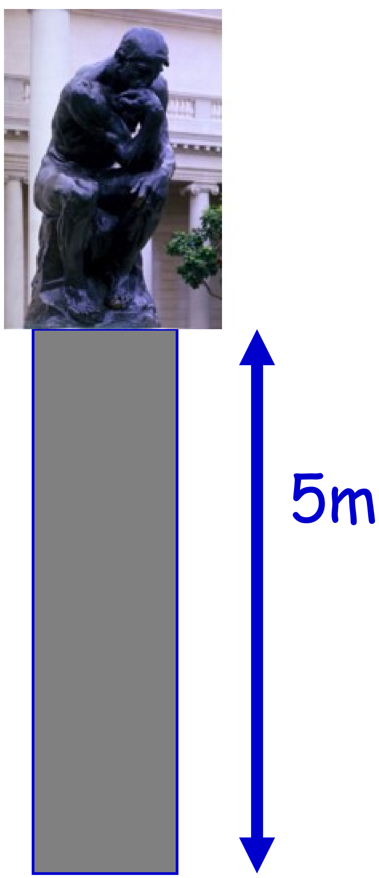
example

An architect wants to design a 5 m high circular pillar with a radius of 0.5 m that holds a bronze statue that weighs 1.0E+04 kg. He chooses concrete for the material of the pillar (Y=1.0E+10 Pa). How much does the pillar compress?



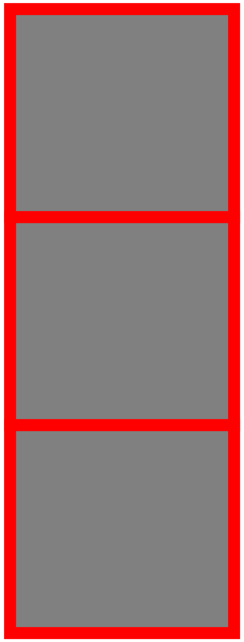
$$Y = \frac{F/A}{\Delta L / L_0} = \frac{M_{statue}\, g / (\pi R^2_{pillar})}{\Delta L / L_0}$$

R=0.5 m L_0=5 m Y=1.0E+10 Pa M=1.0E+04 kg

ΔL=6.2E-05 m

example

A builder is stacking 1 m^3 cubic concrete blocks. Each blocks weighs 5E+03 kg. The ultimate strength of concrete for compression is 2E+07 Pa. How many blocks can he stack before the lowest block is crushed?



The force on the low end of the lowest block is: $F=Nm_{block}g$.

N: total number of blocks

m_{block}=mass of one block

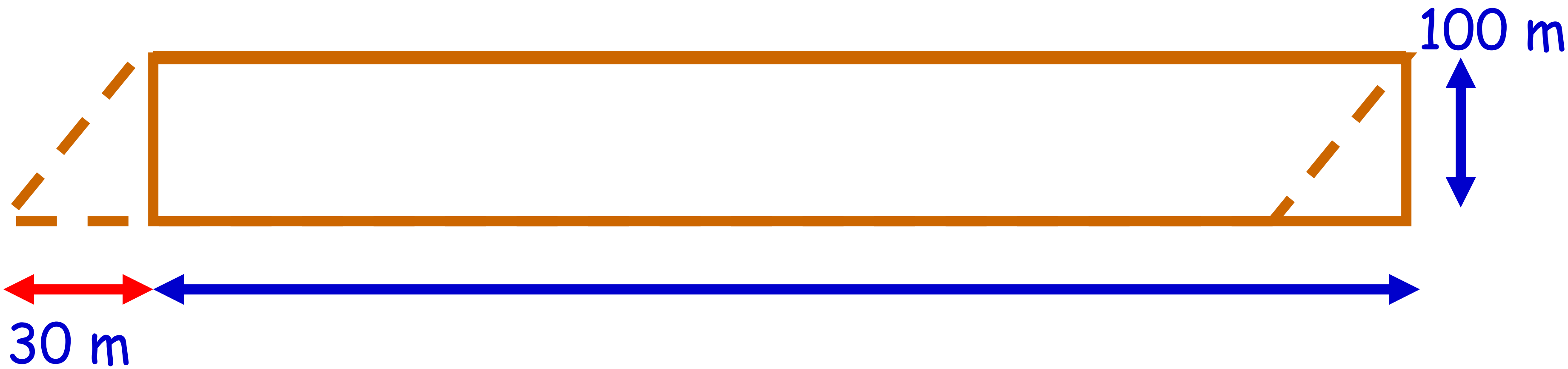
$g=9.81\ m/s^2$

Ultimate strength: $2E+07=F/A$

$=Nm_{block}g/1$

N=408 blocks.

