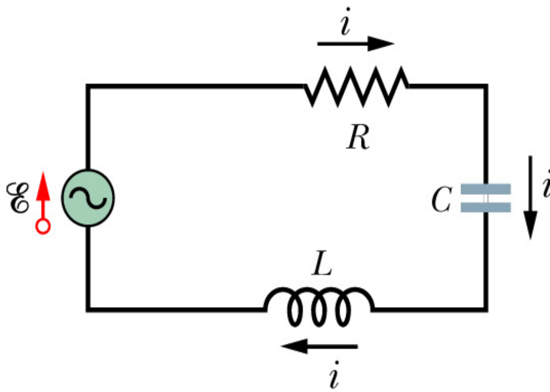
# RLC circuits

- End result:

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + \left( \omega_d L - \frac{1}{\omega_d C} \right)^2}}$$




- I is largest when bottom is small

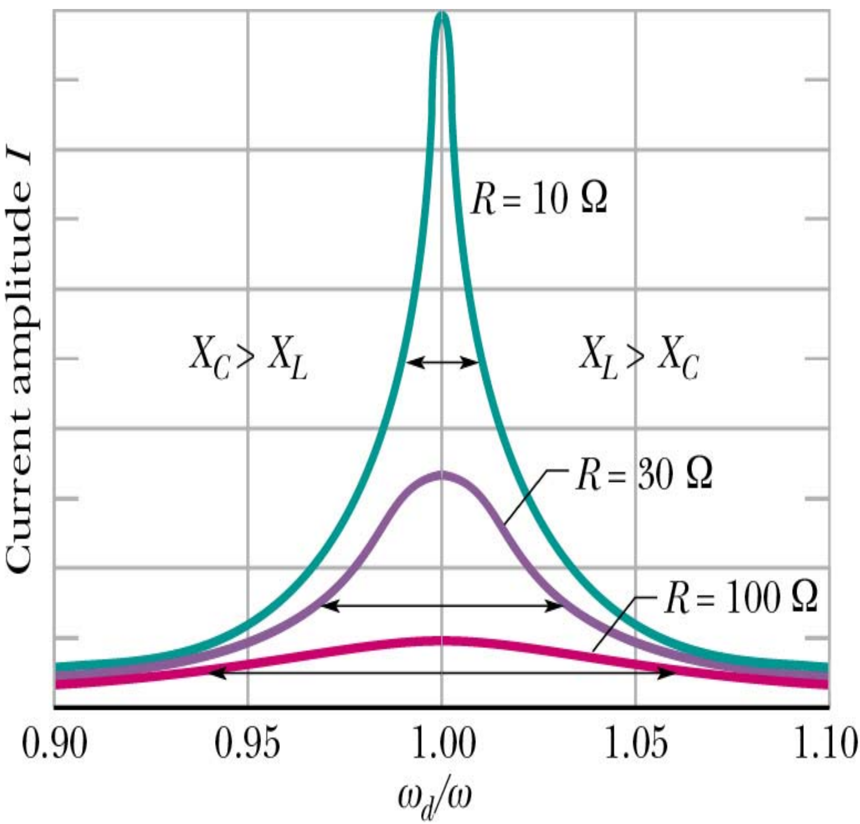
$$\omega_d L = \frac{1}{\omega_d C}$$

- Or

$$\omega_d = \sqrt{\frac{1}{LC}} = \omega$$

$\omega$ is also called the resonance frequency (in the homework)

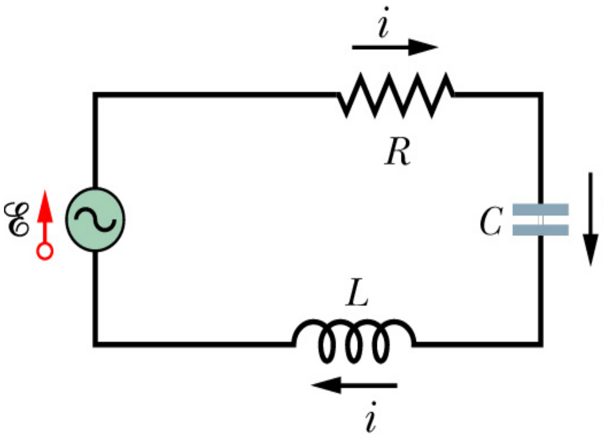
$$\omega = \omega_R = 2\pi f_R$$



# AC circuits

- RLC circuit – resistor, capacitor and inductor in series
- Apply alternating emf

$$\mathrm{E} = \mathrm{E}_m \sin \omega_d t$$




- Elements are in series so same current is driven through each

$$i = I \sin(\omega_d t - \phi)$$

- From the loop rule, at any time *t*, the sum of the voltages across the elements must equal the applied emf

$$\mathrm{E} = v_R + v_C + v_L$$

# AC circuits

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_m}{Z}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

- Define impedance, Z to be

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- Using reactances rewrite current as

$$X_L = \omega_d L$$

$$X_C = \frac{1}{\omega_d C}$$

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$

# AC circuits

- Relationship between reactances and frequencies
- From $$X_L = \omega_d L \qquad X_C = \frac{1}{\omega_d C}$$
- We have $$\frac{X_L}{X_C} = (\omega_d)^2 LC$$
- or $$\frac{X_L}{X_C} = \frac{(\omega_d)^2}{\omega^2}$$

