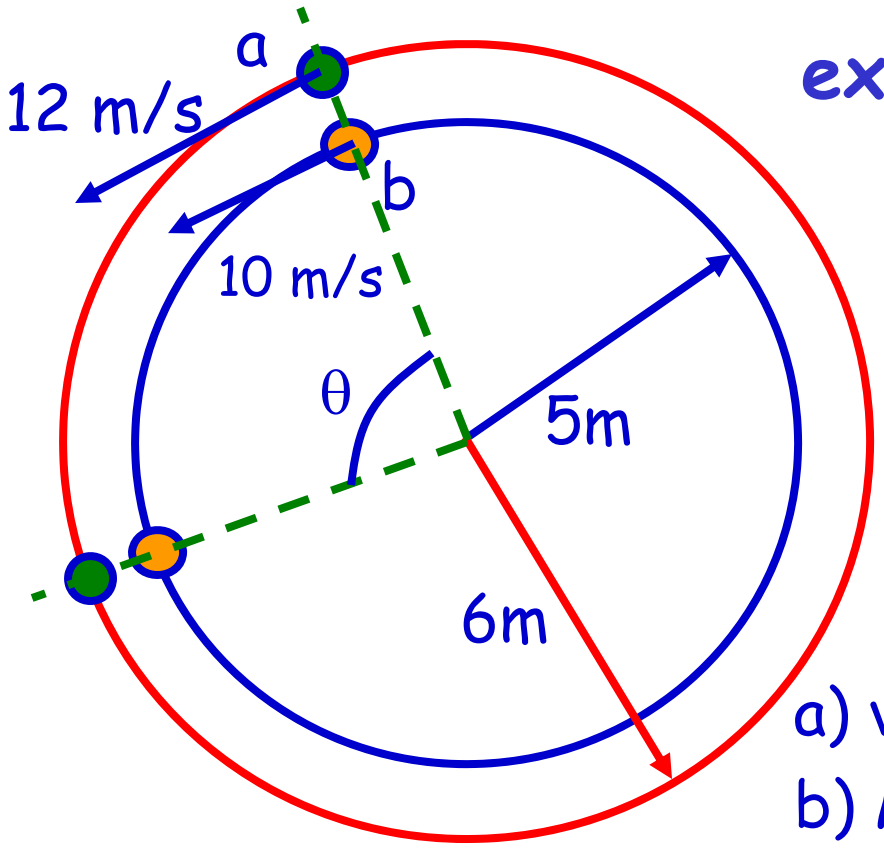
$= \pi/2+12.5=14.1$ rad $(=806^o)$

$\omega(5)= \omega(0)+\alpha t$

$= 1t= 5$ rad/s

Next 5s: $\theta(5)= 14.1+5t$

$=$ 39.1 rad $(=2240^0=6.2$ rev$)$



example

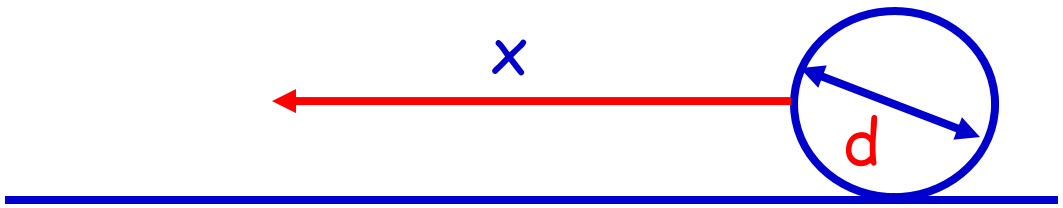
The angular velocity of a is 2 rad/s.
a) What is its tangential velocity?
If b is keeping pace with a,
b) What is its angular velocity?
c) What is its linear velocity?