Moving earth crust



A tectonic plate in the lower crust (100 m deep) of the earth is shifted during an earthquake by 30 m. What is the shear stress involved, if the upper layer of the earth does not move? (S=1.5E+10 Pa)

$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x / h}$$

F/A=4.5E+09 Pa

example

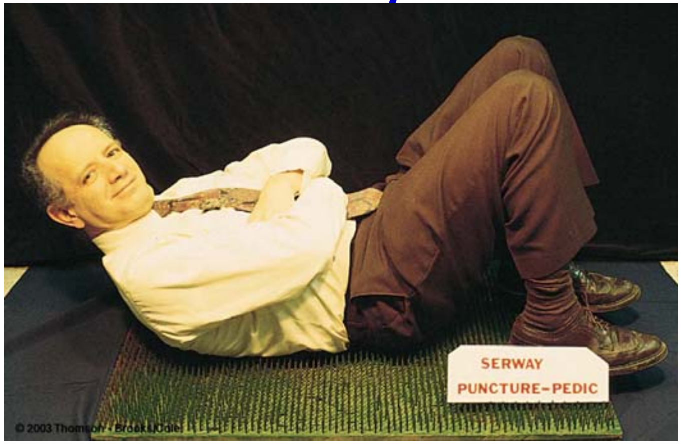
What force per unit area needs to be applied to compress 1 m^3 water by 1%? (B=0.21E+10 Pa)

$$B = -\frac{\Delta F / A}{\Delta V / V_0}$$

$\Delta V/V_0$=0.01 so, $\Delta F/A$=2.1E+07 Pa !!!

example

A nail is driven into a piece of wood with a force of 700 N. What is the pressure on the wood if A_{nail}=1 mm^2?
A person (weighing 700 N) is lying on a bed of such nails (his body covers 1000 nails). What is the pressure exerted by each of the nails?

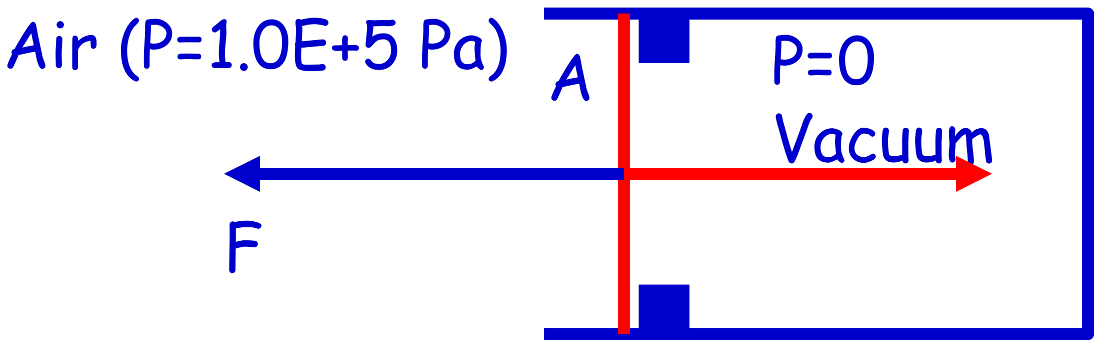


$P_{nail}=F/A_{nail}=700\ N/1E\text{-}06\ m^2=7E+08\ Pa$

$P_{person}=F/(1000A_{nail})=700/1E\text{-}03=7E+05\ Pa$

(about 7 times the atmospheric pressure).

Force and pressure



What is the force needed to move the lit?

Force due to pressure difference: $F_{pressure}=\Delta P A$
If $A=0.01\ m^2$ (about 10 by 10 cm) then
a force F=(1.0E+5)*0.01=1000N is needed to pull the lit.

A submarine

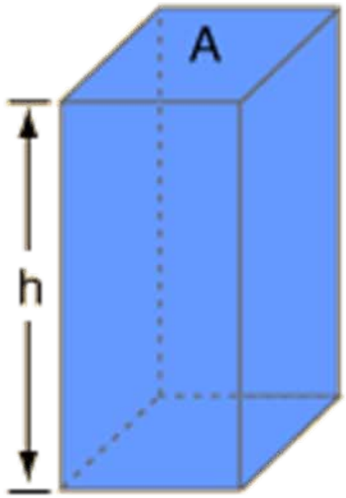
A submarine is built in such a way that it can stand pressures of up to $3x10^6$ Pa (approx 30 times the atmospheric pressure). How deep can it go?

$$P_{depth=h} = P_{depth=0} + \rho g h$$

$$3E{+}06=(1.0E{+}05)+(1.0E{+}03)(9.81)h$$

$$h=296 \text{ m.}$$

Does the shape of the container matter?

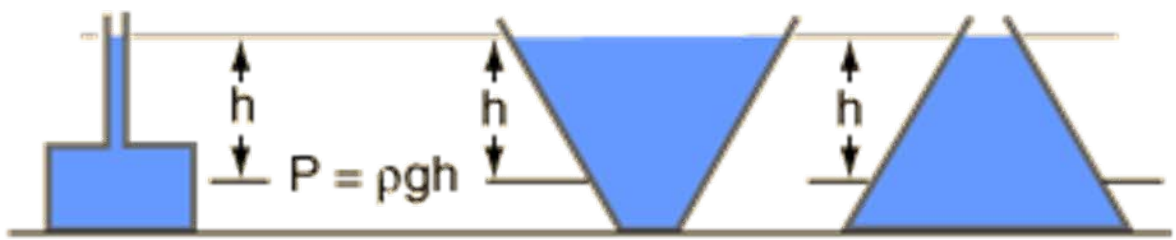


V = hA = volume

weight = mg

Static fluid pressure does not depend on the shape, total mass, or surface area of the liquid.

