TABLE 11.1 Specific Heats of Some Materials at Atmospheric Pressure

Substance	J/kg · °C	cal/g · °C
Aluminum	900	0.215
Beryllium	1 820	0.436
Cadmium	230	0.055
Copper	387	0.092 4
Germanium	322	0.077
Glass	837	0.200
Gold	129	0.030 8
Ice	2 090	0.500
Iron	448	0.107
Lead	128	0.030 5
Mercury	138	0.033
Silicon	703	0.168
Silver	234	0.056
Steam	2 010	0.480
Water	4 186	1.00

Example

A 1 kg block of Copper is raised in temperature by 10 °C. What was the heat transfer Q.?

Answer:

$Q=cm\Delta T$

$=387*1*10=3870$ J

1 cal = 4.186 J

Q=924.5 cal

Another one

A block of Copper is dropped from a height of 10 m. Assuming that all the potential energy is transferred into internal energy (heat) when it hits the ground, what is the raise in temperature of the block (c_{copper}=387 J/(kg ^{o}C))?

Potential energy: mgh=10 mg J

All transferred into heat Q: $Q = cm\Delta T$

$10mg = 387m\Delta T$

$\Delta T = 10\,g/387 = 0.25\ ^{o}C$

An example

The contents of a can of soda (0.33 kg) which is cooled to 4 ^oC is poured into a glass (0.1 kg) that is at room temperature (20 ^0C). What will the temperature of the filled glass be after it has reached full equilibrium (glass and liquid have the same temperature)?

Given c_{water}=4186 J/(kg ^oC) and c_{glass}=837 J/(kg ^0C)

$$Q_{cold} = -Q_{hot}$$

$$m_{water} c_{water} (T_{final} - T_{water}) = -m_{glass} c_{glass} (T_{final} - T_{glass})$$

$$T_{final} = \frac{m_{water} c_{water} T_{water} + m_{glass} c_{glass} T_{glass}}{m_{water} c_{water} + m_{glass} c_{glass}}$$

= (0.33*4186*4+0.1*837*20)/(0.33*4186+0.1*837)=

= 4.9 ^oC

And another

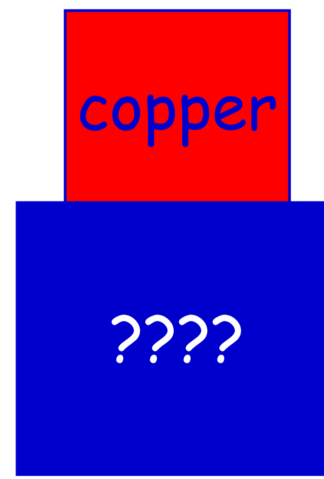
A block of unknown substance with a mass of 8 kg, initially at T=280 K is thermally connect to a block of copper (5 kg) that is at T=320 K (c_{copper}=0.093 cal/g ^{0}C). After the system has reached thermal equilibrium the temperature T equals 290 K. What is the specific heat of the unknown material in cal/g ^{o}C?

$$Q_{cold} = -Q_{hot}$$

$$m_{unknown} c_{unknown} (T_{final} - T_{unknown}) = -m_{copper} c_{copper} (T_{final} - T_{copper})$$

$$c_{unknown} = \frac{-m_{copper} c_{copper} (T_{final} - T_{copper})}{m_{unknown} (T_{final} - T_{unknown})}$$

$$c_{unkown} = \frac{-5000 \cdot 0.093 \cdot (290-320)}{8000 \cdot (290-280)} = 0.17 \text{ cal/g } ^oC$$



