

Example

A 1 kg block of Copper is raised in temperature by 10 °C. What was the heat transfer Q.?

Answer:

$$Q = cm\Delta T$$

$$= 387 * 1 * 10 = 3870 \text{ J}$$

$$1 \text{ cal} = 4.186 \text{ J}$$

$$Q = 924.5 \text{ cal}$$

TABLE 11.1

Specific Heats of Some Materials at Atmospheric Pressure

Substance	J/kg · °C	cal/g · °C
Aluminum	900	0.215
Beryllium	1 820	0.436
Cadmium	230	0.055
Copper	387	0.092 4
Germanium	322	0.077
Glass	837	0.200
Gold	129	0.030 8
Ice	2 090	0.500
Iron	448	0.107
Lead	128	0.030 5
Mercury	138	0.033
Silicon	703	0.168
Silver	234	0.056
Steam	2 010	0.480
Water	4 186	1.00

Another one

A block of Copper is dropped from a height of 10 m. Assuming that all the potential energy is transferred into internal energy (heat) when it hits the ground, what is the raise in temperature of the block ($c_{\text{copper}}=387 \text{ J}/(\text{kg } ^\circ\text{C})$)?

Potential energy: $mgh=10 \text{ mg J}$

All transferred into heat Q: $Q = cm\Delta T$

$$10mg = 387m\Delta T$$

$$\Delta T = 10 \text{ g}/387 = 0.25 \text{ } ^\circ\text{C}$$

An example

The contents of a can of soda (0.33 kg) which is cooled to 4 °C is poured into a glass (0.1 kg) that is at room temperature (20 °C). What will the temperature of the filled glass be after it has reached full equilibrium (glass and liquid have the same temperature)?

Given $c_{\text{water}}=4186 \text{ J}/(\text{kg } ^\circ\text{C})$ and $c_{\text{glass}}=837 \text{ J}/(\text{kg } ^\circ\text{C})$

$$Q_{\text{cold}} = -Q_{\text{hot}}$$
$$m_{\text{water}}c_{\text{water}}(T_{\text{final}} - T_{\text{water}}) = -m_{\text{glass}}c_{\text{glass}}(T_{\text{final}} - T_{\text{glass}})$$

$$T_{\text{final}} = \frac{m_{\text{water}}c_{\text{water}}T_{\text{water}} + m_{\text{glass}}c_{\text{glass}}T_{\text{glass}}}{m_{\text{water}}c_{\text{water}} + m_{\text{glass}}c_{\text{glass}}}$$

$$= (0.33 \cdot 4186 \cdot 4 + 0.1 \cdot 837 \cdot 20) / (0.33 \cdot 4186 + 0.1 \cdot 837) =$$
$$= 4.9 \text{ } ^\circ\text{C}$$

And another

A block of unknown substance with a mass of 8 kg, initially at $T=280$ K is thermally connect to a block of copper (5 kg) that is at $T=320$ K ($c_{\text{copper}}=0.093$ cal/g $^{\circ}\text{C}$). After the system has reached thermal equilibrium the temperature T equals 290 K. What is the specific heat of the unknown material in cal/g $^{\circ}\text{C}$?

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$m_{\text{unknown}} c_{\text{unknown}} (T_{\text{final}} - T_{\text{unknown}}) = -m_{\text{copper}} c_{\text{copper}} (T_{\text{final}} - T_{\text{copper}})$$

$$c_{\text{unknown}} = \frac{-m_{\text{copper}} c_{\text{copper}} (T_{\text{final}} - T_{\text{copper}})}{m_{\text{unknown}} (T_{\text{final}} - T_{\text{unknown}})}$$

$$c_{\text{unkown}} = \frac{-5000 \cdot 0.093 \cdot (290 - 320)}{8000 \cdot (290 - 280)} = 0.17 \text{ cal/g } ^{\circ}\text{C}$$



copper

????