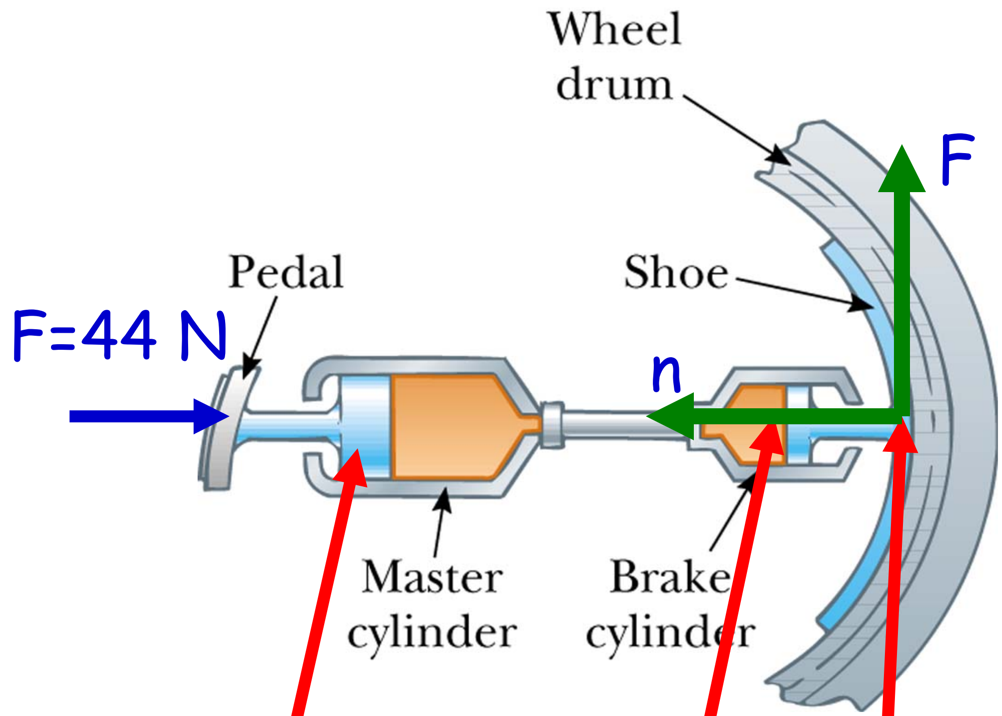
$$\text{Pressure} = \frac{\text{weight}}{\text{area}} = \frac{mg}{A} = \frac{\rho V g}{A} = \rho g h$$



NO!!

Hydraulic brake

What is the frictional torque about the axle exerted by the shoe on the wheel drum when a force of 44 N is applied to the pedal?