Heating water with a ball of Lead

A ball of Lead at T=100 °C with mass 300 g is dropped in a glass of water (0.3 L) at T=20 °C. What is the final (after thermal equilibrium has occurred) temperature of the system? (c_{water}=1 cal/g °C, c_{lead}=0.03 cal/g °C $\rho_{water}=10^3$ kg/m^3)

$$Q_{cold} = -Q_{hot}$$

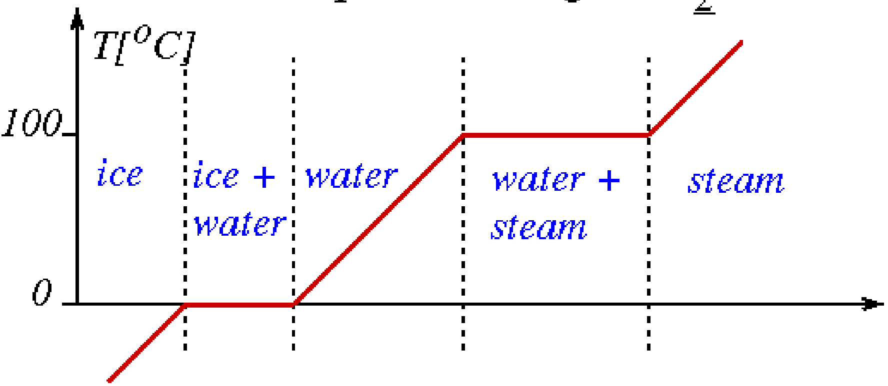
$$m_{water} c_{water} (T_{final} - T_{water}) = -m_{lead} c_{lead} (T_{final} - T_{lead})$$

$$T_{final} = \frac{m_{water} c_{water} T_{water} + m_{lead} c_{lead} T_{lead}}{m_{water} c_{water} + m_{lead} c_{lead}}$$

= (0.3*1*20+0.3*0.03*100)/(0.3*1+0.3*0.03)=

= 6.9/0.309=22.3°C

Example: Heating of H_2O

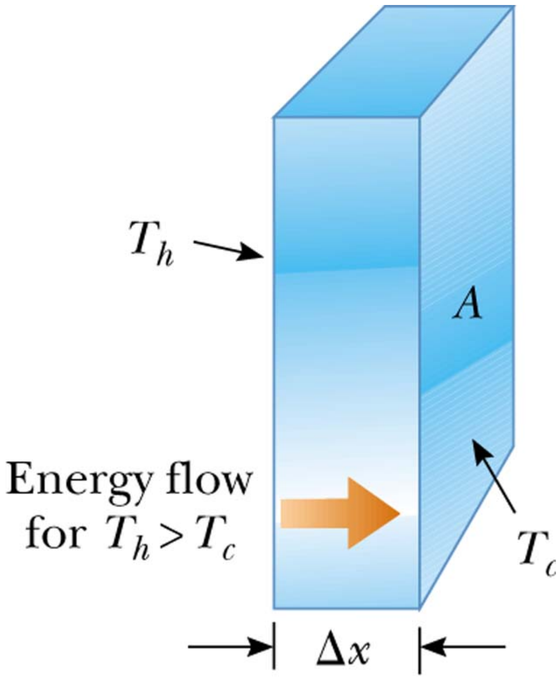


(a)	ice	$Q = m\, c_{ice} \Delta T$	raises T of ice
(b)	ice+water	$Q = m\, L_f$	melts ice
(c)	water	$Q = m\, c_{water} \Delta T$	raises T of water
(d)	water + steam	$Q = m\, L_v$	vaporizes water
(e)	steam	$Q = m\, c_{steam} \Delta T$	raises T of steam

Ice with T=-30 °C is heated to steam of T=150 °C.
How many heat (in cal) has been added in total?

c_{ice}=0.5 cal/g °C
c_{water}=1.0 cal/g °C
c_{steam}=0.480 cal/g °C
L_f=540 cal/g
L_v=79.7 cal/g
m=1 kg=1000g

A) Ice from -30 to 0 °C	Q=1000*0.5*30= 15000 cal
B) Ice to water	Q=1000*540= 540000 cal
C) water from 0 °C to 100 °C	Q=1000*1.0*100=100000 cal
D) water to steam	Q=1000*79.7= 79700 cal
E) steam from 100 °C to 150 °C	Q=1000*0.48*50=24000 cal
TOTAL	Q= =758700 cal



Example

A glass window (A=4 m^2,Δx=0.5 cm) separates a living room (T=20 ^{0}C) from the outside (T=0 ^{o}C). A) What is the rate of heat transfer through the window (k_{glass}=0.84 J/(m.s.^{o}C))? B) By what fraction does it change if the surface becomes 2x smaller and the temperature drops to -20 ^{0}C?