a) $v = \omega r = 2*6 = 12$ m/s

b) Must be the same: 2 rad/s

c) $v = \omega r = 2*5 = 10$ m/s

A rolling coin



ω_0=18 rad/s
α=-1.80 rad/s
d=0.02 m

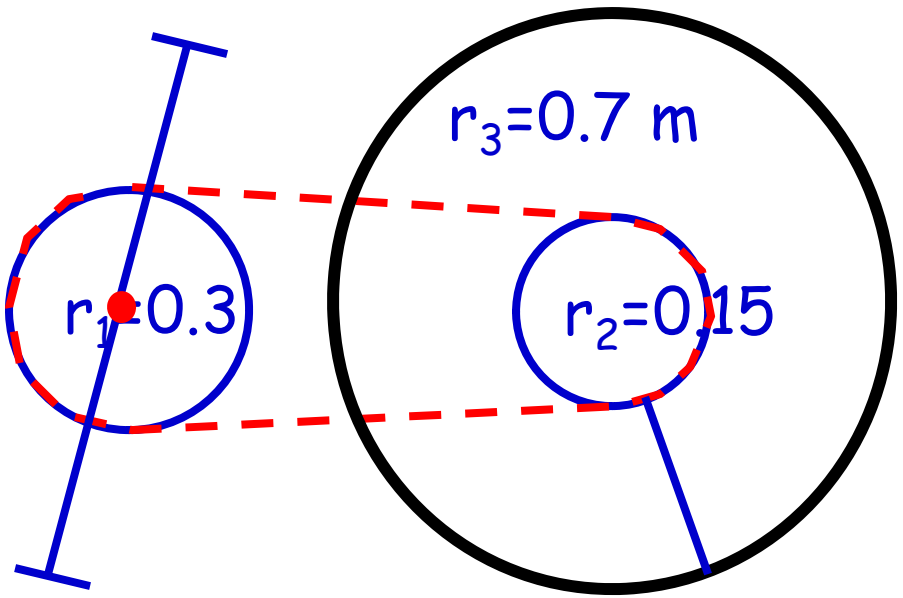
a) For how long does the coin roll?
b) What is the average angular velocity?
c) How far does the coin roll before coming to rest?

a) $\omega(t)= \omega(0)+\alpha t$ $0=18-1.8t$ $t=10$ s

b) $\overline{\omega} = (\omega(0)+ \omega(10))/2=18/2=9$ rad/s

c) $\overline{v}=\overline{\omega}r = 9*0.01=0.09$ m/s $x=vt=0.09*10=0.9$ m

gears



If ω_1=3 rad/s, how fast is the bike going?
b) What if r_1=0.1 m ?

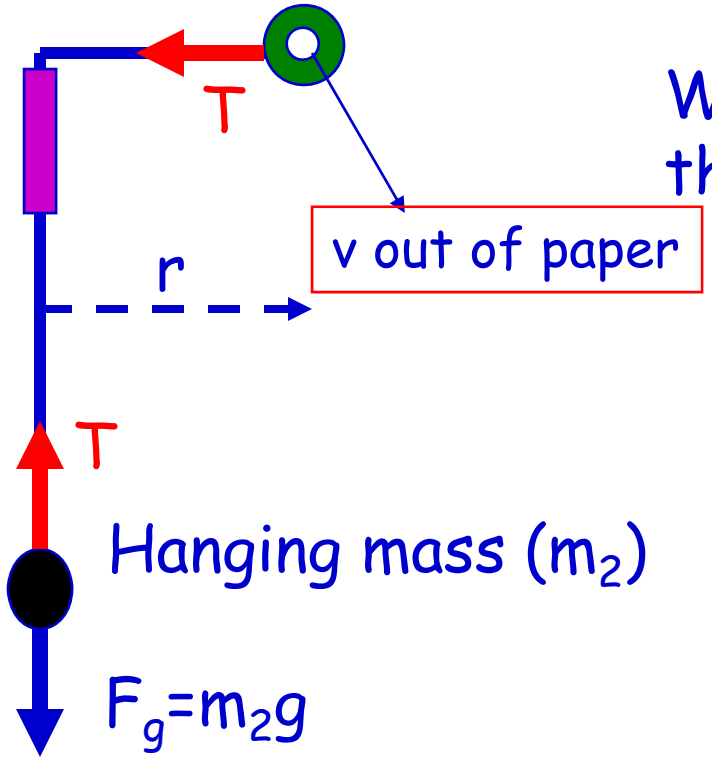
$V_1 = \omega_1 r_1 = 3*0.3 = 0.9$ m/s
$V_1 = V_2$ (because of the chain)
$\omega_2 = V_2/r_2 = 0.9/0.15 = 6$ rad/s
$\omega_3 = \omega_2$ (connected)
$V_3 = \omega_3 r_3 = 6*0.7 = 4.2$ m/s