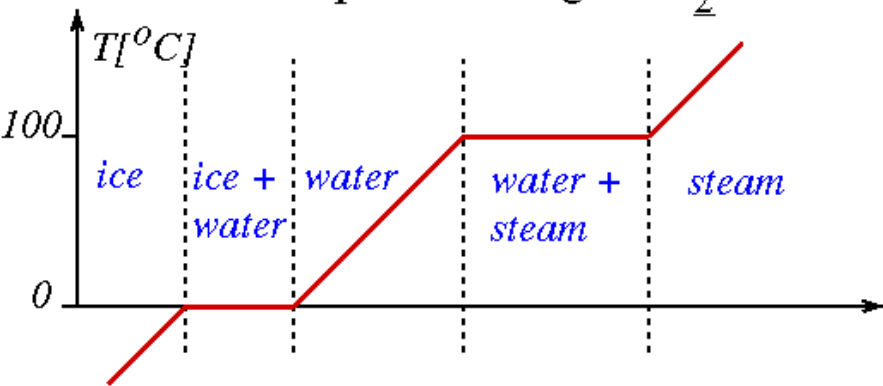
Heating water with a ball of Lead

A ball of Lead at $T=100\text{ }^{\circ}\text{C}$ with mass 300 g is dropped in a glass of water (0.3 L) at $T=20\text{ }^{\circ}\text{C}$. What is the final (after thermal equilibrium has occurred) temperature of the system? ($c_{\text{water}}=1\text{ cal/g }^{\circ}\text{C}$, $c_{\text{lead}}=0.03\text{ cal/g }^{\circ}\text{C}$ $\rho_{\text{water}}=10^3\text{ kg/m}^3$)

$$Q_{\text{cold}} = -Q_{\text{hot}}$$
$$m_{\text{water}}c_{\text{water}}(T_{\text{final}} - T_{\text{water}}) = -m_{\text{lead}}c_{\text{lead}}(T_{\text{final}} - T_{\text{lead}})$$
$$T_{\text{final}} = \frac{m_{\text{water}}c_{\text{water}}T_{\text{water}} + m_{\text{lead}}c_{\text{lead}}T_{\text{lead}}}{m_{\text{water}}c_{\text{water}} + m_{\text{lead}}c_{\text{lead}}}$$

$$= (0.3 \cdot 1 \cdot 20 + 0.3 \cdot 0.03 \cdot 100) / (0.3 \cdot 1 + 0.3 \cdot 0.03) =$$
$$= 6.9 / 0.309 = 22.3^{\circ}\text{C}$$

Example: Heating of H₂O



- | | | |
|--------------------------|----------------------------|-------------------|
| (a) <i>ice</i> | $Q = m c_{ice} \Delta T$ | raises T of ice |
| (b) <i>ice+water</i> | $Q = m L_f$ | melts ice |
| (c) <i>water</i> | $Q = m c_{water} \Delta T$ | raises T of water |
| (d) <i>water + steam</i> | $Q = m L_v$ | vaporizes water |
| (e) <i>steam</i> | $Q = m c_{steam} \Delta T$ | raises T of steam |

A) Ice from -30 to 0 °C

B) Ice to water

C) water from 0 °C to 100 °C

D) water to steam

E) steam from 100 °C to 150 °C

TOTAL

Ice with $T = -30$ °C is heated to steam of $T = 150$ °C.

How many heat (in cal) has been added in total?

$$c_{ice} = 0.5 \text{ cal/g } ^\circ\text{C}$$

$$c_{water} = 1.0 \text{ cal/g } ^\circ\text{C}$$

$$c_{steam} = 0.480 \text{ cal/g } ^\circ\text{C}$$

$$L_f = 540 \text{ cal/g}$$

$$L_v = 79.7 \text{ cal/g}$$

$$m = 1 \text{ kg} = 1000\text{g}$$

$$Q = 1000 * 0.5 * 30 = 15000 \text{ cal}$$

$$Q = 1000 * 540 = 540000 \text{ cal}$$

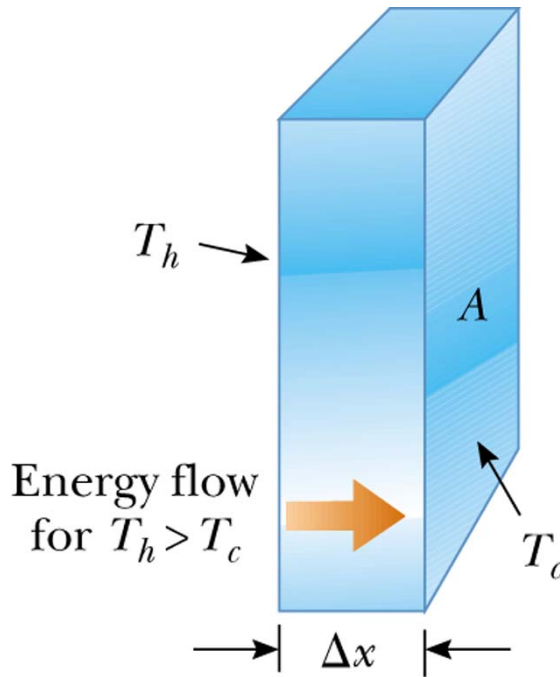
$$Q = 1000 * 1.0 * 100 = 100000 \text{ cal}$$