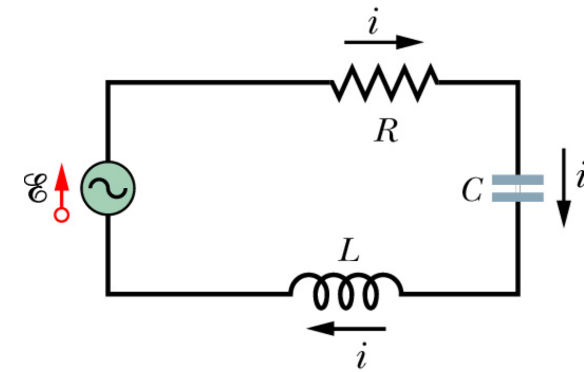
# AC circuits

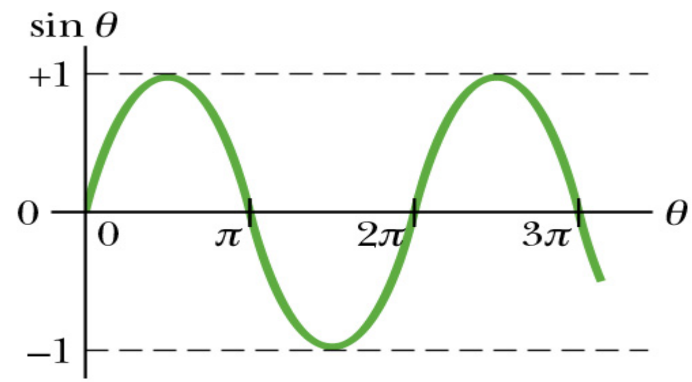
- Instantaneous rate which energy is dissipated (power) in a resistor is $P = i^2 R$
- But $i = I\sin(\omega_d t - \phi)$

$$P = I^2 R \sin^2(\omega_d t - \phi)$$

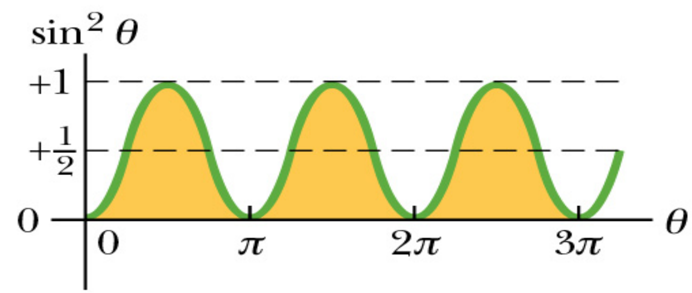
- Want average (rms) rate
  - Average over complete cycle T

$$\langle \sin^2\theta \rangle = 1/2$$




(a)



# AC circuits

- For alternating current circuits define root-mean-square or rms values for *i, V* and emf

$$I_{rms} = \frac{I}{\sqrt{2}} \qquad V_{rms} = \frac{V}{\sqrt{2}} \qquad \mathrm{E}_{rms} = \frac{\mathrm{E}}{\sqrt{2}}$$

- Ammeters, voltmeters - give rms values
- The average (rms) power dissipated by resistor in an ac circuit is

$$P_{rms} = \frac{I^2 R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R \qquad P_{rms} = I_{rms}^2 R$$

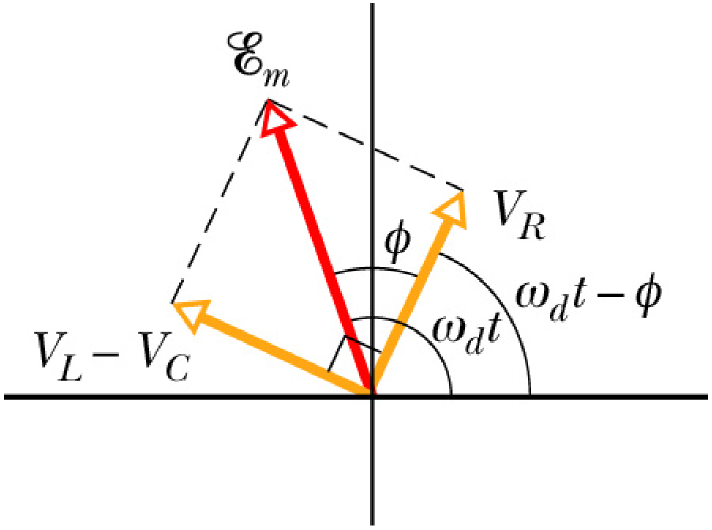
# EM Oscillations

- Write average power in another form using

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} \qquad P_{avg} = I_{rms}^2 R = \frac{\mathcal{E}_{rms}}{Z} I_{rms} R = \mathcal{E}_{rms} I_{rms} \frac{R}{Z}$$

- Using phasor and amplitude relations

$$\cos\phi = \frac{V_R}{\mathcal{E}_m} = \frac{IR}{IZ} = \frac{R}{Z}$$




- Rewrite average power as

$$P_{avg} = \mathcal{E}_{rms} I_{rms} \cos\phi$$

# AC circuits

- If ac circuit has only resistive load *Z=R* (e.g. at the resonance frequency)

$$P_{rms} = \mathrm{E}_{rms} I_{rms}$$

- Trade-off between current and voltage
  - For general use want low voltage
  - Means high current but

$$P_{rms} = I_{rms}^2 R$$

- General energy transmission rule: Transmit at the highest possible voltage and the lowest possible current