b)

$V_1 = \omega_1 r_1 = 3*0.1 = 0.3$ m/s
$V_1 = V_2$ (because of the chain)
$\omega_2 = V_2/r_2 = 0.3/0.15 = 2$ rad/s
$\omega_3 = \omega_2$ (connected)
$V_3 = \omega_3 r_3 = 2*0.7 = 1.4$ m/s

Lifting by swinging

Swinging mass (m_1) with velocity v



What is the relation between v and r that will keep m_2 stationary?

m_1: $T=m_1a$

m_2: $T=m_2g$

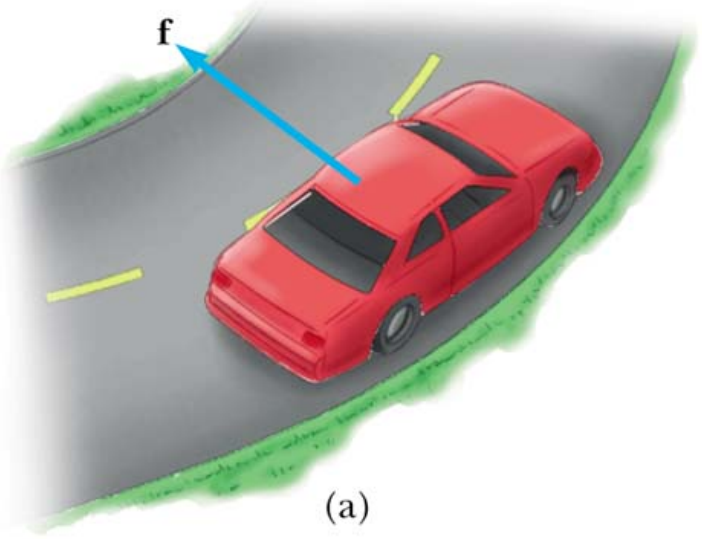
$a=m_2g/m_1$

Also: $a_c=v^2/r$

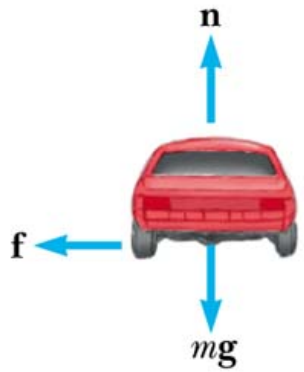
$v^2/r= m_2g/m_1$

If m_1 slows down, r must go down so m_2 sinks.

A car going through a bend



(a)



(b)

A car is passing through a bend with radius 100 m. The kinetic coefficient of friction of the tires on the road is 0.5. What is the maximum velocity the car can have without flying out of the bend?

