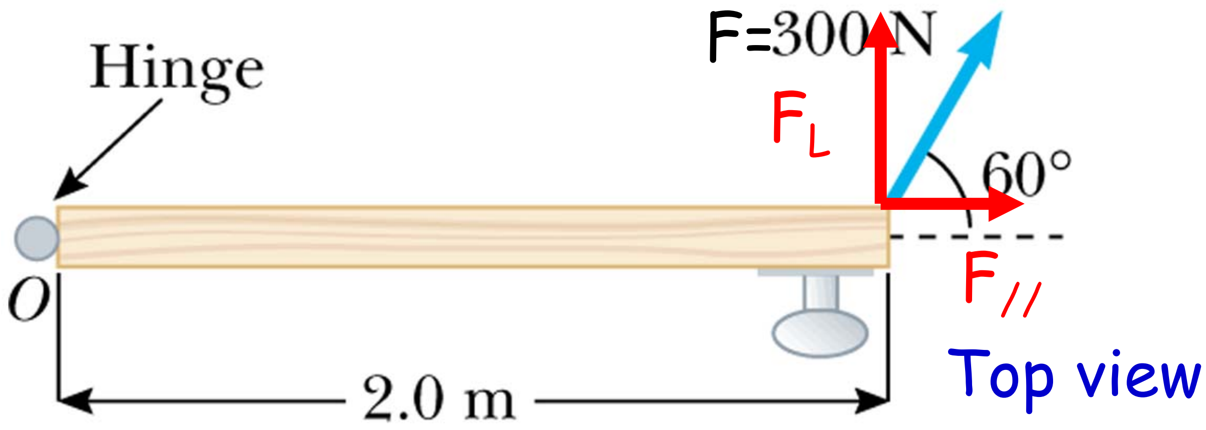
Decompositions



What is the torque applied to the door?

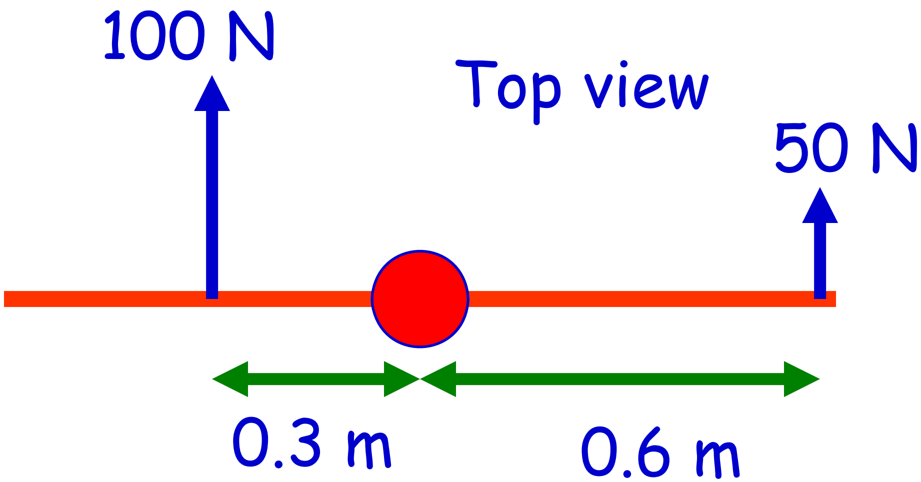
Force parallel to the rotating door: $F_{//}=F\cos 60^0=150$ N

Force perpendicular to rotating door: $F_{\perp}=F\sin 60^0=260$ N

Only $F_{\perp}$ is effective for opening the door:

$\tau=F_{\perp}\cdot d=260*2.0=520$ N m

Multiple forces causing torque



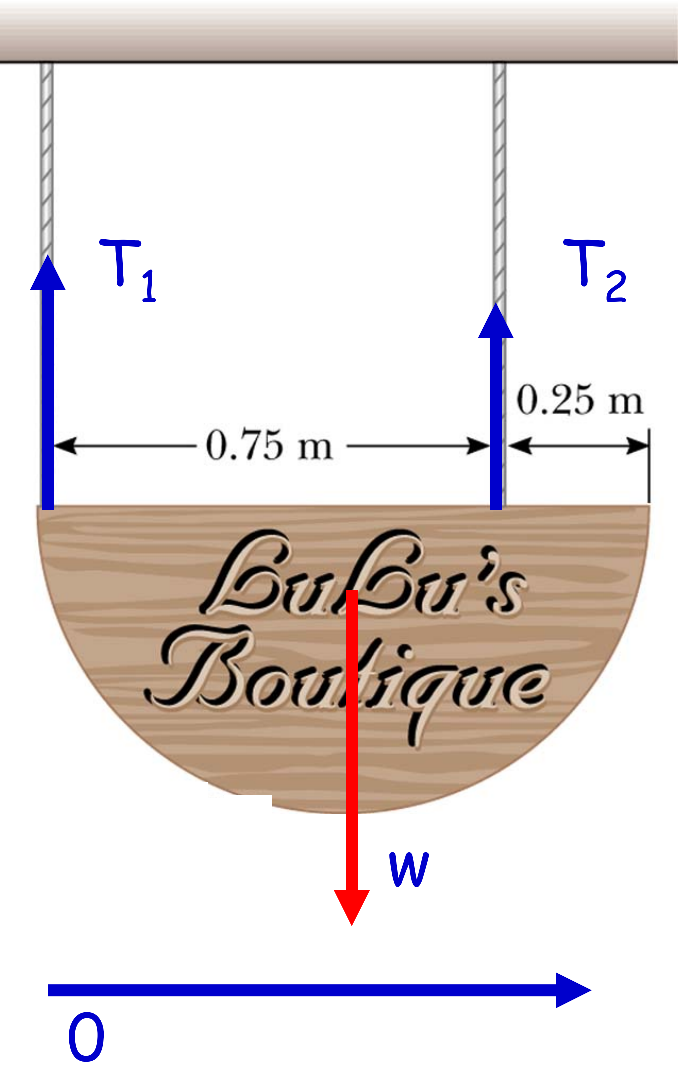
Two persons try to go through a rotating door at the same time, one on the l.h.s. of the rotator and one the r.h.s. of the rotator. If the forces are applied as shown in the drawing, what will happen?

$\tau_1 = F_1 \cdot d_1 = -100 * 0.3 = -30$ N m

$\tau_2 = F_2 \cdot d_2 = 50 * 0.6 = 30$ N m

$\Sigma\tau$ 0 N m +

Nothing will happen! The 2 torques are balanced.



Weight of board: w

What is the tension in each of the wires (in terms of w)?