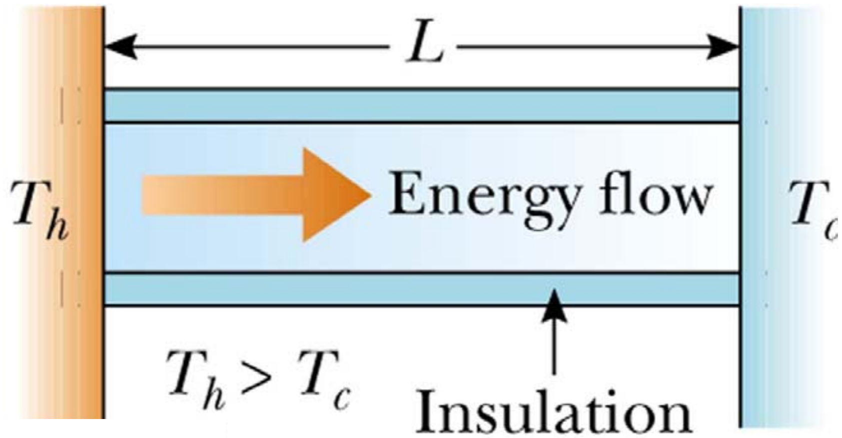
A) $P = kA\Delta T/\Delta x = 0.84*4*20/0.005 = 13440$ Watt

B) $P_{orig} = kA\Delta T/\Delta x \quad P_{new} = k(0.5A)(2\Delta T)/\Delta x = P_{orig}$

The heat transfer is the same

Another one.

Heat reservoir



Heat sink

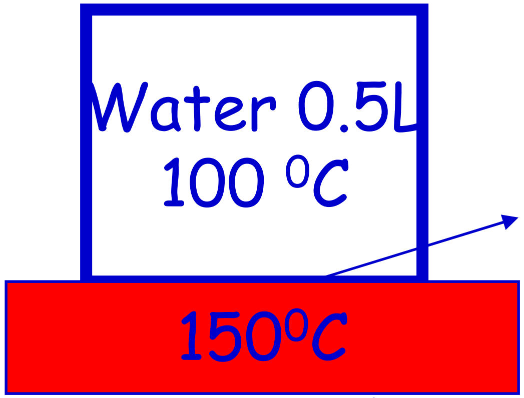
An insulated gold wire (i.e. no heat lost to the air) is at one end connected to a heat reservoir (T=100 ^{0}C) and at the other end connected to a heat sink (T=20 ^{0}C). If its length is 1m and P=200 W what is its cross section (A)?

k_{gold}=314 J/(m.s.^{0}C).

$P=kA\Delta T/\Delta x=314*A*80/1=25120*A=200$

$A=8.0E-03\ m^2$

And another



A=0.03 m^2 thickness: 0.5 cm.

A student working for his exam feels hungry and starts boiling water (0.5L) for some noodles. He leaves the kitchen when the water just boils.The stove's temperature is 150 ^0C. The pan's bottom has dimensions given above. Working hard on the exam, he only comes back after half an hour. Is there still water in the pan? (L_v=540 cal/g, k_{pan}=1 cal/(m.s.^0C))

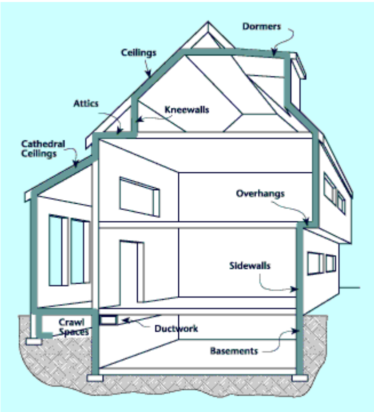
To boil away 0.5L (=500 g) of water: $Q=L_v*500=270000$ cal

