Review

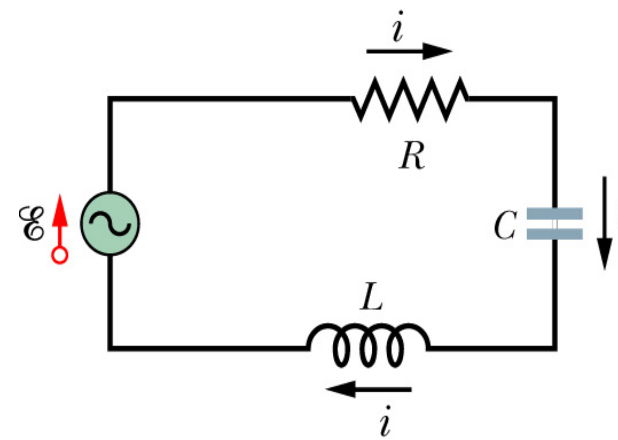
- Characterized ideal LC circuit
  - Charge, current and voltage vary sinusoidally
- Added resistance to LC circuit
  - Oscillations become damped
  - Charge, current and voltage still vary sinusoidally but decay exponentially
- Added ac generator to circuits with just a
  - Resistor
  - Capacitor
  - Inductor

Review

Element	Reactance/ Resistance	Phase of Current	Phase angle ϕ	Amplitude Relation
Resistor	R	In phase	0°	$V_R = I_R R$
Capacitor	$X_C = 1/\omega_d C$	Leads v_C	-90°	$V_C = I_C X_C$
Inductor	$X_L = \omega_d L$	Lags v_L	+90°	$V_L = I_L X_L$

EM Oscillations

- RLC circuit – resistor, capacitor and inductor in series
- Apply alternating emf



$$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$$

- Elements are in series so same current is driven through each
- From the loop rule, at any time *t*, the sum of the voltages across the elements must equal the applied emf

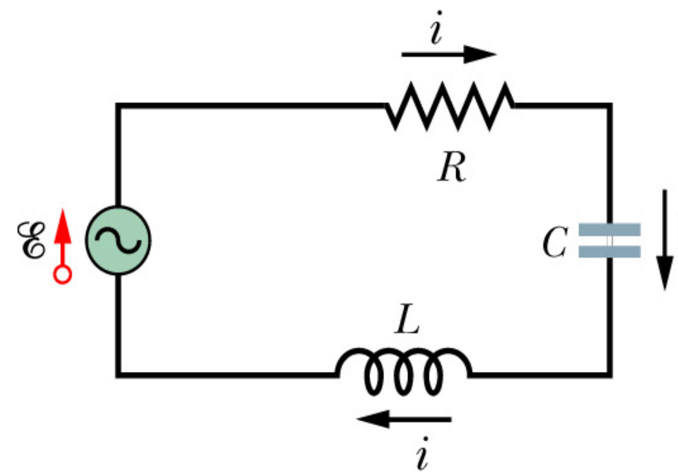
$$i = I \sin(\omega_d t - \phi)$$

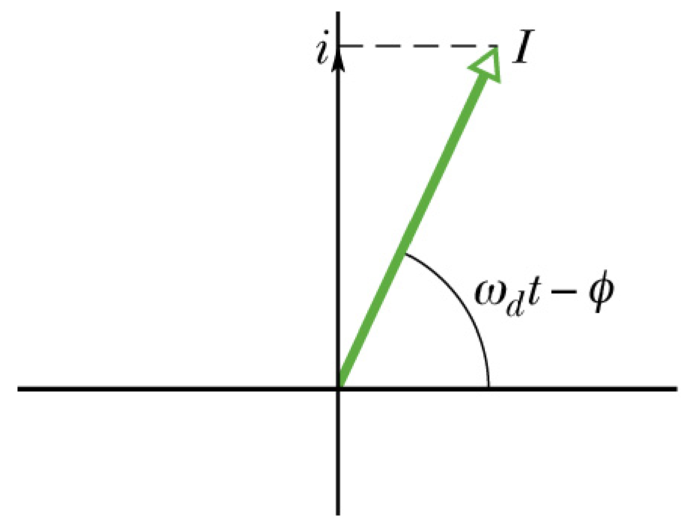
$$\mathcal{E} = v_R + v_C + v_L$$

EM Oscillations

- Want to find amplitude I and the phase constant ϕ
- Using phasors, represent the current at time t
  - Length is amplitude I
  - Projection on vertical axis is current i at time t
  - Angle of rotation is the phase at time t