Translational equilibrium

$\Sigma F = ma = 0$

$T_1 + T_2 - w = 0$ so $T_1 = w - T_2$

Rotational equilibrium

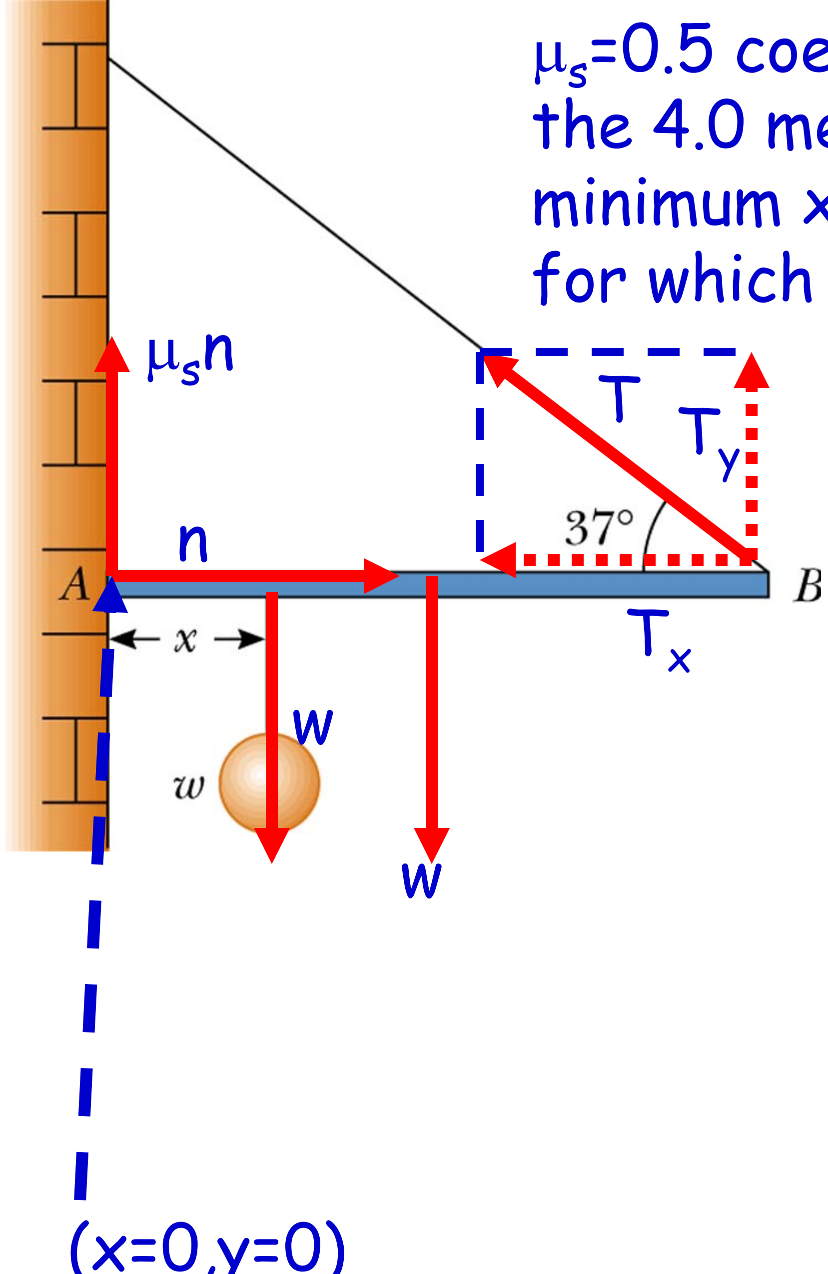
$\Sigma \tau = 0$

$T_1 0 - 0.5 * w + 0.75 * T_2 = 0$

$T_2 = 0.5/0.75 * w = 2/3 w$

$T_1 = w/3$

$T_2 = 2w/3$



μ_s=0.5 coef of friction between the wall and the 4.0 meter bar (weight w). What is the minimum x where you can hang a weight w for which the bar does not slide?

Translational equilibrium (Hor.)

$\Sigma F_x = ma = 0$

$n - T_x = n - T\cos 37^\circ = 0$ so $n = T\cos 37^\circ$

Translational equilibruim (vert.)

$\Sigma F_y = ma = 0$

$\mu_s n - w - w + T_y = 0$

$\mu_s n - 2w + T\sin 37^\circ = 0$

$\mu_s T\cos 37^0 - 2w + T\sin 37^0 = 0$

$1.00T = 2w$

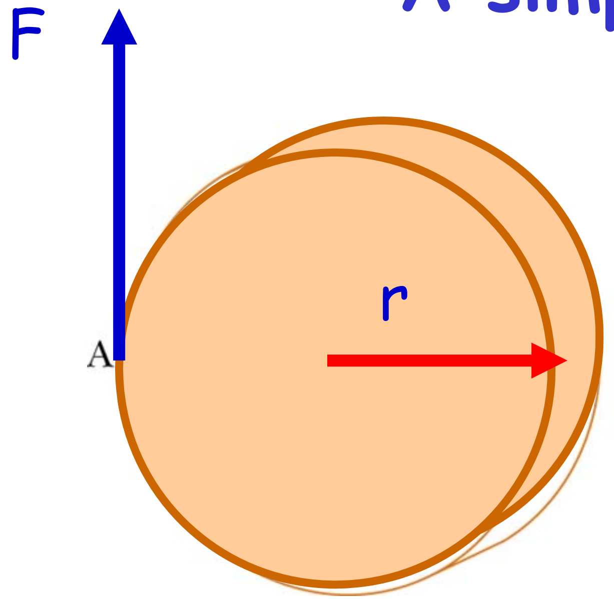
Rotational equilibrium:

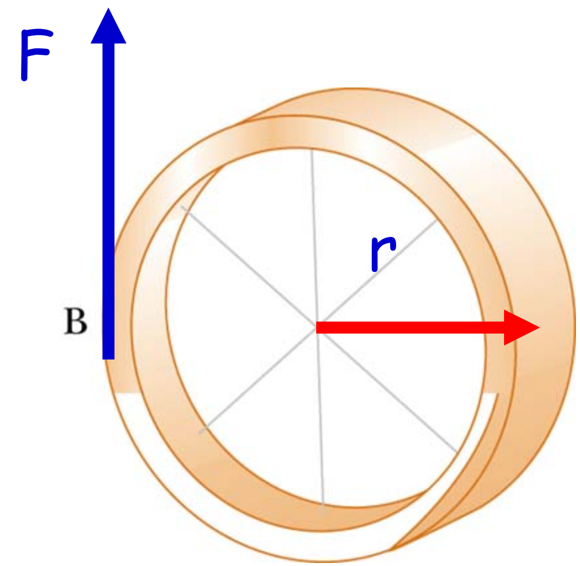
$\Sigma\tau = 0$

$xw + 2w - 4T\sin 37^0 = 0$ so $w(x+2-4.8) = 0$

$x = 2.8$ m

A simple example





A and B have the same total mass. If the same torque is applied, which one accelerates faster?

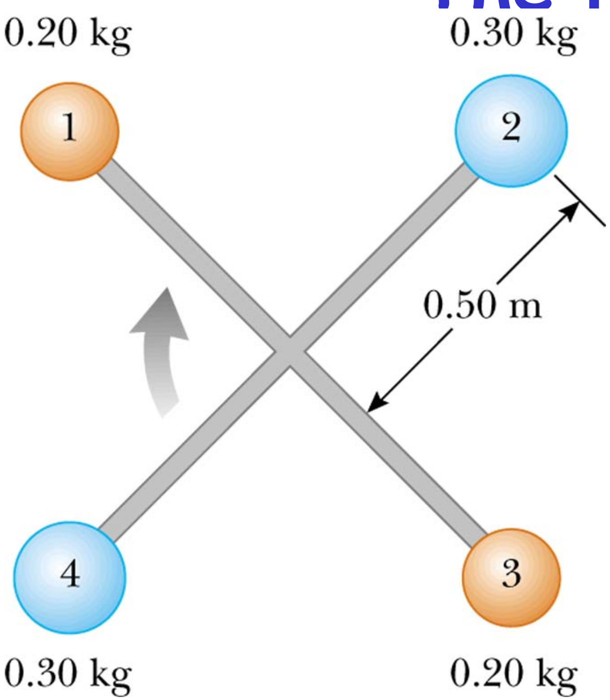
Answer: A

$\tau = I\alpha$

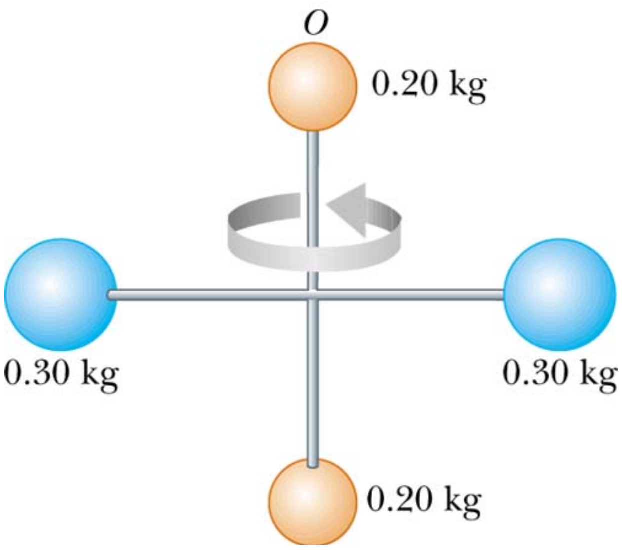
Moment of inertia I:

$I = (\Sigma m_i r_i^2)$

The rotation axis matters!