Heat added by the stove: $P=kA\Delta T/\Delta x=1*0.03*50/0.005=$

$=300$ cal

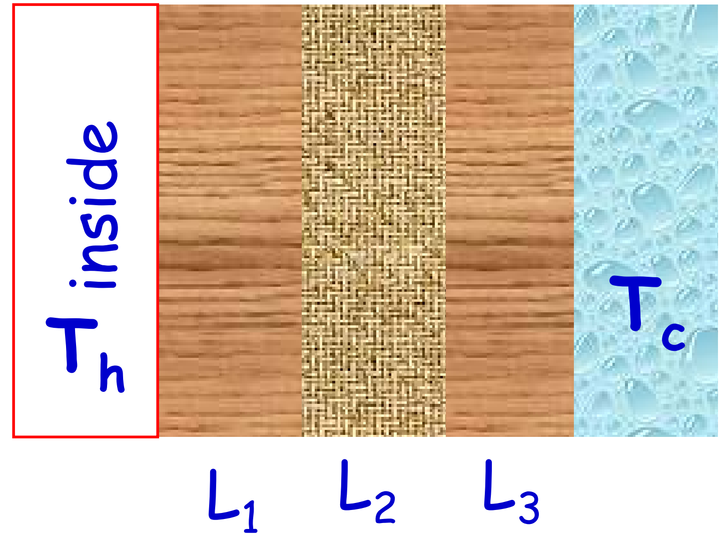
$P=Q/\Delta t$ $\Delta t=Q/P=270000/300=900$ s (15 minutes)

He'll be hungry for a bit longer…



Isolation

$$P = \frac{Q}{\Delta t} = \frac{A(T_h - T_c)}{\sum_i (L_i / k_i)}$$



A house is built with 10 cm thick wooden walls and roofs. The owner decides to install insulation. After installation the walls and roof are 4 cm wood+2 cm isolation+4 cm wood. If k_{wood}=0.10 J/(m.s.^{0}C) and $k_{isolation}$=0.02 J/(m.s.^{0}C), by what factor does he reduce his heating bill?

$P_{before}=A\Delta T/[0.10/0.10]=A\Delta T$

$P_{after}=A\Delta T/[0.04/0.10+0.02/0.02+0.04/0.10]=0.55A\Delta T$

Almost a factor of 2 (1.81)!

Radiation

Nearly all objects emit energy through radiation:

$P=\sigma AeT^4$: Stefan's law (J/s)

$\sigma=5.6696\times10^{-8}$ $W/m^2.K^4$

A: surface area

e: object dependent constant emissivity (0-1)

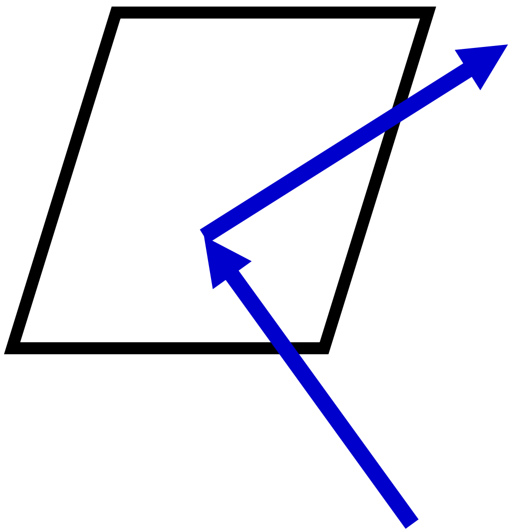
T: temperature (K)

P: energy radiated per second.

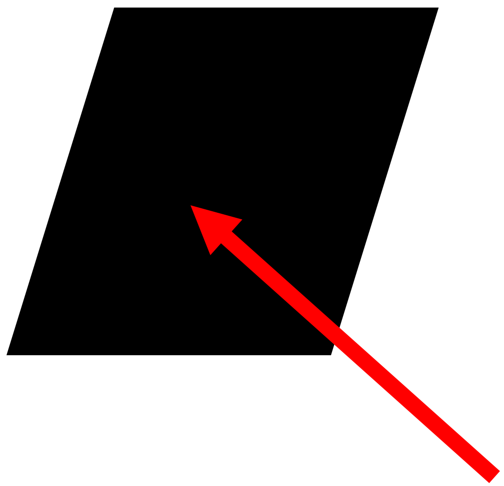
If an object is at Temperature T and its surroundings are at T_0, then the net energy gained/lost is:

$P=\sigma Ae(T^4-T_o^4)$

emissivity



Ideal reflector
e=0
no energy is absorbed



Ideal absorber (black body)
e=1
all energy is absorbed
also ideal radiator!