$F_{MC}/A_{MC}=F_{BC}/A_{BC}$

$F_{BC}=44*1.8/6.4=12.4$ N

$F_{shoe\text{-}drum}=F_{drum\text{-}shoe}=$normal force

$F_{friction}=\mu N=0.5*12.4=6.2$ N

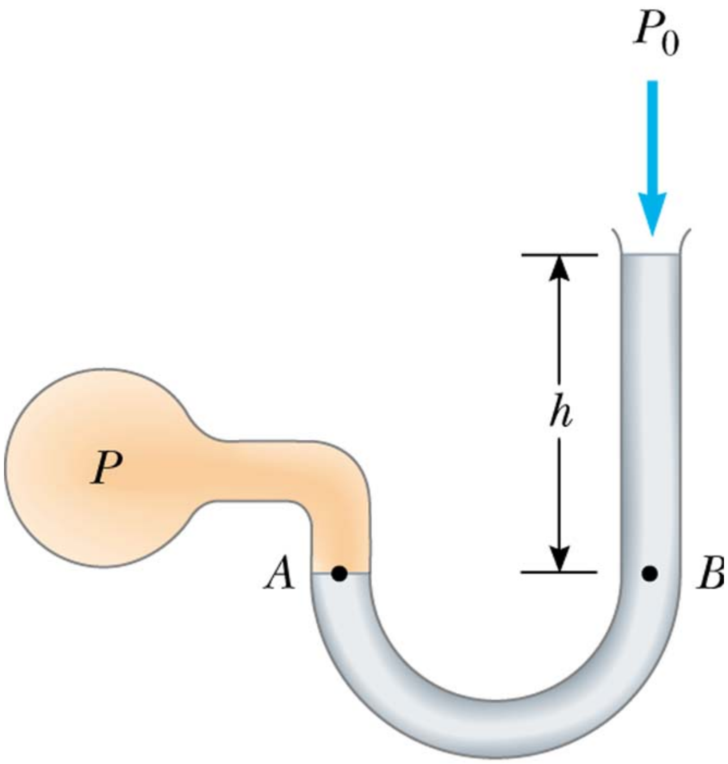
$\tau=F*R=6.2*0.34=2.1$ N m

6.4 cm^2

1.8 cm^2

R=0.34 m
coef of friction: 0.5

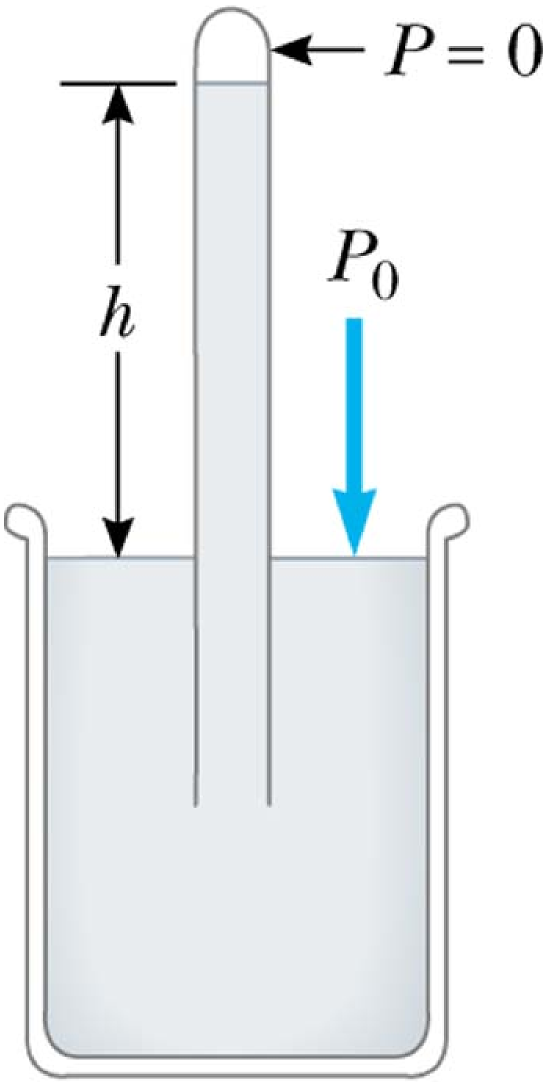
Pressure measurement



The open-tube manometer.
The pressure at A and B is the same:
$P=P_0+\rho gh$
so $h=(P-P_0)/(\rho g)$

If the pressure P=1.01 atm, what is h? (the liquid is water)
h=(1.01-1)*(1.0E+05)/(1.0E+03*9.81)=
=0.1 m

Pressure Measurement: the mercury barometer



$P_0 = \rho_{mercury} g h$

$\rho_{mercury} = 13.6E+03$ kg/m^3

$\rho_{mercury,specific} = 13.6$

Pressure at different altitudes

The pressure in the lecture room equals 1 atm (1.013E+05 Pa). What will the pressure on the 6th floor of the building be (h=20 m)? And at the top of mount Everest (h=8500 m)?

Just like the case for a fluid, the pressure changes with depth (height in the case of air).

$P_{lecture\ room}=P_{6th\ floor}+\rho_{air}gh$

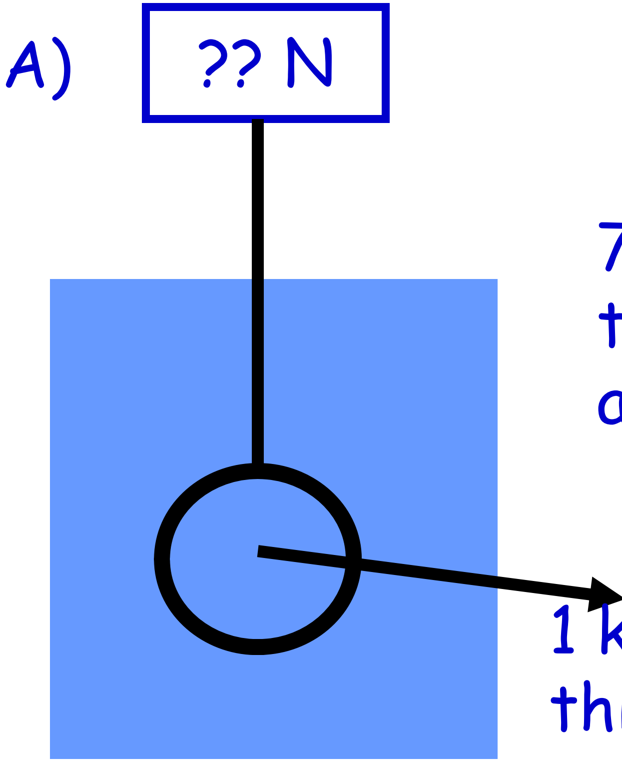
$P_{6th\ floor}=P_{lecture\ room}-\rho_{air}gh=1.013E+05-1.29*9.81*20=$

$=1.010+05$ Pa (3 promille change)

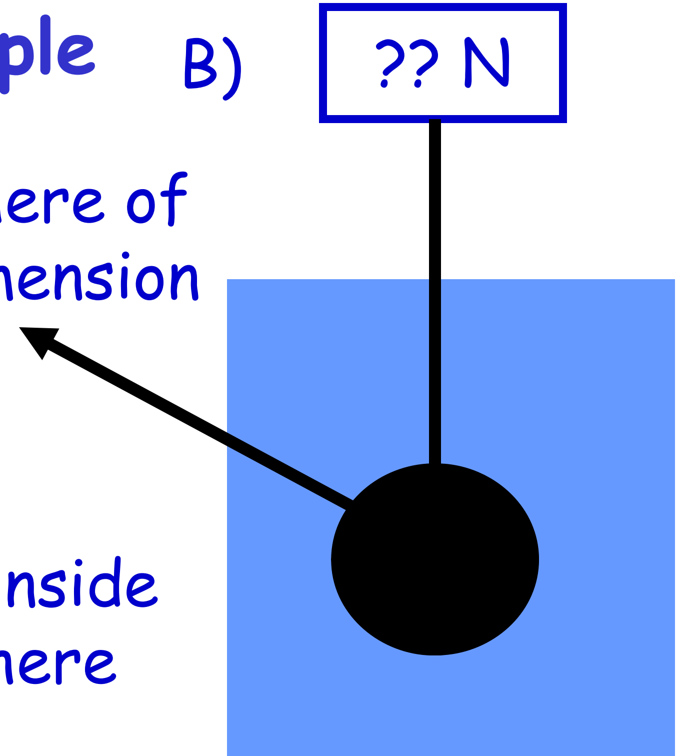
$P_{mount\ everest}=1.013E+05-1.29*9.81*8500=-6.3E+03$ Pa????

