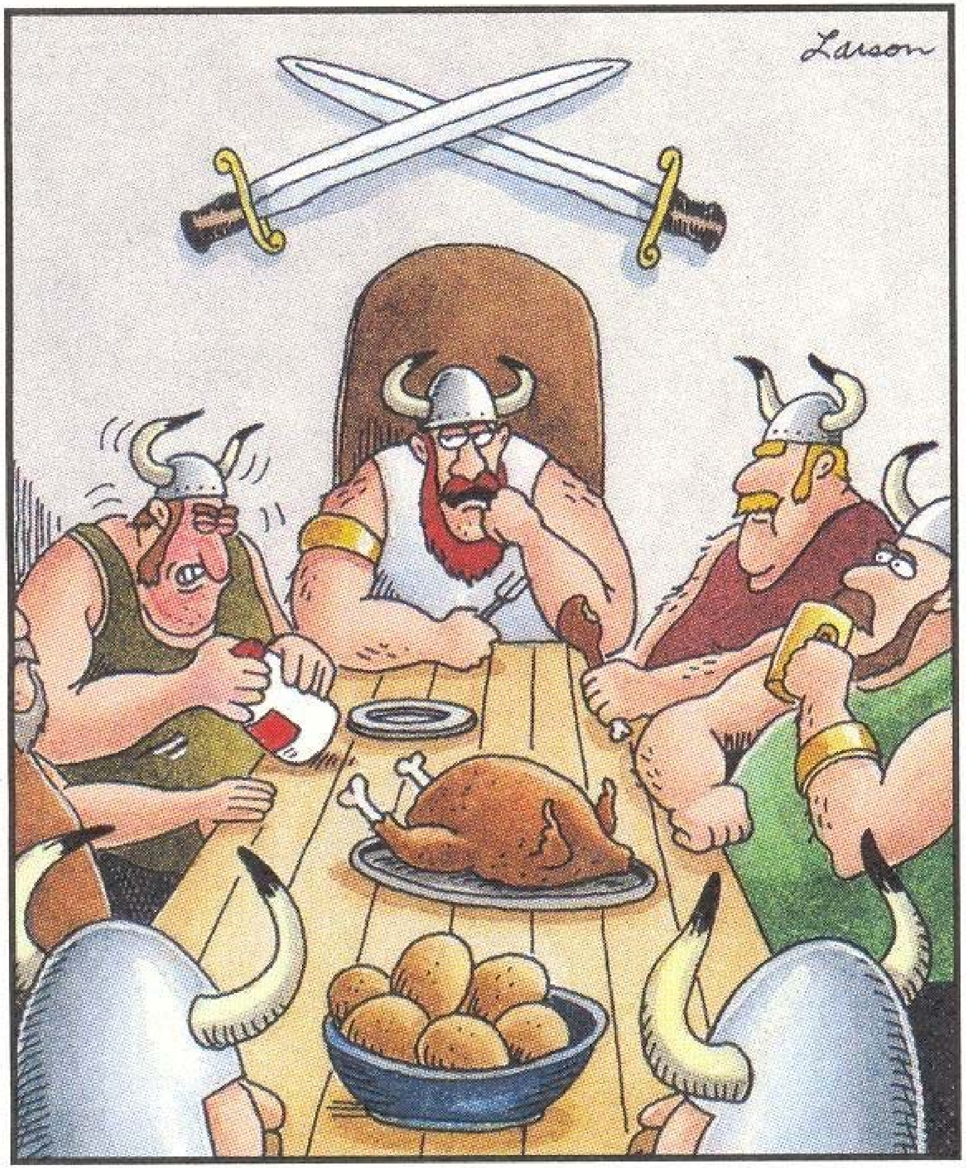


"I can't believe this! ... Can't *anyone* here get the lid off the mayonnaise?"