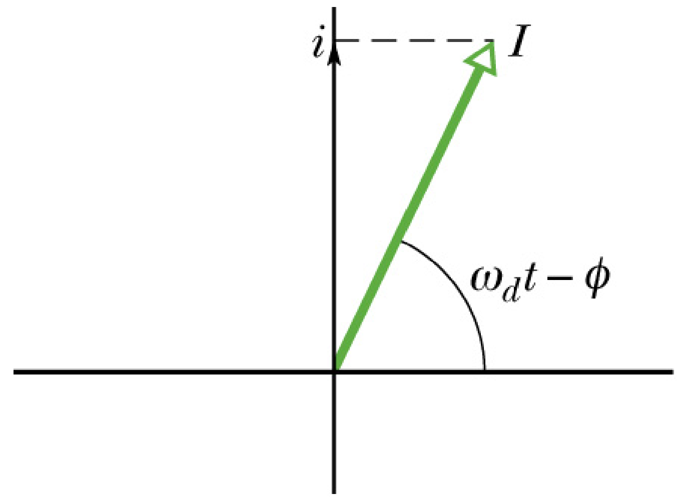
$$\omega_d t - \phi$$

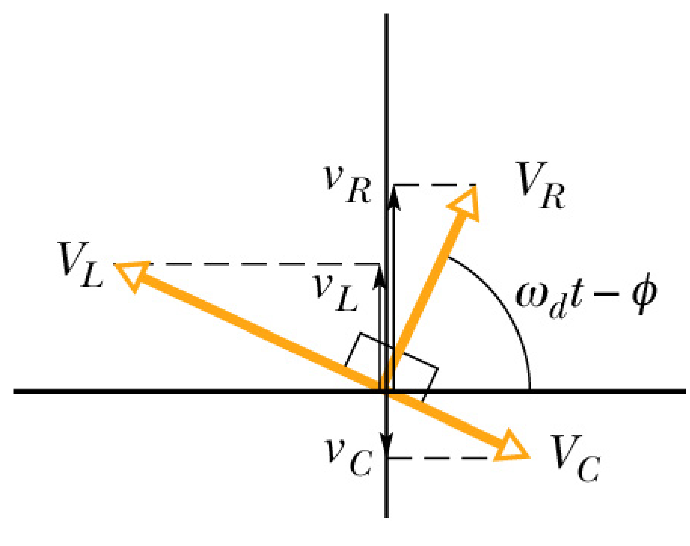




EM Oscillations

- Draw phasors for voltages of *R, C* and *L* at same time *t*
- Orient V_R, V_L, & V_C phasors relative to current phasor
- Resistor – V_R and I are in phase
- Inductor – V_L is ahead of I by 90°
- Capacitor – I is ahead of V_C by 90°
- v_R, v_C, & v_L are projections