A BBQ

The coal in a BBQ cover an area of 0.25 m^2. If the emissivity of the burning coal is 0.95 and their temperature 500 ^0C, how much energy is radiated every minute?

$P=\sigma AeT^4$ J/s

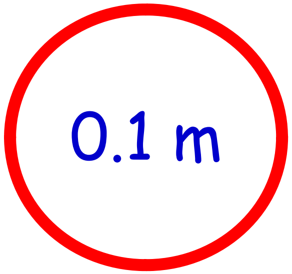
$=5.67\times10^{-8}*0.25*0.95*(773)^4=4808$ J/s

1 minute: 2.9×10^5 J (to cook 1 L of water 3.3×10^5 J)

Metal hoop

A metal (thermal expansion coefficient α=17×10^{-6} /^0C) hoop of radius 0.10 m is heated from 20 ^0C to 100 ^0C. By how much does its radius change?

$$\Delta L = \alpha L_0 \Delta T$$
$$= 17\times10^{-6}(2\pi r_0)80 = 8.5\times10^{-4}\text{ m}$$
$$r_{new} = (L_0+\Delta L)/2\pi = L_0/2\pi + \Delta L/2\pi = r_0 + 1.35\times10^{-4}\text{ m}$$



Diving Bell

A cylindrical diving bell (diameter 3 m and 4 m tall, with an open bottom is submerged to a depth of 220 m in the sea. The surface temperature is 25 ^0C and at 220 m, T=5 ^0C. The density of sea water is 1025 kg/m^3. How high does the sea water rise in the bell when it is submerged?

Consider the air in the bell.

$P_{surf}=1.0\text{x}10^5$Pa $V_{surf}=\pi r^2 h=28.3$ m^3 $T_{surf}=25+273=298$ K

$P_{sub}=P_0+\rho_w g*\text{depth}=2.3\text{x}10^6$ Pa $V_{sub}=?$ $T_{sub}=5+273=278$ K

Next, use PV/T=constant

$P_{surf}V_{surf}/T_{surf}=P_{sub}V_{sub}/T_{sub}$ plug in the numbers and find:

$V_{sub}=1.15$ m^3 (this is the amount of volume taken by the air left)

$V_{\text{taken by water}}=28.3-1.15=27.15$ m^3= $\pi r^2 h$

$h=27.15/\pi r^2=3.8$ m rise of water level in bell.

Moles

Two moles of Nitrogen gas (N_2) are enclosed in a cylinder with a moveable piston. A) If the temperature is 298 K and the pressure is 1.01×10^6 Pa, what is the volume (R=8.31 J/mol.K)?

b) What is the average kinetic energy of the molecules?

$k_B=1.38\times10^{-23}$ J/K

A) $PV=nRT$

$V=nRT/P$

$=2*8.31*298/1.01\times10^6=4.9E-03\ m^3$

B) $E_{kin,average}=\frac{1}{2}m\overline{v^2}=3/2k_BT=3/2*1.38\times10^{-23}*298=6.2\times10^{-21}$ J

$3*10^3$ J of heat is transferred to a 1 cm^3 cube of gold at T=20 °C. Will all the gold have melted afterwards?

$L_f = 6.44 \times 10^4$ J/kg $T_{melt} = 1063$ °C $c_{specific} = 129$ J/kg ^{0}C

$\rho = 19.3 \times 10^3$ kg/m^3.

When solid: $m = \rho V = (19.3 \times 10^3) \times (1 \times 10^{-6}\ m^3) = 1.93 \times 10^{-2}$ kg.