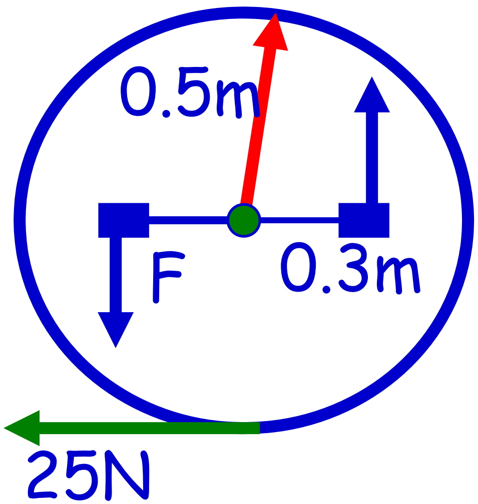
$I=(\Sigma m_i r_i^2)$
$=0.2*0.5+0.3*0.5+$
$0.2*0.5+0.3*0.5$
$=0.5\ kg\ m^2$



$I=(\Sigma m_i r_i^2)$
$=0.2*0.+0.3*0.5+$
$0.2*0+0.3*0.5$
$=0.3\ kg\ m^2$

Example

A monocycle (bicycle with one wheel) has a wheel that has a diameter of 1 meter. The mass of the wheel is 5 kg (assume all mass is sitting at the outside of the wheel). The friction force from the road is 25 N. If the cycle is accelerating with 0.3 m/s, what is the force applied on each of the paddles if the paddles are 30 cm from the center of the wheel?



$\Sigma\tau = I\alpha$

$\alpha = a/r$ so $\alpha = 0.3/0.5 = 0.6$ rad/s

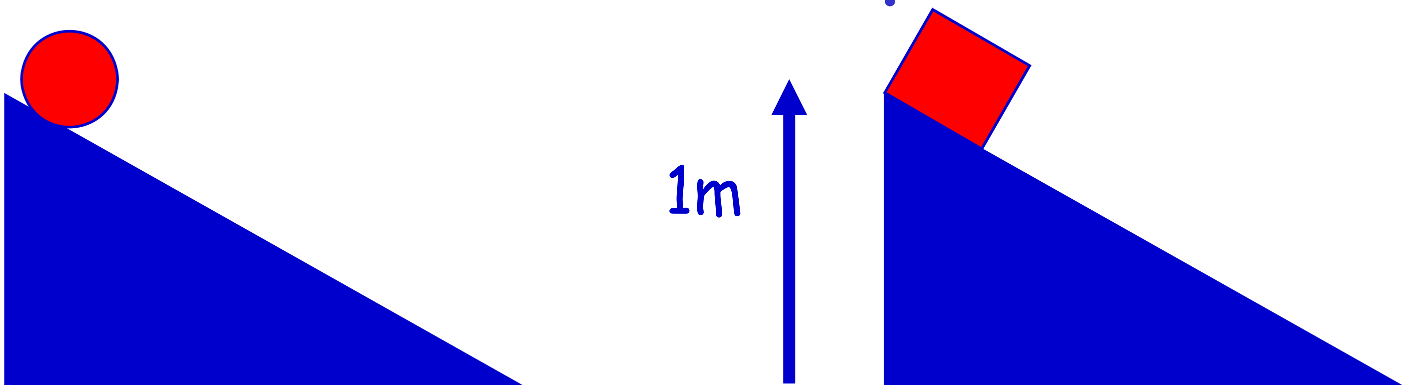
$I = (\Sigma m_i r_i^2) = MR^2 = 5(0.5^2) = 1.25$ kg m^2

$\tau_{friction} = -25*0.5 = -12.5$

$\tau_{paddles} = F*0.3 + F*0.3 = 0.6F$

$0.6F - 12.5 = 1.25*0.6$, so $F = 22.1$ N

Example.



Consider a ball and a block going down the same 1m-high slope. The ball rolls and both objects do not feel friction. If both have mass 1kg, what are their velocities at the bottom (I.e. which one arrives first?). The diameter of the ball is 0.4 m.