Emf for an Inductor

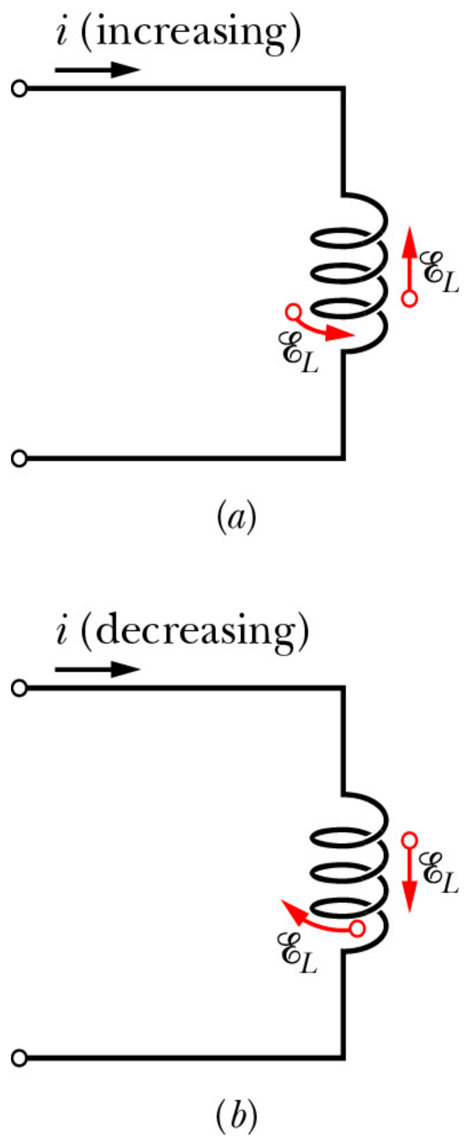
$$N\Phi_B = Li$$

- Self-inductance – changing current through an inductor gives an emf (voltage change) across the inductor denoted by

$$\mathcal{E}_L = -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt}$$

$$\boxed{\mathcal{E}_L = -L\frac{di}{dt}}$$

- Note the sign of this emf goes against the current change



Energy stored in fields

- Energy stored in the electric field of a capacitor

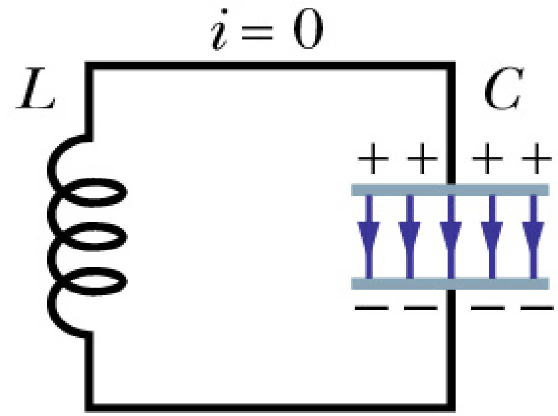
$$U_E = \frac{1}{2}\frac{q^2}{C}$$

- Energy stored in the *B* field of an inductor

$$U_B = \frac{1}{2}Li^2$$

LC Circuits

- LC Circuit – inductor & capacitor in series
- Find *q*, *i* and *V* vary sinusoidally with period *T* (angular frequency ω)
- *E* field of capacitor and *B* field of inductor oscillate
- The energy oscillates between E field stored in the capacitor and the B field stored in the inductor