$Q = cm\Delta T = 129 * 1.93 \times 10^{-2} * \Delta T = 2.5\Delta T$

To raise to melting point: $\Delta T = (1063-20) = 1043$ °C, so Q=2608 J

During the phase change from solid to liquid: $Q = L_f m$

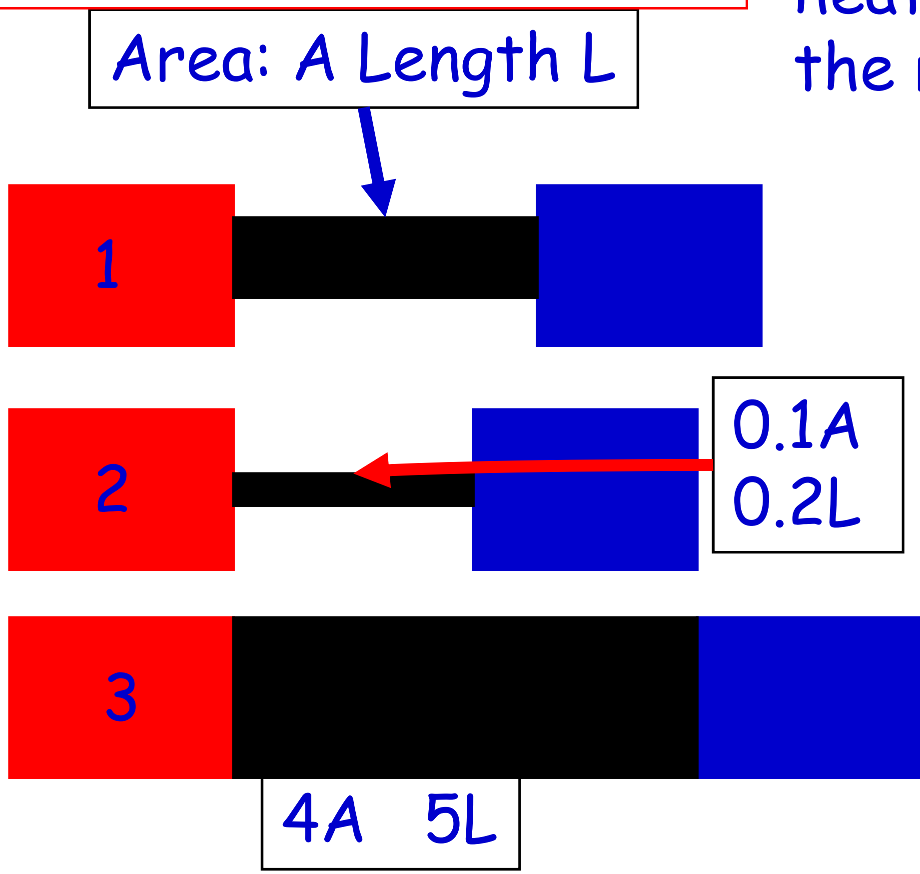
$Q = 6.44 \times 10^4 * 1.93 \times 10^{-2} = 1.24 \times 10^3$ J to make liquid.

Total needed: 3850 J, only 3000 J available, doesn't melt completely.

Heat transfer

A hot block and a cold block are thermally connected. Three different methods to transfer heat are proposed as shown. Which one is the most efficient way (fastest) to transfer heat from hot to cold and what are the relative rates of transfer?

A: cross section surface of black wire,L:its length



Use: $P=kA\Delta T/L$

Case 1: $P \sim A/L$

Case 2: $P \sim 0.1A/0.2L=0.5A/L$

Case 3: $P \sim 4A/5L=0.8L$

$P_1:P_2:P_3$ = 1:0.5:0.8

First case is most efficient.

Thermal equilibrium

20 g of a solid at 70 ^{0}C is placed in 100 g of a fluid at 20 ^{o}C. After waiting a while the temperature of the whole system is 30 ^{o}C and stays that way. The specific heat of the solid is:

a) Equal to that of the fluid
b) Less than that of the fluid
c) Larger than that of the fluid
d) Unknown; different phases cannot be compared
e) Unknown; different materials cannot be compared

$$Q_{fluid} = -Q_{solid}$$

$$m_{fluid} c_{fluid} (T_{final} - T_{fluid}) = -m_{solid} c_{solid} (T_{final} - T_{solid})$$

$$\frac{c_{fluid}}{c_{solid}} = \frac{-m_{solid}(T_{final} - T_{solid})}{m_{fluid}(T_{final} - T_{fluid})} = \frac{-20(30-70)}{100(30-20)} = 0.8$$

$$c_{solid} > c_{fluid}$$