Block: $[\frac{1}{2}mv^2+mgh]_{initial}= [\frac{1}{2}mv^2+mgh]_{final}$

$$1*9.8*1 = 0.5*1*v^2 \quad \text{so } v=4.4 \text{ m/s}$$

Ball: $[\frac{1}{2}mv^2+mgh+\frac{1}{2}I\omega^2]_{initial}= [\frac{1}{2}mv^2+mgh+\frac{1}{2}I\omega^2]_{final}$

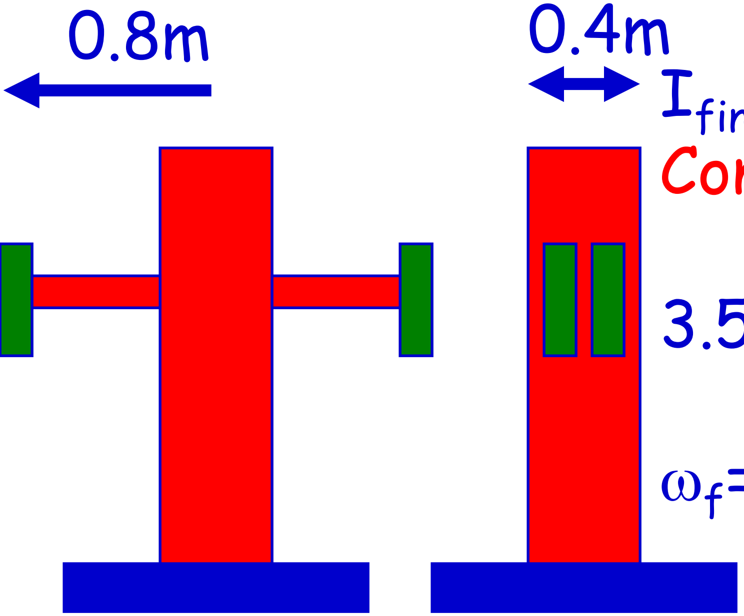
$I=0.4*MR^2=0.064$ kgm^2 and $\omega=v/R=2.5v$

$$1*9.8*1 = 0.5*1*v^2+0.5*0.064*(2.5v)^2$$

so $v=3.7$ m/s Part of the energy goes to the rotation: slower!

The spinning lecturer...

A lecturer (60 kg) is rotating on a platform with $\omega=2\pi$ rad/s (1 rev/s). He is holding two masses of 0.8 m away from his body. He then puts the masses close to his body (R=0.0 m). Estimate how fast he will rotate.