$$Q = 1000 * 79.7 = 79700 \text{ cal}$$

$$Q = 1000 * 0.48 * 50 = 24000 \text{ cal}$$

$$Q = \underline{\underline{758700 \text{ cal}}}$$

Example



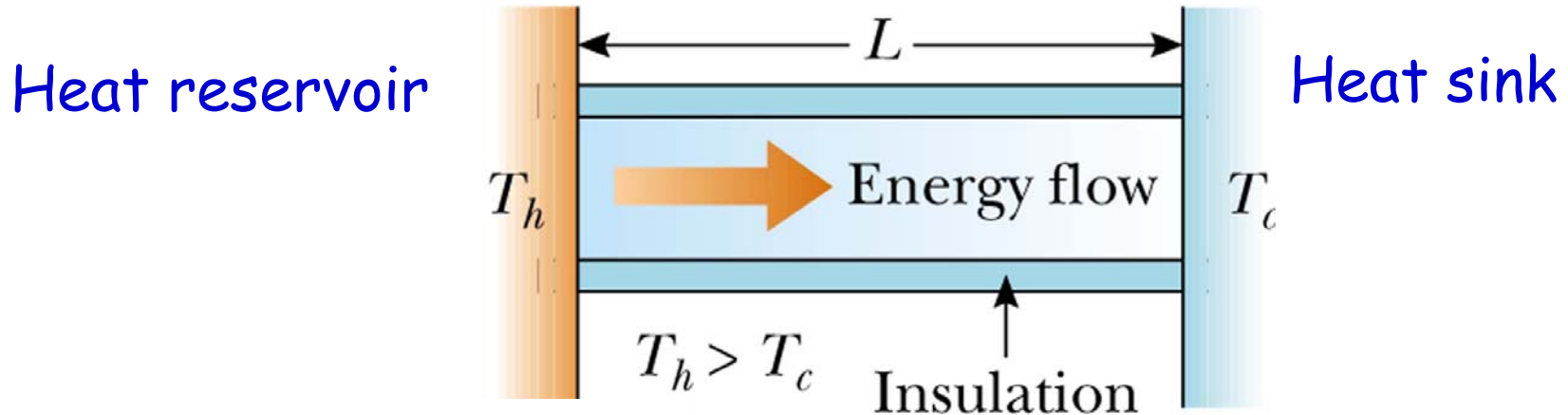
A glass window ($A=4 \text{ m}^2, \Delta x=0.5 \text{ cm}$) separates a living room ($T=20 \text{ }^\circ\text{C}$) from the outside ($T=0 \text{ }^\circ\text{C}$). A) What is the rate of heat transfer through the window ($k_{\text{glass}}=0.84 \text{ J}/(\text{m}\cdot\text{s}\cdot^\circ\text{C})$)? B) By what fraction does it change if the surface becomes 2x smaller and the temperature drops to $-20 \text{ }^\circ\text{C}$?

A) $P = kA\Delta T / \Delta x = 0.84 * 4 * 20 / 0.005 = 13440 \text{ Watt}$

B) $P_{\text{orig}} = kA\Delta T / \Delta x$ $P_{\text{new}} = k(0.5A)(2\Delta T) / \Delta x = P_{\text{orig}}$

The heat transfer is the same

Another one.