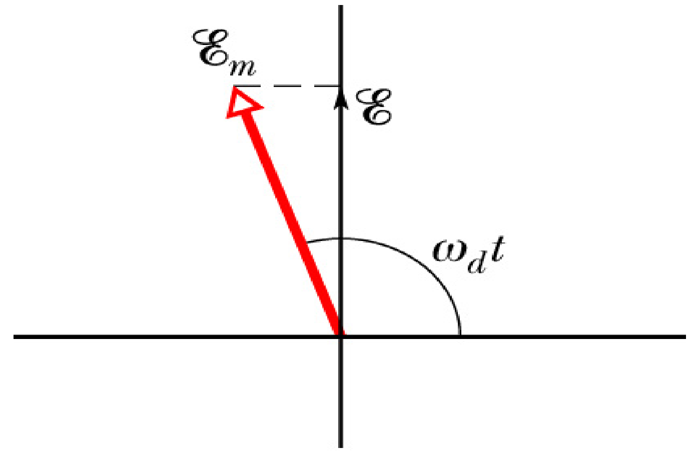
EM Oscillations

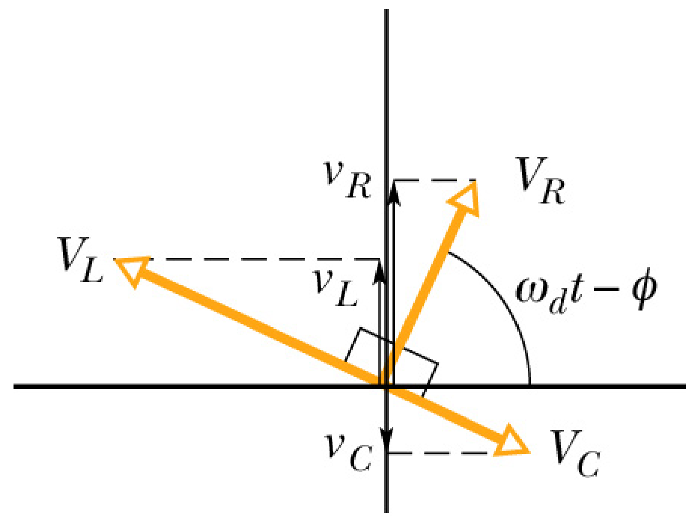
- Draw phasor for applied emf

$$E = E_m \sin \omega_d t$$

- Length is amplitude E_m
- Projection is E at time *t*
- Angle is phase of emf $\omega_d t$
- From loop rule the projection E = the algebraic sum of projections v_R, v_L & v_C

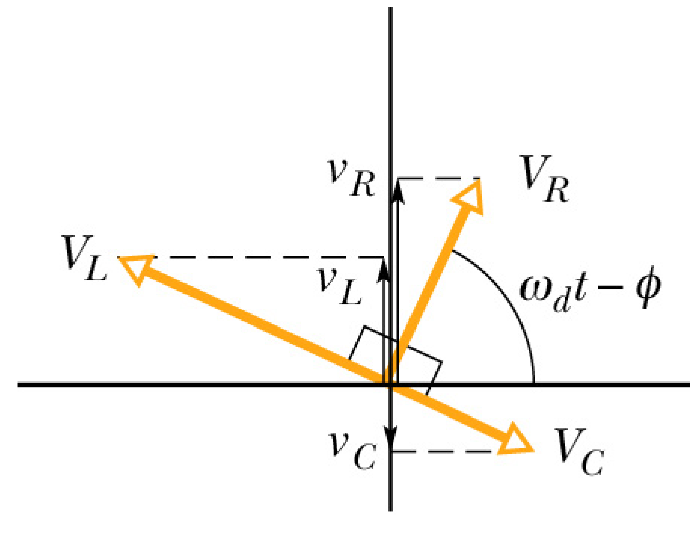
$$E = V_R + V_C + V_L$$





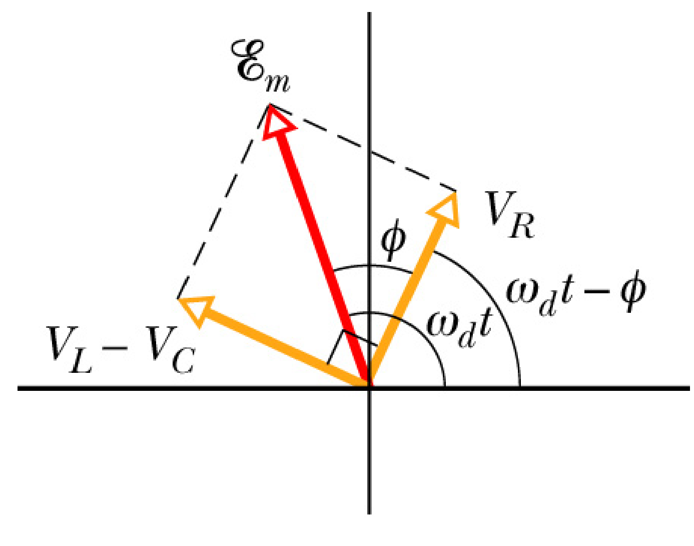
EM Oscillations

$$\mathcal{E} = V_R + V_C + V_L$$



- Phasors rotate together so equality always holds
- Phasor $\mathcal{E}_m$ = vector sum of voltage phasors

