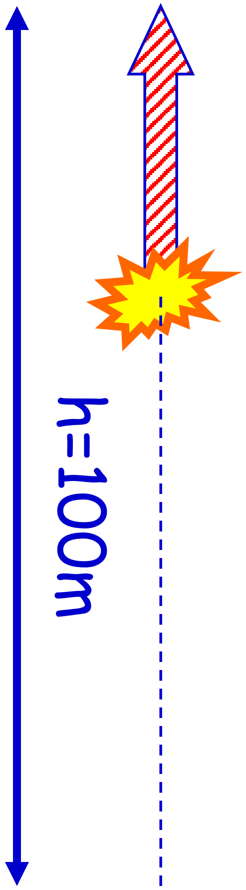
Example



A toy-rocket of 5.0 kg, after the initial acceleration stage, travels 100 m in 2 seconds.
What is the work done by the engine?
What is the power of the engine?

$W=(F\cos\theta)\Delta h=m_{rocket}g\ \Delta h=4905\ J$

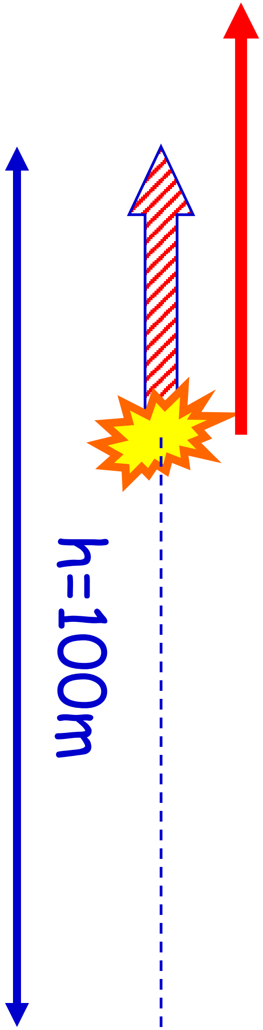
(Force by engine must balance gravity!)

$P=W/\Delta t=4905/2=2453\ W$ (=3.3 horsepower)

or

$P=(F\cos\theta)v=mgv=5.0\times 9.81\times 100/2=2453\ W$

Another rocket



A toy rocket (5kg) is launched from rest and reaches a height of 100 m within 2 seconds. What is the work done by the engine during acceleration?

$h(t)=h(0)+v_0t+0.5at^2$ $100=0.5a2^2$ so $a=50\ m/s^2$

$V(t)=V(0)+at$ $V(2)=0+50*2=100\ m/s$

Force by engine=$(50+9.81)m=59.81*5=299.05$ N

($9.81\ m/s^2$ due to balancing of gravitation)

$W=F\Delta h=299.05*100=29905$ J

Change in potential energy:

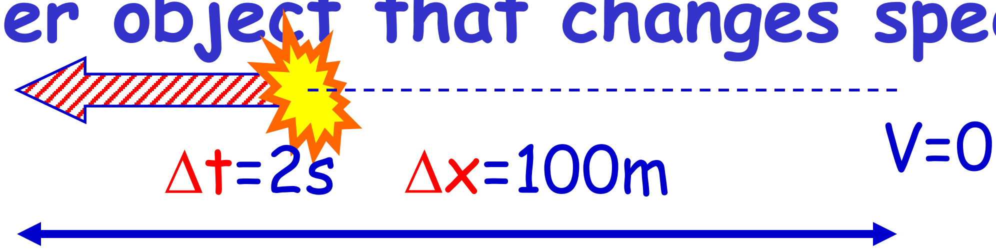
$PE_f-PE_i=mgh_f-mgh_i=4905-0=4905$ J

Where did all the work (29905-4905=25000 J)*go*?

Into the acceleration: energy of motion (kinetic energy)

Kinetic energy

Consider object that changes speed only



a) $W=F\Delta x=(ma)\Delta x$ … used Newton's second law

b) $v=v_0+at$ so $t=(v-v_0)/a$

c) $x=x_0+v_0t+0.5at^2$ so $x-x_0=\Delta x=v_0t+0.5at^2$

Combine b) & c)

d) $a\Delta x=(v^2-v_0^2)/2$

Combine a) & d)

$$W=\tfrac{1}{2}m(v^2-v_0^2)$$

Rocket:

$W=\tfrac{1}{2}5(100^2-0^2)$

$=25000$ J!!

That was missing!