$\Sigma F = ma$

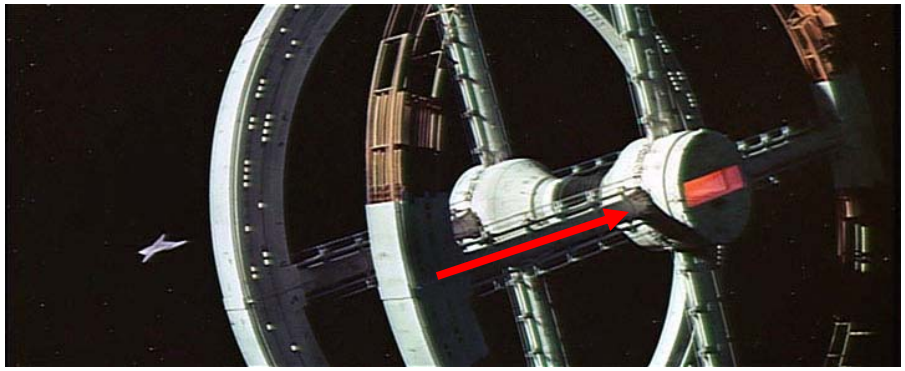
$\mu_k n = ma_c$

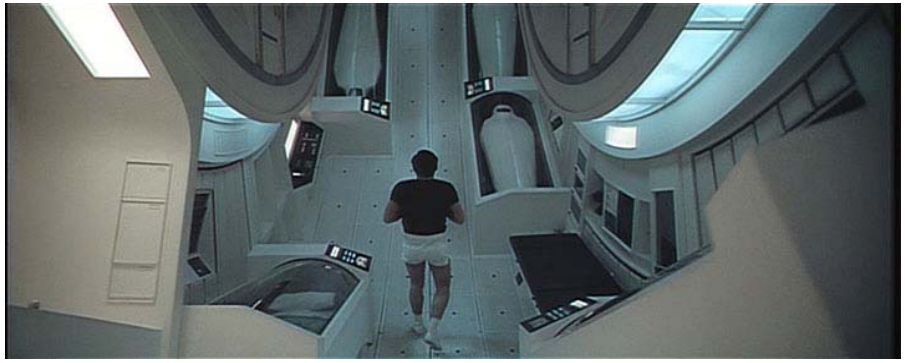
$\mu_k mg = mv^2/r$

$0.5 * 9.81 = v^2/100$

$v = 22$ m/s

2001: A space odyssey



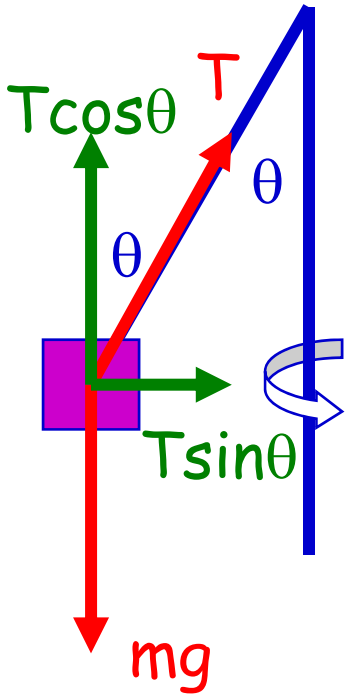


A space ship rotates with a linear velocity of 50 m/s. What should the distance from the central axis to the crew's cabin's be so that the crew feels like they are on earth? (the floor of the cabins is the inside of the outer edge of the spaceship)

The rotating spaceship has an acceleration directed towards the center of the ship: the 'lack' of forces acting on the crew pushes them against the ship.

$\Sigma F=ma \quad mg=mv^2/r \quad$ so $\quad r=v^2/9.8 \quad$ and thus $r=255$ m

Conical motion



What is the centripetal acceleration if the mass is 1 kg and $\theta=20^o$?

Vertical direction:

$\Sigma F=ma$

$T\cos\theta-mg=0$

So $T=mg/\cos\theta$

Horizontal direction:

$\Sigma F=ma_c$

$T\sin\theta=ma_c$

$mg\sin\theta/\cos\theta=mg\tan\theta=ma_c$

$a_c=g\tan\theta=9.8*0.36=3.6\ m/s^2$