$$\vec{\mathcal{E}}_m = \vec{V}_R + \vec{V}_C + \vec{V}_L$$



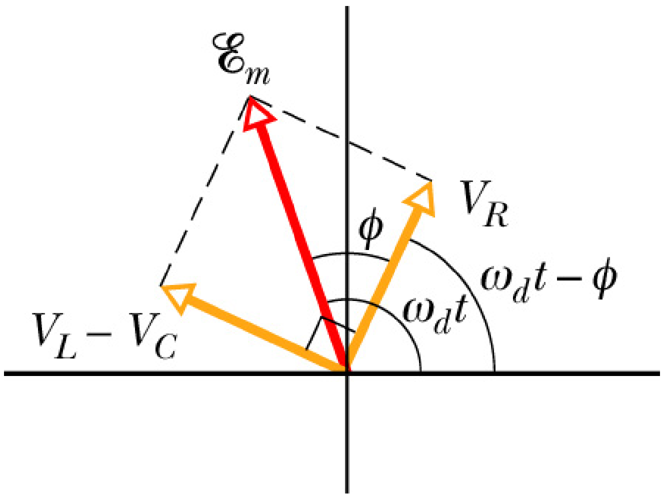
- Combine V_L & V_C to form single phasor

$$V_L - V_C$$

EM Oscillations

- Using Pythagorean theorem

$$\mathcal{E}_m^2 = V_R^2 + (V_L - V_C)^2$$



- From amplitude relations replace voltages with

$$V_R = IR \qquad V_L = IX_L \qquad V_C = IX_C$$

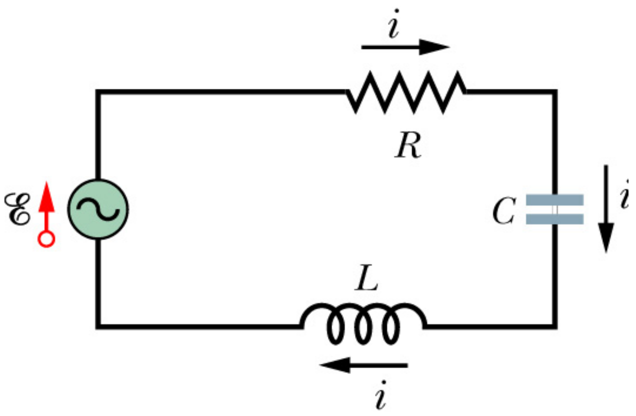
$$\mathcal{E}_m^2 = (IR)^2 + (IX_L - IX_C)^2$$

- Rearrange to find amplitude I

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

EM Oscillations

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$



- Define impedance, Z to be

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- Using reactances rewrite current as

$$X_L = \omega_d L$$

$$X_C = \frac{1}{\omega_d C}$$

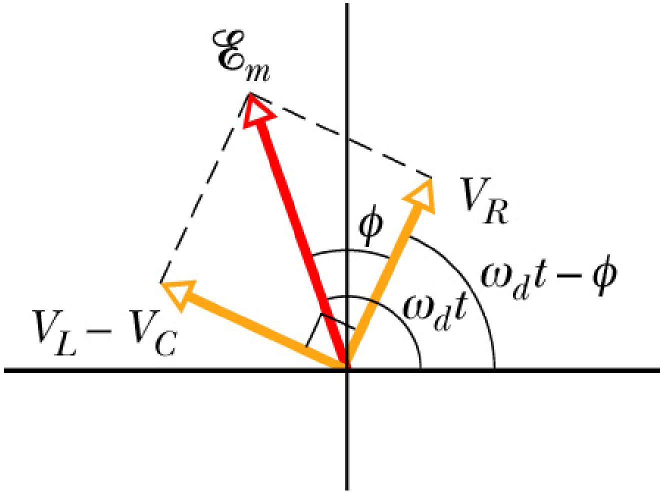
$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

$$I = \frac{\mathcal{E}_m}{Z}$$

EM Oscillations

- Using trig find the phase constant ϕ

$$\tan\phi = \frac{V_L - V_C}{V_R}$$



- Using amplitude relations

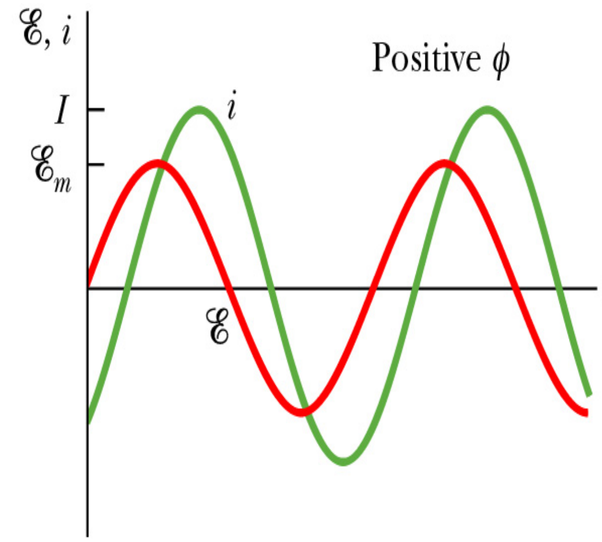
$$\tan\phi = \frac{IX_L - IX_C}{IR}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