The density of air changes with altitude, and so the equation does not hold; it is very easy to compress air (small bulk modulus) compared to e.g. water.

An example



1 kg of water inside thin hollow sphere



7 kg iron sphere of the same dimension as in A)

Two weights of equal size and shape, but different mass are submerged in water. What are the weight read out?

$B = \rho_{water} V_{displaced} g \quad w = \rho_{sphere} V_{sphere} g$

A) $B = \rho_{water} V_{sphere} g \quad w = \rho_{water} V_{sphere} g$ so B=w and 0 N is read out!

B) $B = \rho_{water} V_{sphere} g = M_{water\ sphere}$

$w = \rho_{iron} V_{sphere} g = M_{iron\ sphere} g = 7 M_{water\ sphere} g$

$T = w - B = 6*1*9.8$

$= 58.8$ N

Another one

An air mattress 2 m long 0.5 m wide and 0.08 m thick and has a mass of 2.0 kg. A) How deep will it sink in water? B) How much weight can you put on top of the mattress before it sinks? ρ_{water}=1.0E+03 kg/m^3

A) $h = \rho_{object}V_{object}/(\rho_{water}A)$

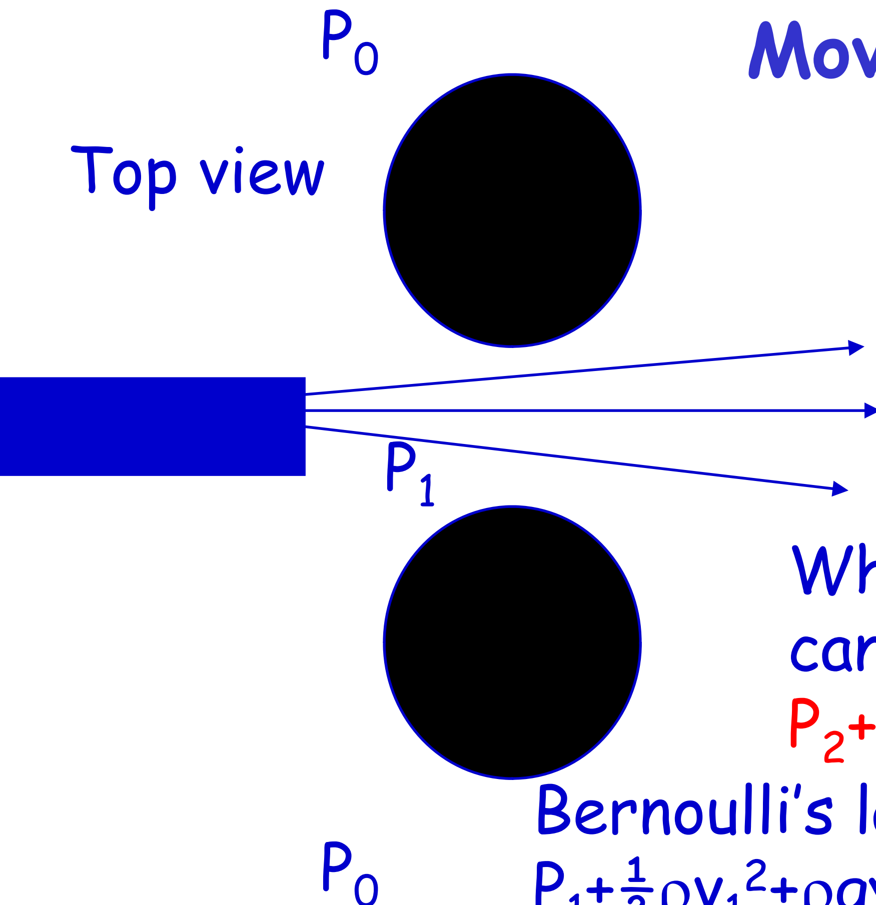
$h = M_{object}/(1.0E{+}03*2*0.5) = 2.0/1.0E{+}03 = 2.0E{-}03$ m=2 mm

B) if the objects sinks the mattress is just completely submerged: h=thickness of mattress.

$0.08 = (M_{weight}+2.0)/(1.0E{+}03*2*0.5)$

So M_{weight}=78 kg

Moving cans



Before air is blown in between the cans, $P_0 = P_1$; the cans remain at rest and the air in between the cans is at rest (0 velocity)

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_o$

When air is blown in between the cans, the velocity is not equal to 0.