Kinetic energy: $KE=\tfrac{1}{2}mv^2$

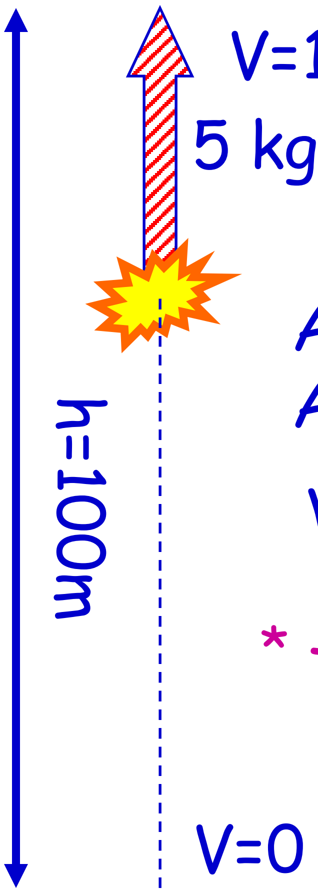
When work is done on an object and the only change is its speed: The work done is equal to the change in KE:

$$W=KE_{final}-KE_{initial}$$

Conservation of mechanical energy

Mechanical energy = potential energy + kinetic energy

In a closed system, mechanical energy is conserved*



$ME = mgh + \frac{1}{2}mv^2 = \text{constant}$

What about the accelerating rocket?

At launch: $ME = 5*9.81*0 + \frac{1}{2}5*0^2 = 0$

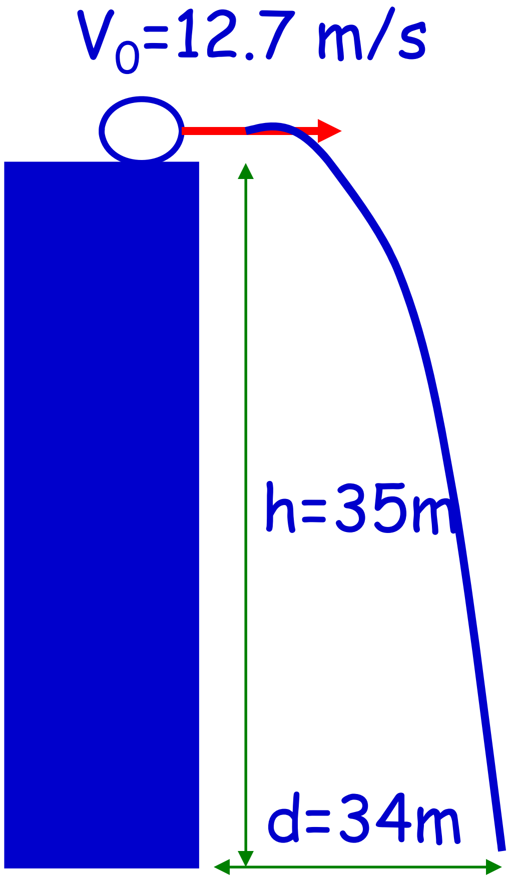
At 100 m height: $ME = 5*9.81*100 + \frac{1}{2}5*100^2 = 29905$

We did not consider a closed system! (Fuel burning)

* There is an additional condition, see slides 12,13,14

Example of closed system

A snowball is launched horizontally from the top of a building at v=12.7 m/s. The building is 35 m high. The mass is 0.2 kg. Is mechanical energy conserved?



At launch:

$ME = mgh + \frac{1}{2}mv^2$

$= 0.2*9.81*35 + \frac{1}{2}*0.2*12.7^2 = 84.7$ J

At ground:

$a\Delta h = (v^2 - v_{0,ver}^2)/2$ so v=26.2 m/s

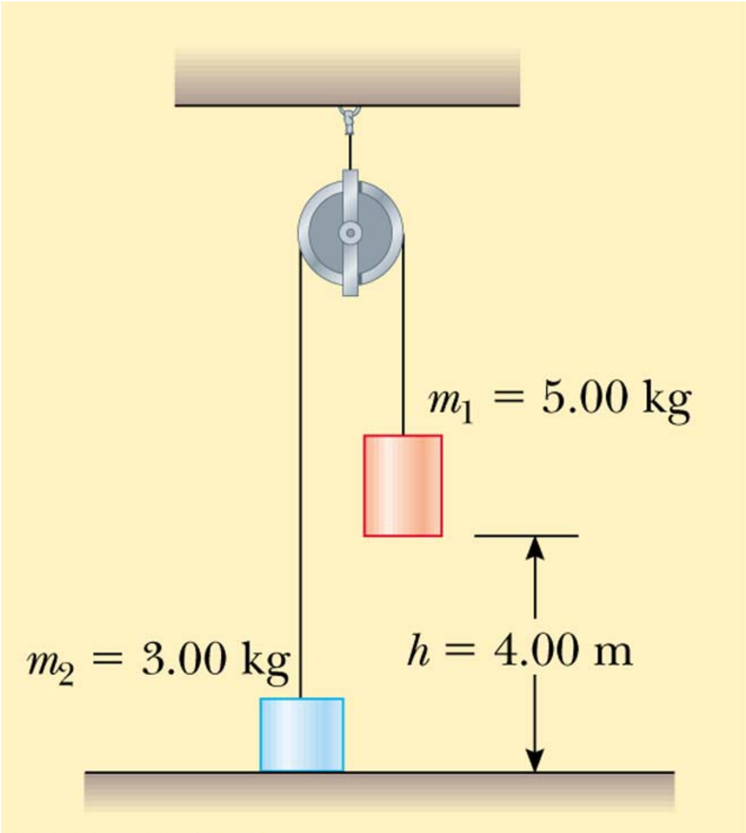
$v = \sqrt{(v_{hor}^2 + v_{ver}^2)} = \sqrt{(12.7^2 + 26.2^2)} = 29.1$

$ME = mgh + \frac{1}{2}mv^2$

$= 0.2*9.81*0 + \frac{1}{2}*0.2*29.1^2 = 84.7$ J

ME is conserved!!

Conservation of mechanical energy



A) what is the speed of m_1 and m_2 when they pass each other?

$(PE_1+PE_2+KE_1+KE_2)$=constant

At time of release:

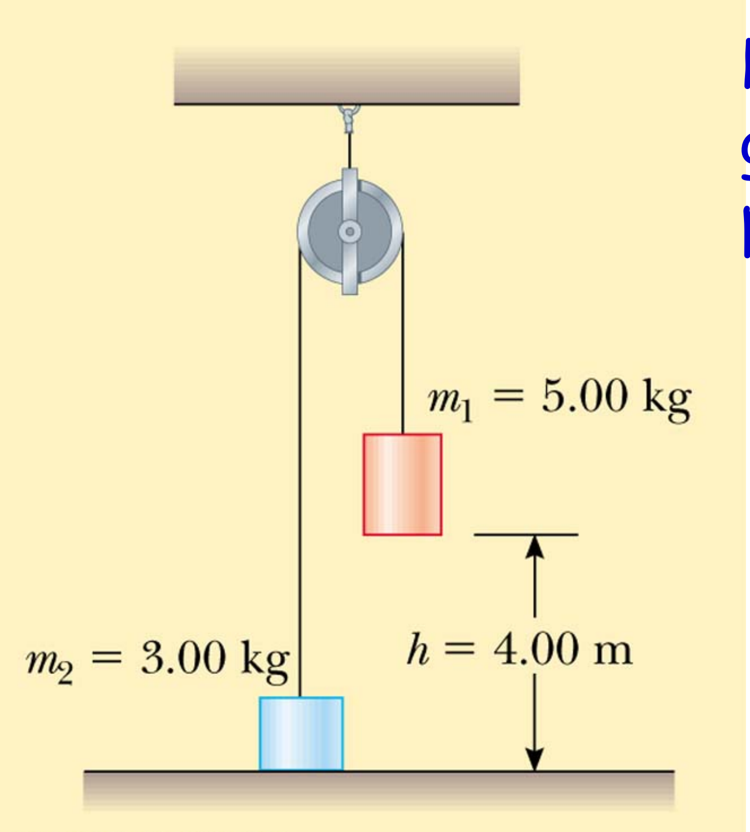
$PE_1=m_1gh_1=5.00*9.81*4.00$	=196. J
$PE_2=m_2gh_2=3.00*9.81*0.00$	=0.00 J
$KE_1=0.5*m_1*v^2=0.5*5.00*(0.)^2$	=0.00 J
$KE_2=0.5*m_1*v^2=0.5*3.00*(0.)^2$	=0.00 J
Total	=196. J

At time of passing:

$PE_1=m_1gh_1=5.00*9.81*2.00$	=98.0 J
$PE_2=m_2gh_2=3.00*9.81*2.00$	=58.8 J
$KE_1=0.5*m_1*v^2=0.5*5.00*(v)^2$	$=2.5v^2$ J
$KE_2=0.5*m_2*v^2=0.5*3.00*(v)^2$	$=1.5v^2$ J
Total	$=156.8+4.0v^2$