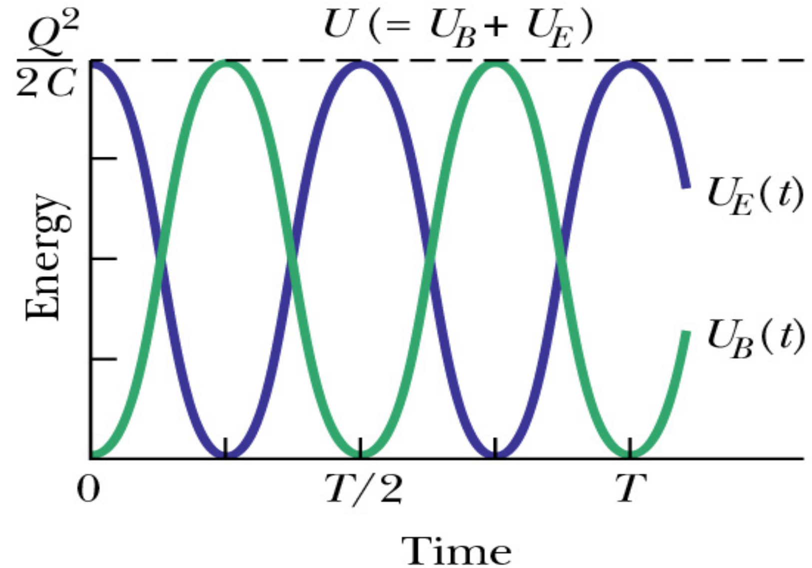
$$U_E = \frac{q^2}{2C}$$

$$U_B = \frac{Li^2}{2}$$

LC Circuits

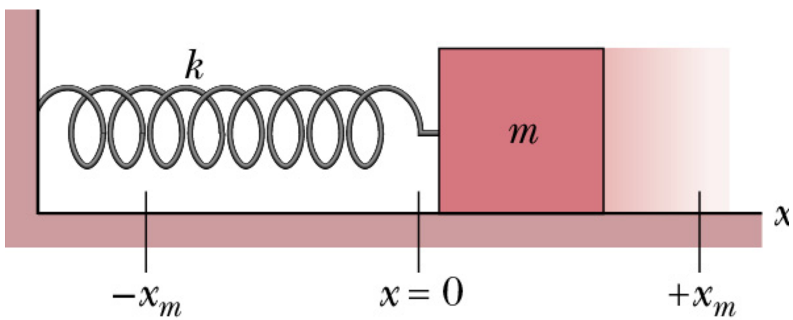
- Total energy of LC circuit

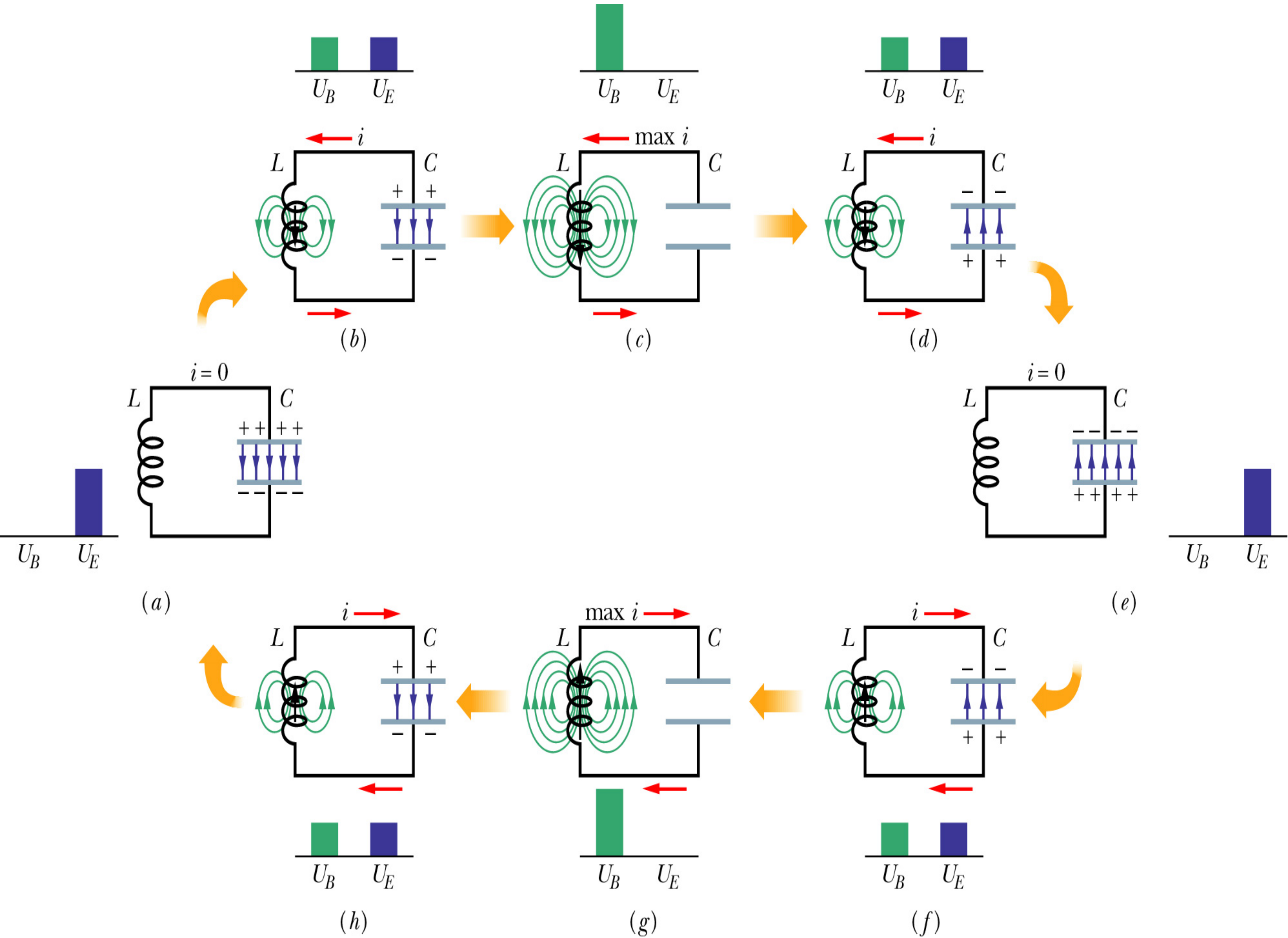
$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$