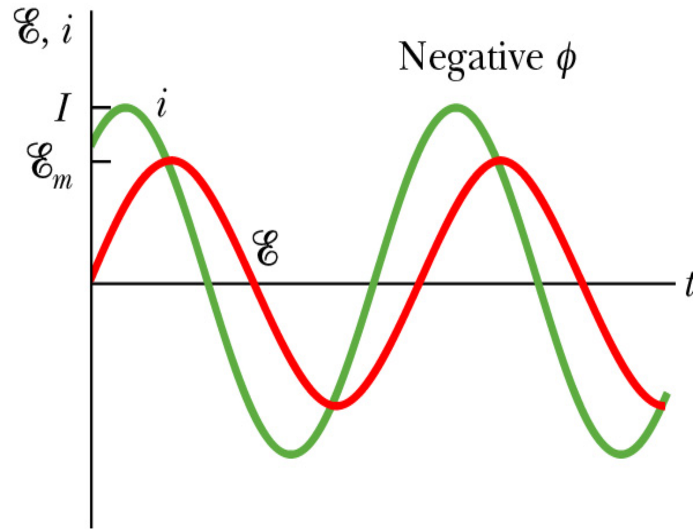
- Examine 3 cases:
  - $X_L > X_C$
  - $X_L < X_C$
  - $X_L = X_C$

EM Oscillations

$$\tan\phi = \frac{X_L - X_C}{R}$$

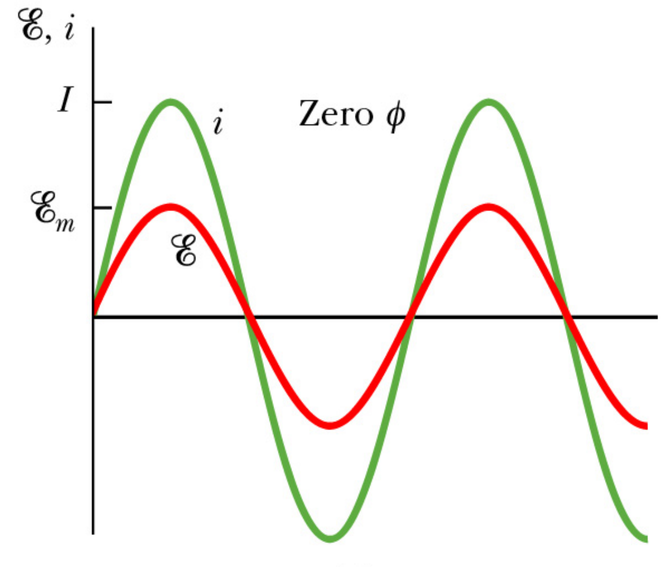


- If $X_L > X_C$ the circuit is more inductive than capacitive
  - ϕ is positive
  - Emf is before current
- If $X_L < X_C$ the circuit is more capacitive than inductive
  - ϕ is negative
  - Current is before emf



EM Oscillations

$$\tan\phi = \frac{X_L - X_C}{R}$$



- If $X_L = X_C$ the circuit is in resonance – emf and current are in phase
- Current amplitude I is max when impedance, Z is min

$$X_L - X_C = 0$$

$$Z = R$$

$$I = \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_m}{R}$$

EM Oscillations

- When $X_L = X_C$ the driving frequency is

$$\omega_d L = \frac{1}{\omega_d C}$$

$$\omega_d = \frac{1}{\sqrt{LC}}$$

- This is the same as the natural frequency, ω

$$\omega_d = \omega = \frac{1}{\sqrt{LC}}$$

- For RLC circuit, resonance and the max current I occurs when $\omega_d = \omega$

EM Oscillations

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

- For small driving frequency, $\omega_d < \omega$
  - X_L is small but X_C is large
  - Circuit capacitive
- For large driving frequency, $\omega_d > \omega$
  - X_C is small but X_L is large
  - Circuit inductive
- For $\omega_d = \omega$, circuit is in resonance