$196=156.8+4.0v^2$ so $v=3.13$ m/s

work



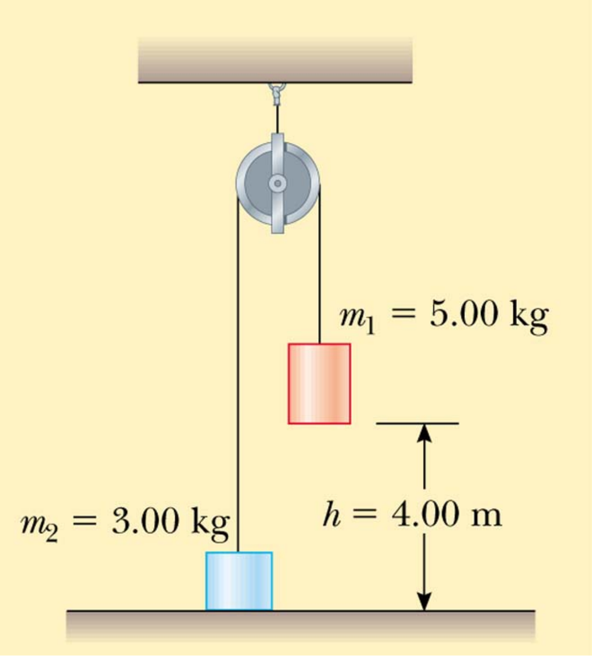
How much work is done by the gravitational force when the masses pass each other?

$W = F\Delta x = m_1 g 2.00 + m_2 g(-2.00) = 39.2$ J

$\Sigma Pe_{start} - \Sigma Pe_{passing} = (196. - 98. - 58.8) =$ 39.2 J

The work done by F_g is the same as the change in potential energy

Friction (non-conservative)



The pulley is not completely frictionless. The friction force equals 5 N. What is the speed of the objects when they pass?

$(\Sigma PE+ \Sigma KE)_{start}-(\Sigma PE+\Sigma KE)_{passing}=W_{nc}$

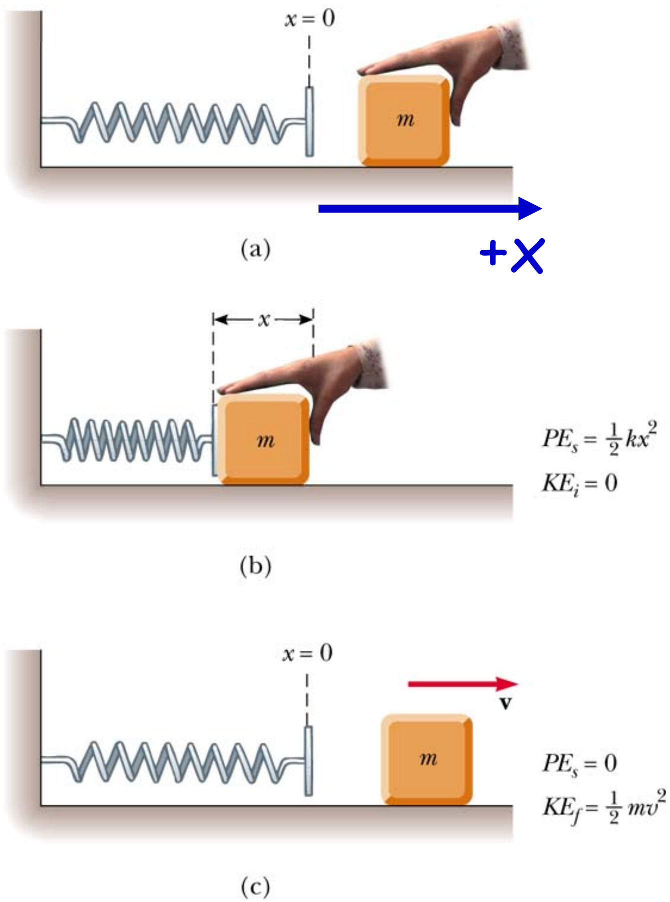
$W_{nc}=F_{friction}\Delta x=5.00*2.00=10.0$ J

$(196.)-(156.8+\Sigma KE)=10$ J

$\Sigma KE=29.2$ J$=0.5*(m_1+m_2)v^2=4v^2$

$v=2.7$ m/s

A spring

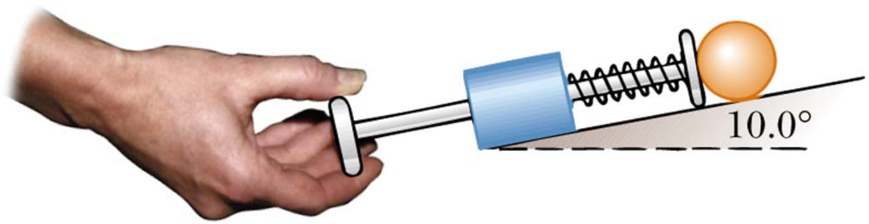


$F_s=-kx$ k: spring constant (N/m)

$F_s(x=0)=0$ N
$F_s(x=-a)=ka$
$\bar{F}_s=(0+ka)/2=ka/2$

$W_s=\bar{F}_s\,\Delta x=(ka/2)*(a)=ka^2/2$
The energy stored in a spring depends on the location of the endpoint: elastic potential energy.