$P_2 + \frac{1}{2}\rho v_2^2$

Bernoulli's law:

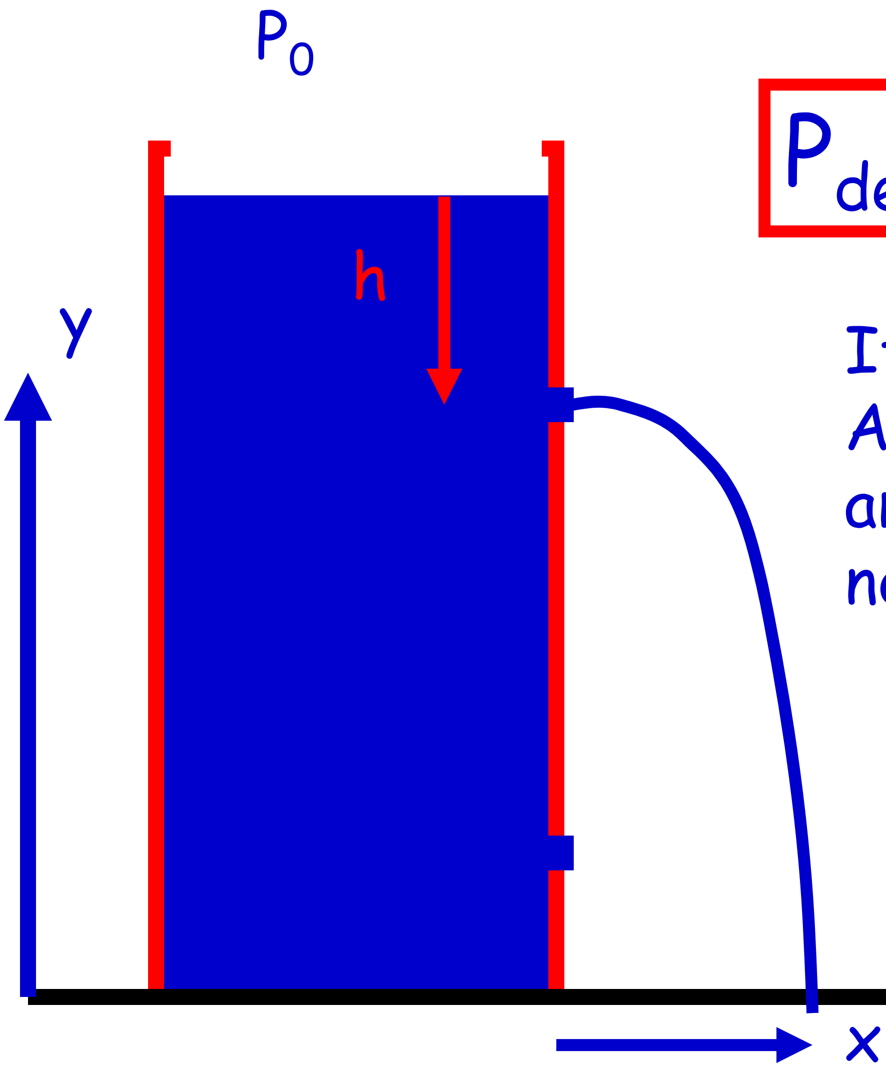
$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

$P_0 = P_2 + \frac{1}{2}\rho v_2^2$ so $P_2 = P_0 - \frac{1}{2}\rho v_2^2$

So $P_2 < P_0$

Because of the pressure difference left and right of each can, they move inward

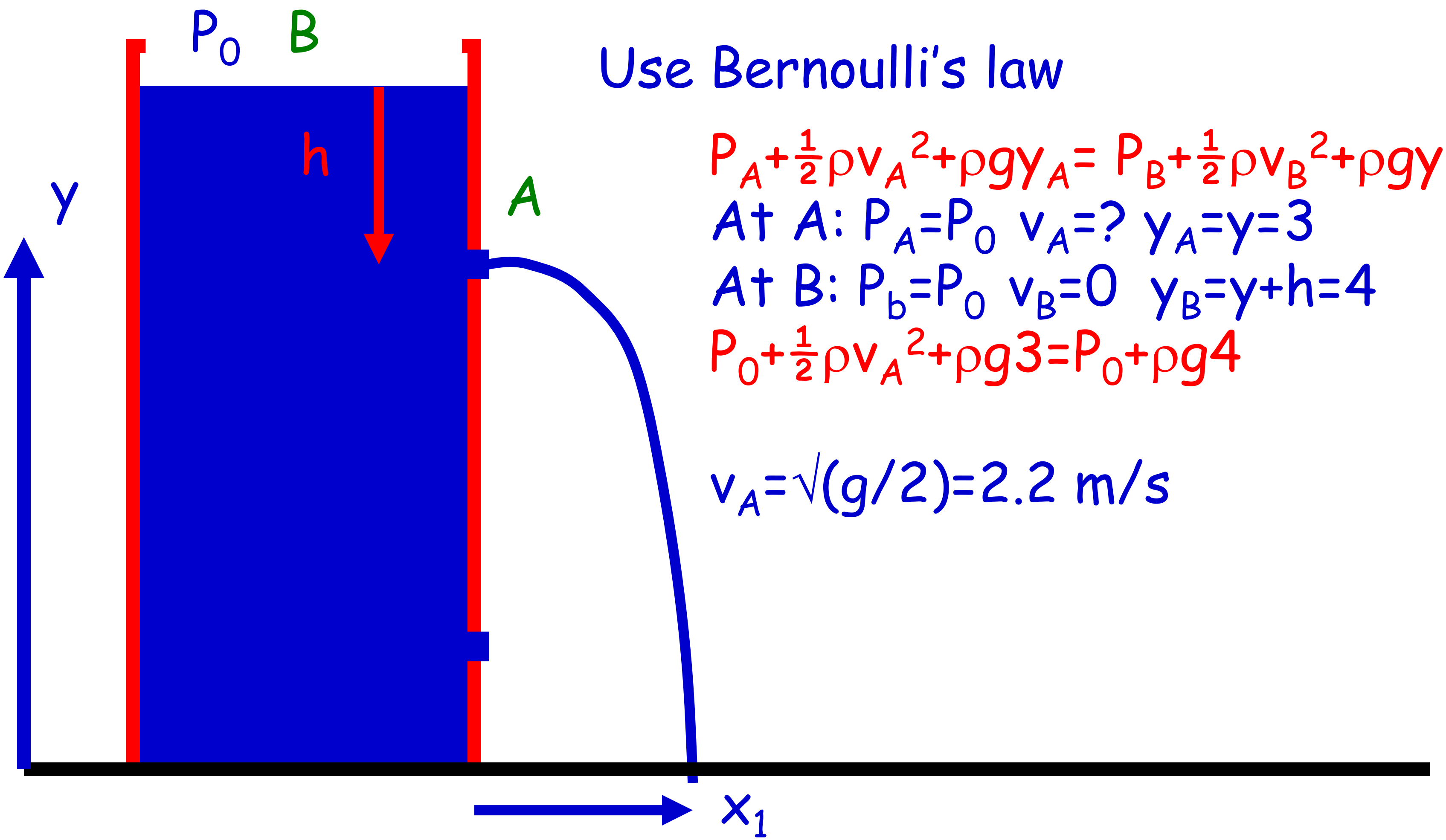
How far does the water go?