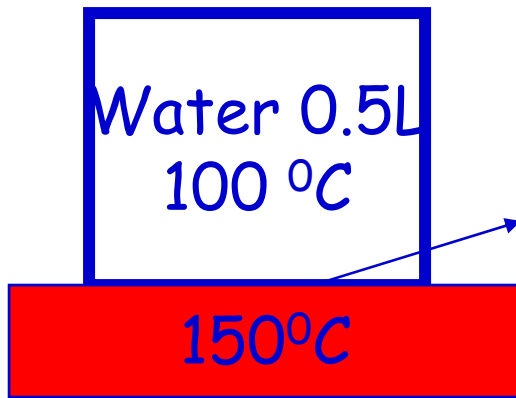


An insulated gold wire (i.e. no heat lost to the air) is at one end connected to a heat reservoir ($T=100\text{ }^{\circ}\text{C}$) and at the other end connected to a heat sink ($T=20\text{ }^{\circ}\text{C}$). If its length is 1m and $P=200\text{ W}$ what is its cross section (A)?

$$k_{\text{gold}}=314\text{ J}/(\text{m}\cdot\text{s}\cdot^{\circ}\text{C}).$$

$$P=kA\Delta T/\Delta x=314*A*80/1=25120*A=200$$

$$A=8.0\text{E}-03\text{ m}^2$$



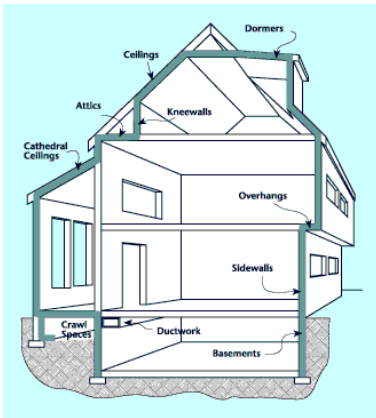
And another
 $A=0.03 \text{ m}^2$ thickness: 0.5 cm.

A student working for his exam feels hungry and starts boiling water (0.5L) for some noodles. He leaves the kitchen when the water just boils. The stove's temperature is 150 °C. The pan's bottom has dimensions given above. Working hard on the exam, he only comes back after half an hour. Is there still water in the pan? ($L_v=540 \text{ cal/g}$, $k_{\text{pan}}=1 \text{ cal}/(\text{m}\cdot\text{s}\cdot^\circ\text{C})$)

To boil away 0.5L (=500 g) of water: $Q=L_v \cdot 500=270000 \text{ cal}$
 Heat added by the stove: $P=kA\Delta T/\Delta x=1 \cdot 0.03 \cdot 50/0.005=$
 $=300 \text{ cal}$

$P=Q/\Delta t$ $\Delta t=Q/P=270000/300=900 \text{ s}$ (15 minutes)

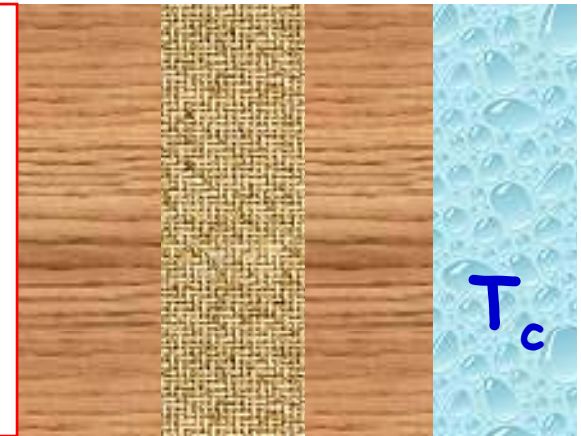
He'll be hungry for a bit longer...



Isolation

$$P = \frac{Q}{\Delta t} = \frac{A(T_h - T_c)}{\sum_i (L_i / k_i)}$$

T_h inside



L_1 L_2 L_3

A house is built with 10 cm thick wooden walls and roofs. The owner decides to install insulation. After installation the walls and roof are 4 cm wood+2 cm isolation+4 cm wood. If $k_{\text{wood}}=0.10 \text{ J}/(\text{m}\cdot\text{s}\cdot^\circ\text{C})$ and $k_{\text{isolation}}=0.02 \text{ J}/(\text{m}\cdot\text{s}\cdot^\circ\text{C})$, by what factor does he reduce his heating bill?

$$P_{\text{before}} = A\Delta T / [0.10/0.10] = A\Delta T$$

$$P_{\text{after}} = A\Delta T / [0.04/0.10 + 0.02/0.02 + 0.04/0.10] = 0.55A\Delta T$$

Almost a factor of 2 (1.81)!