PINBALL



The ball-launcher spring has a constant k=120 N/m. A player pulls the handle 0.05 m. The mass of the ball is 0.1 kg. What is the launching speed?

$(PE_{gravity}+PE_{spring}+KE_{ball})_{pull}=(PE_{gravity}+PE_{spring}+KE_{ball})_{launch}$

$mgh_{pull}+\frac{1}{2}kx_{pull}^2+\frac{1}{2}mv_{pull}^2 = mgh_{launch}+\frac{1}{2}kx_{launch}^2+\frac{1}{2}mv_{launch}^2$

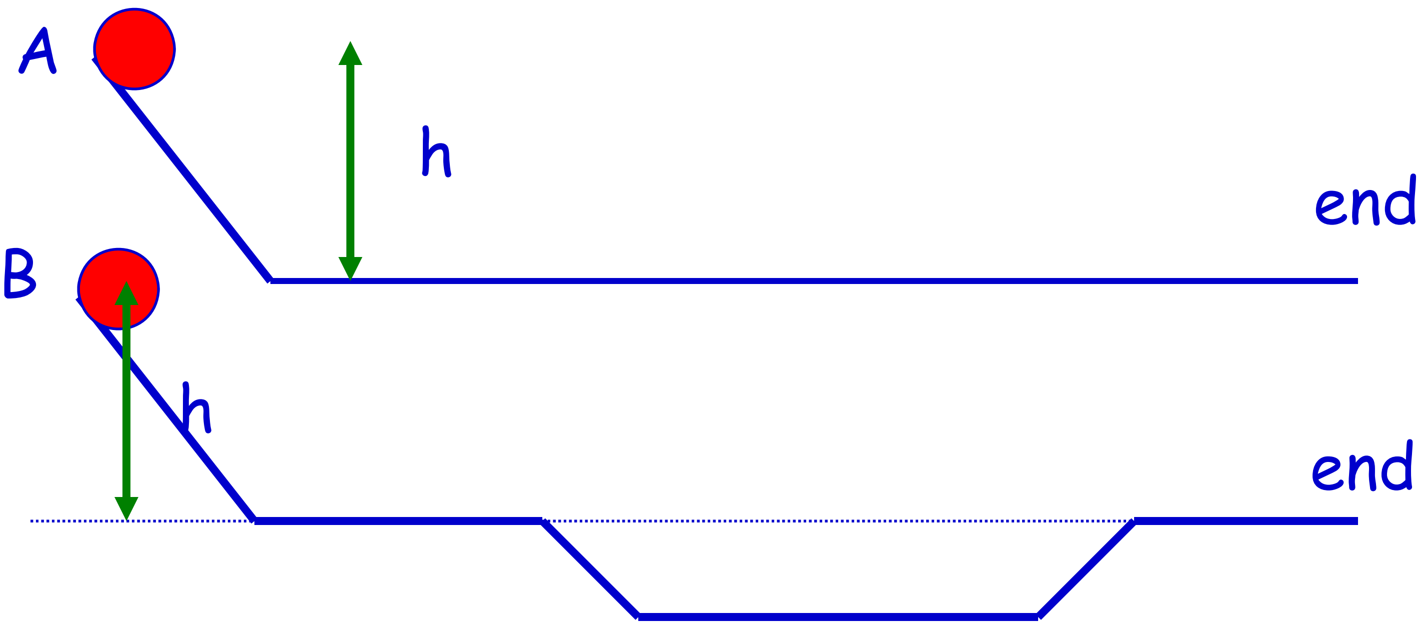
$0.1*9.81*0+\frac{1}{2}120(0.05)^2+\frac{1}{2}0.1(0)^2=$

$0.1*9.81*(0.05*\sin(10^o))+\frac{1}{2}120*(0)^2+\frac{1}{2}0.1v_{launch}^2$

$0.15=8.5E\text{-}03+0.05v^2$

$v=1.7$ m/s

Ball on a track



In which case has the ball the highest velocity at the end?

A) Case A B) Case B C) Same speed

In which case does it take the longest time to get to the end?

A) Case A B) Case B C) Same time