- Analogy to block-spring system

$$U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$





U_B U_E
i
L C
(b)
max i
(c)
(d)
$i = 0$
(a)
(e)
(h)
(g)
(f)

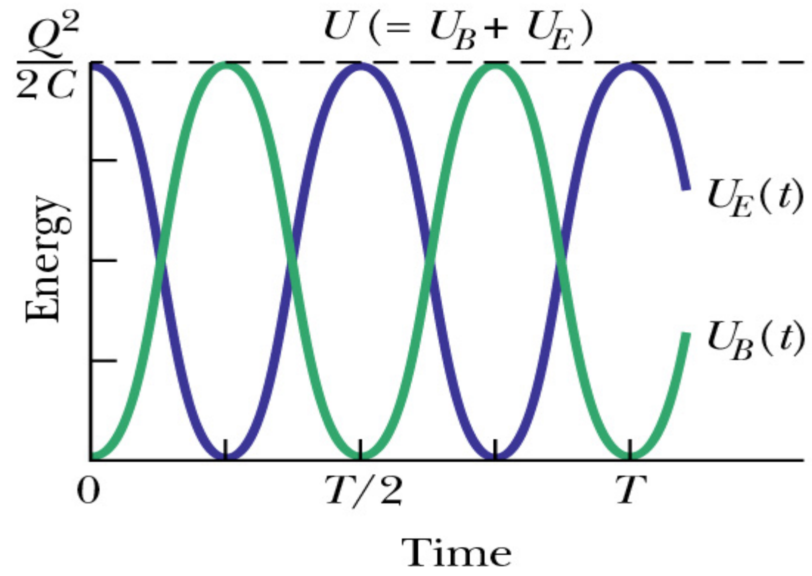
LC Circuits

- Total energy of LC circuit

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

- Total energy is constant

$$\frac{dU}{dt} = 0$$



- Differentiating gives

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0$$

LC Circuits

- Using $i = \frac{dq}{dt}$ $\frac{di}{dt} = \frac{d^2q}{dt^2}$

- We obtain

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

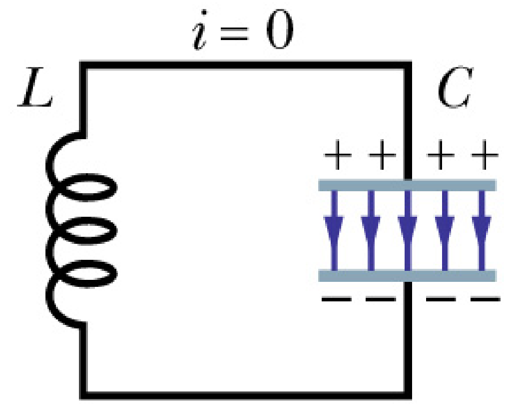
- Solution is

$$\boxed{q = Q\cos(\omega t + \phi)}$$

LC Circuits

- The phase constant, ϕ, is determined by the conditions at time t=0 (or some other time)

$$q = Q\cos(\omega t + \phi)$$



- If $\phi = 0$ then at $t = 0$

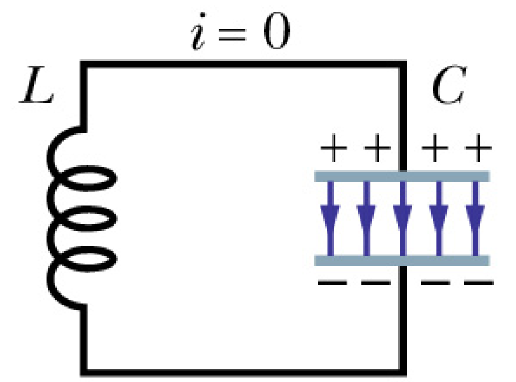
  $q = Q$

- I will take $\phi = 0$ for the rest of the lecture notes. To get the full result when you see ωt replace by $\omega t + \phi$