$$\boxed{P_{depth=h} = P_{depth=0} + \rho g h}$$

If h=1m & y=3m what is x? Assume that the holes are small and the water level doesn't drop noticeably.

If h=1m and y=3m what is X?



Use Bernoulli's law

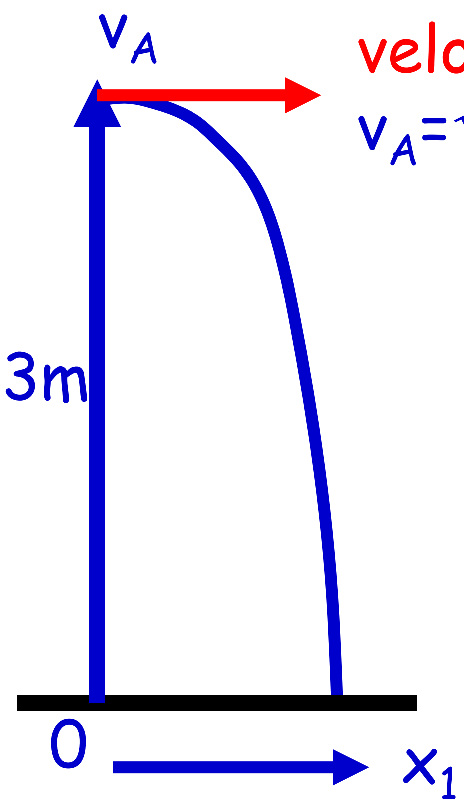
$P_A+\frac{1}{2}\rho v_A^2+\rho g y_A= P_B+\frac{1}{2}\rho v_B^2+\rho g y_B$

At A: $P_A=P_0$ $v_A=?$ $y_A=y=3$

At B: $P_b=P_0$ $v_B=0$ $y_B=y+h=4$

$P_0+\frac{1}{2}\rho v_A^2+\rho g3=P_0+\rho g4$

$v_A=\sqrt{(g/2)}=2.2$ m/s



Each water element of mass m has the same velocity v_A. Let's look at one element m.

$v_A=\sqrt{(g/2)}=2.2$ m/s

In the horizontal direction:

$x(t)=x_0+v_{0x}t+\frac{1}{2}at^2=2.2t$

In the vertical direction:

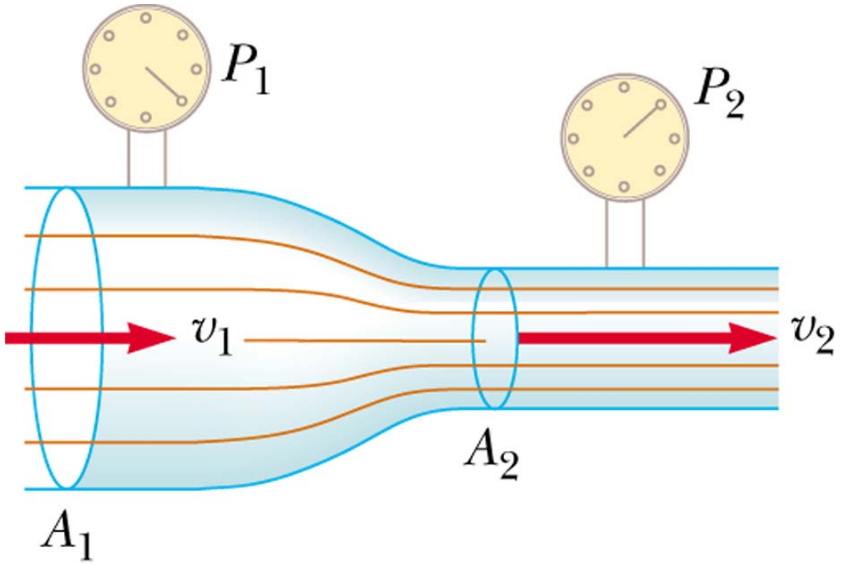
$y(t)=y_0+v_{0y}t+\frac{1}{2}at^2=3-0.5gt^2$