$I_{initial}=0.5M_{lec}R^2+2(M_wR_w{}^2)+2(0.33M_{arm}0.8^2)$

$=1.2+1.3+1.0= 3.5$ kg m^2

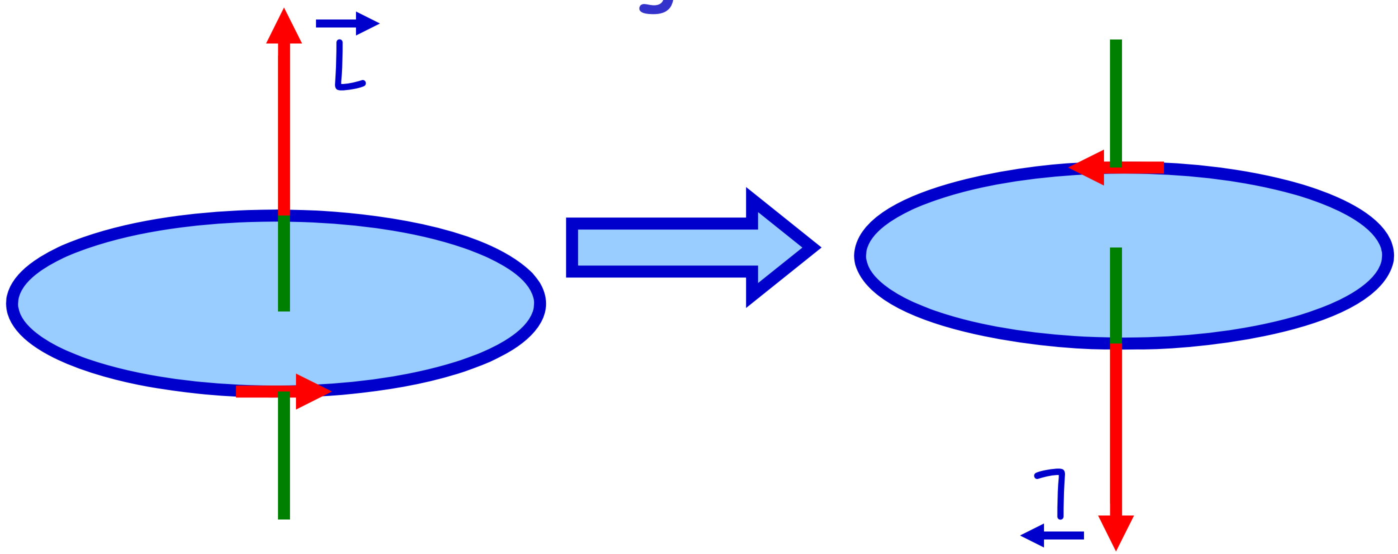
I_{final} $=0.5M_{lec}R^2=1.2$ kg m^2

Conservation of angular mom. $I_i\omega_i=I_f\omega_f$

$3.5*2\pi=1.2*\omega_f$

$\omega_f=18.3$ rad/s (approx 3 rev/s)

Rotating a bike wheel!



A person on a platform that can freely rotate is holding a spinning wheel and then turns the wheel around.
What will happen?

Initial: angular momentum: $I_{wheel}\omega_{wheel}$
Closed system, so L must be conserved.

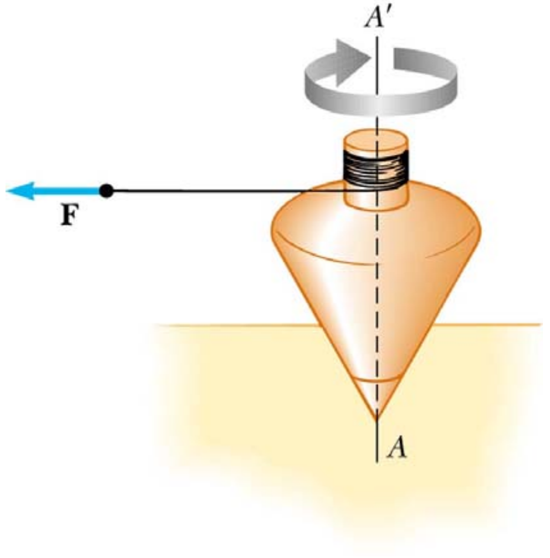
Final: $-I_{wheel}\,\omega_{wheel}+I_{person}\omega_{person}$

$$\omega_{person}=\frac{2I_{wheel}\,\omega_{wheel}}{I_{person}}$$

Global warming

The polar ice caps contain 2.3×10^{19} kg of ice. If it were all to melt, by how much would the length of a day change?
$M_{earth}=6\times10^{24}$ kg $R_{earth}=6.4\times10^{6}$ m

Before global warming: ice does not give moment of inertia
$I_i=2/5*M_{earth}R^2_{earth}=2.5\times10^{38}$ kg m^2
$\omega_i=2\pi/(24*3600\ s)=7.3\times10^{-5}$ rad/s
After ice has melted:
$I_f=I_i+2/3*MR^2_{ice}=2.5\times10^{38}+2.4\times10^{33}=2.500024\times10^{38}$
$\omega_f=\omega_i I_i/I_f=7.3\times10^{-5}*0.9999904$
The length of the day has increased by
0.9999904*24 hrs=0.83 s.



A top

A top has $I=4.00\times10^{-4}$ kg m^2. By pulling a rope with constant tension F=5.57 N, it starts to rotate along the axis AA′. What is the angular velocity of the top after the string has been pulled 0.8 m?

Work done by the tension force:

$W=F\Delta x=5.57*0.8=4.456$ J

This work is transformed into kinetic energy of the top:

$KE=0.5I\omega^2=4.456$ so $\omega=149$ rad/s=23.7 rev/s