LC Circuits

- Charge of LC circuit

$$q = Q\cos(\omega t)$$



- Find current by $i = \frac{dq}{dt}$

$$i = \frac{d}{dt}[Q\cos(\omega t)] = -Q\omega\sin(\omega t)$$

- Amplitude I is

$$I = \omega Q$$

$$i = -I\sin(\omega t)$$

LC Circuits

- What is ω for an LC circuit?

$$q = Q\cos(\omega t)$$

$$\frac{d^2q}{dt^2} = -\omega^2 Q\cos(\omega t)$$

- Substitute into

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

$$-L\omega^2 Q\cos(\omega t) + \frac{1}{C}Q\cos(\omega t) = 0$$

- Find ω for LC circuit is

$$\boxed{\omega = \sqrt{\frac{1}{LC}}}$$

LC Circuits

- The energy stored in an LC circuit at any time, *t*

$$U = U_B + U_E$$

$$q = Q\cos(\omega t) \qquad i = -I\sin(\omega t)$$

- Substitute

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t)$$

$$U_B = \frac{Li^2}{2} = \frac{L}{2}\omega^2 Q^2 \sin^2(\omega t)$$

- Using

$$\omega = \sqrt{\frac{1}{LC}}$$

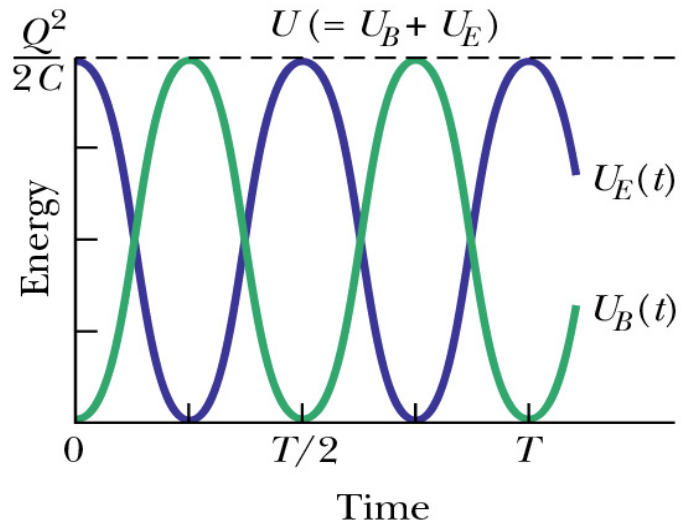
$$U_B = \frac{Q^2}{2C}\sin^2(\omega t)$$

LC Circuits

- Thus

$$U_E = \frac{Q^2}{2C} \cos^2(\omega t)$$

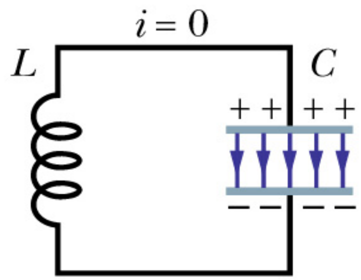
$$U_B = \frac{Q^2}{2C} \sin^2(\omega t)$$



- Maximum value for both $U_{E,\max} = U_{B,\max} = Q^2/2C$
- At any instant, sum is $U = U_B + U_E = Q^2/2C$
- When U_E = max, U_B = 0, and conversely, when U_B = max, U_E = 0

LC Circuits

- Capacitor in LC circuit has $V_{C,max} = 15\ V$ and $U_{E,max} = 150\ J$. When capacitor has $V_C = 5\ V$ and $U_E = 50\ J$, what are the
  - 1) emf across the inductor?
  - 2) the energy stored in the B field?



- Apply the loop rule
  - Net potential difference around the circuit must be zero

$$v_L(t) = v_C(t)$$

$$U_{E,\max} = U_E(t) + U_B(t)$$

1) 5 V

2) 100 J