$= 0$ when the water hits the ground, so

$t=0.78$ s

so $x(0.78)=2.2*0.78=1.72$ m

Fluid flow



(a)

$$P_1+\tfrac{1}{2}\rho v_1^2+\rho g y_1= P_2+\tfrac{1}{2}\rho v_2^2+\rho g y_2$$
$$P_1+\tfrac{1}{2}\rho v_1^2 = P_2+\tfrac{1}{2}\rho v_2^2$$
$$(v_2^2-v_1^2)=2(P_1-P_2)/\rho$$

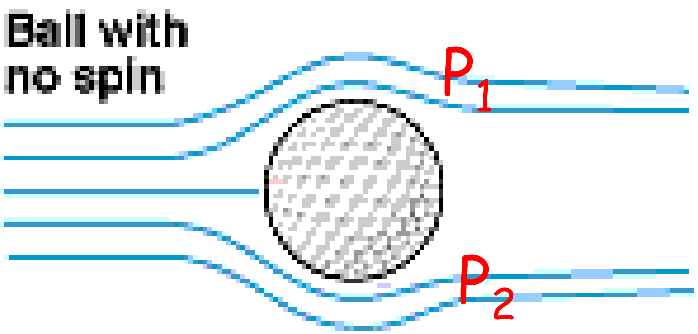
If $P_1=4.0*10^5$ Pa, $P_2=2.0*10^5$ Pa and by counting the amount of water coming from the right v_2 is found to be 30 m/s, what is v_1? (ρ=1E+03 kg/m^3)

$900-v_1^2=2*(2.0E+5)/(1E+03)$ $v_1=22.3$ m/s

Applications of Bernoulli's law: the golf ball

Neglecting the small change in height between the top and bottom of the golf ball:

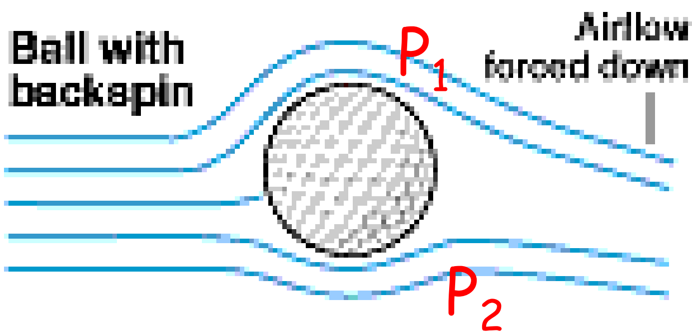
$$P_1+\tfrac{1}{2}\rho v_1^2= P_2+\tfrac{1}{2}\rho v_2^2$$
$$P_1-P_2= \tfrac{1}{2}\rho(v_2^2- v_1^2)$$



$P_1-P_2= \tfrac{1}{2}\rho(v_2^2- v_1^2)=0$

$v_2=v_1$

No pressure difference, no lift



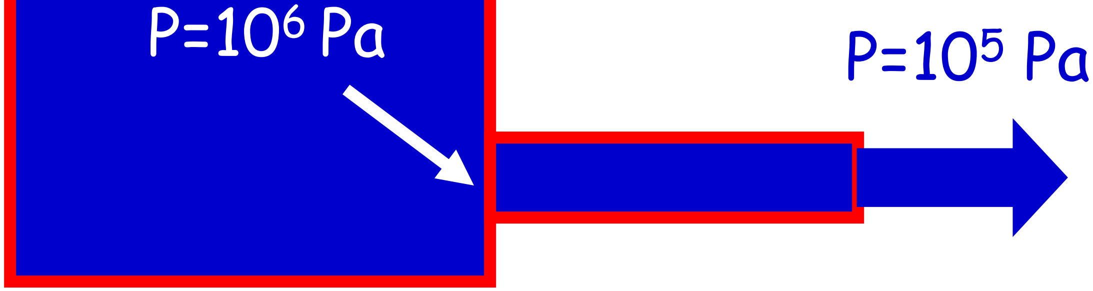
$P_1-P_2= \tfrac{1}{2}\rho(v_2-\mathbf{v})^2-(v_1+ \mathbf{v})^2=0$

$P_2>P_1$ so:

Upward force: the ball goes higher and thus travels faster

Example

Flow rate $Q=0.5\ m^3/s$
Tube length: 3 m
$\eta=1500E\text{-}03\ Ns/m^2$



What should the radius of the tube be?

$$\text{Rate of flow } Q = \frac{\pi R^4(P_1-P_2)}{8\eta L}$$

$$R=[8Q\eta L/(\pi(P_1-P_2))]^{1/4}=0.05\ m$$