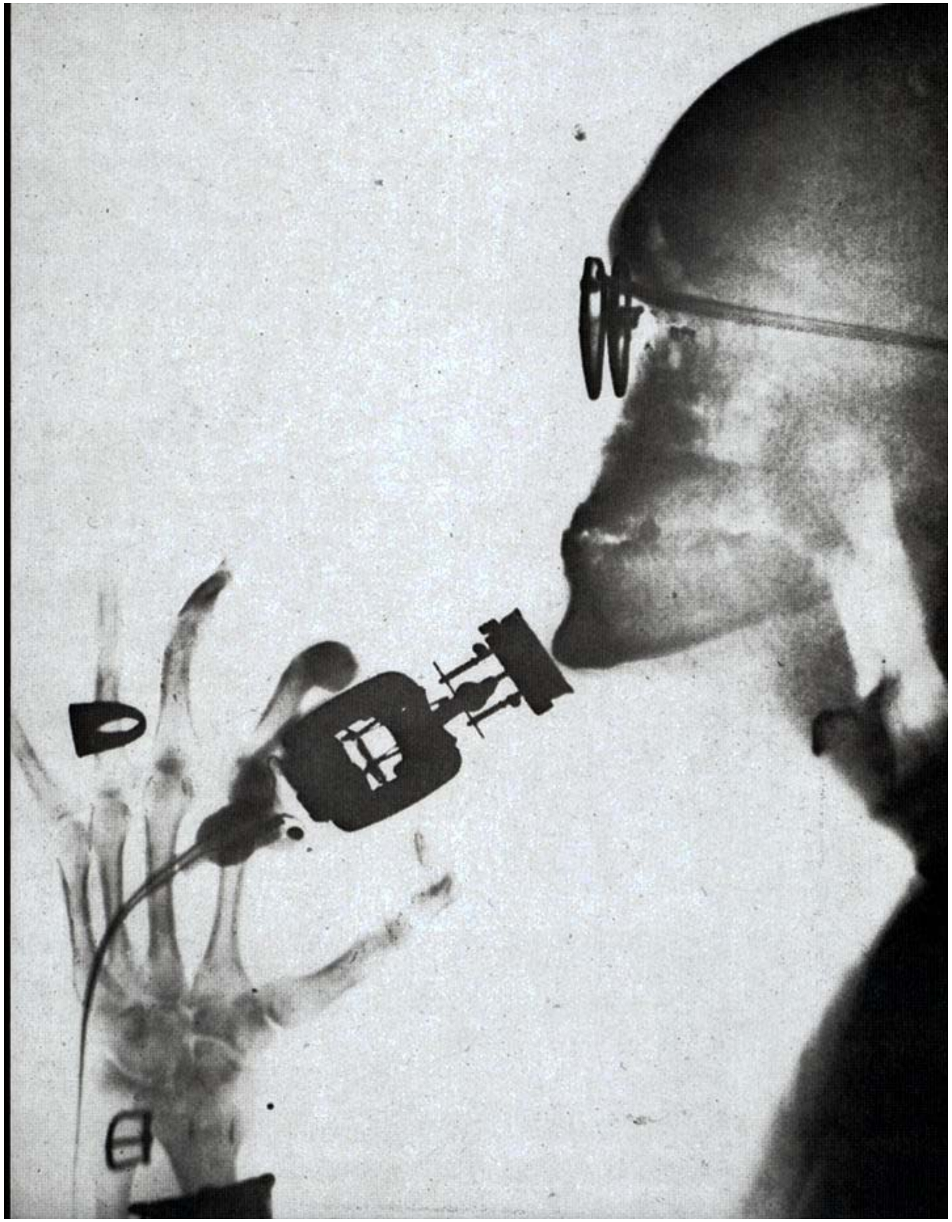
Electromagnetic Waves

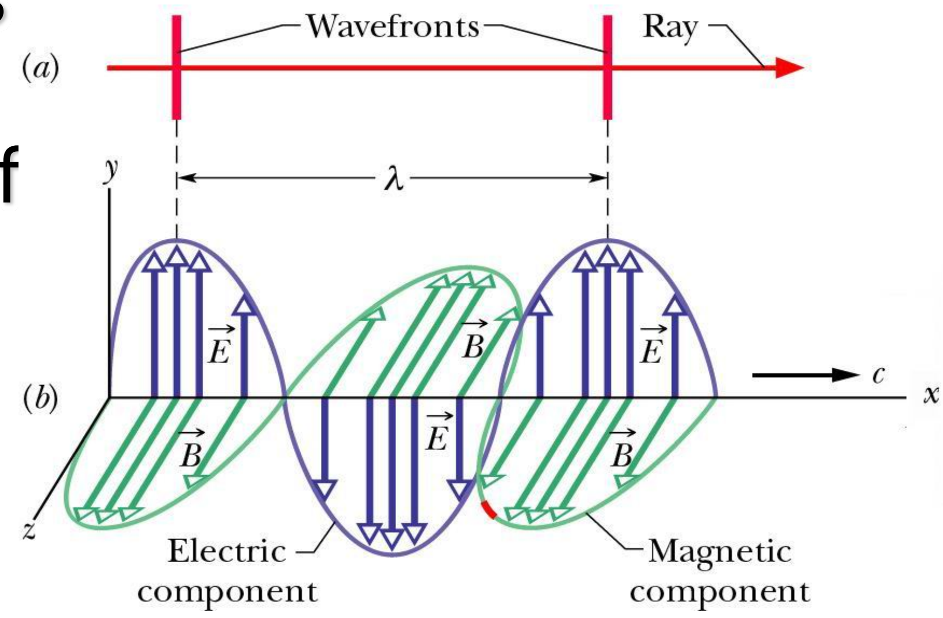


E & B fields of EM Waves

- Write E and B fields as sinusoidal functions of position x (along path of wave) and time t

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$



- Angular frequency ω and angular wave number k

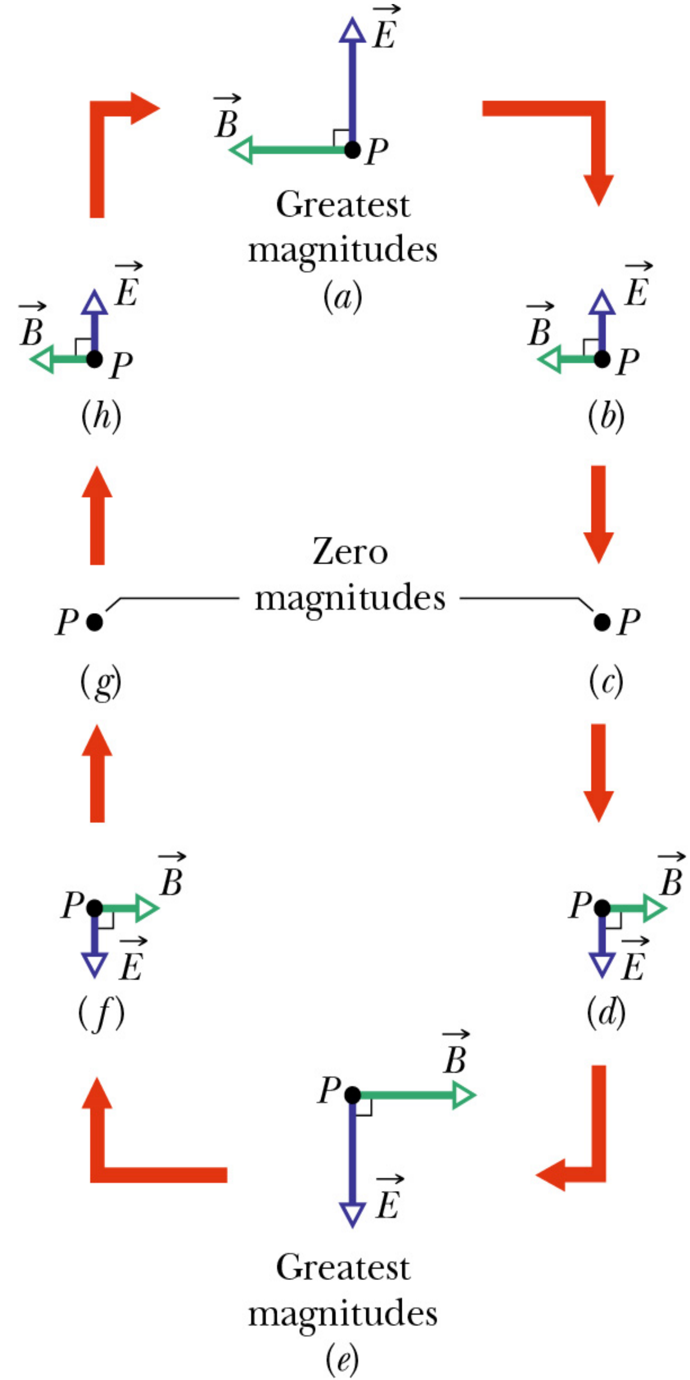
$$\omega = 2\pi f$$

$$k = \frac{2\pi}{\lambda}$$

- E and B components cannot exist independently

Time-dependence of E & B fields

- *E* and *B* fields change with time and have features:
  - *E* and *B* fields ⊥ to direction of wave's travel – transverse wave
  - *E* field is ⊥ *B* field
  - Direction of wave's travel is given by cross product $\vec{E} \times \vec{B}$
  - *E* and *B* fields vary
    - Sinusoidally
    - With same frequency and in phase



Frequency, Wavelength & Velocity

- Speed of wave is

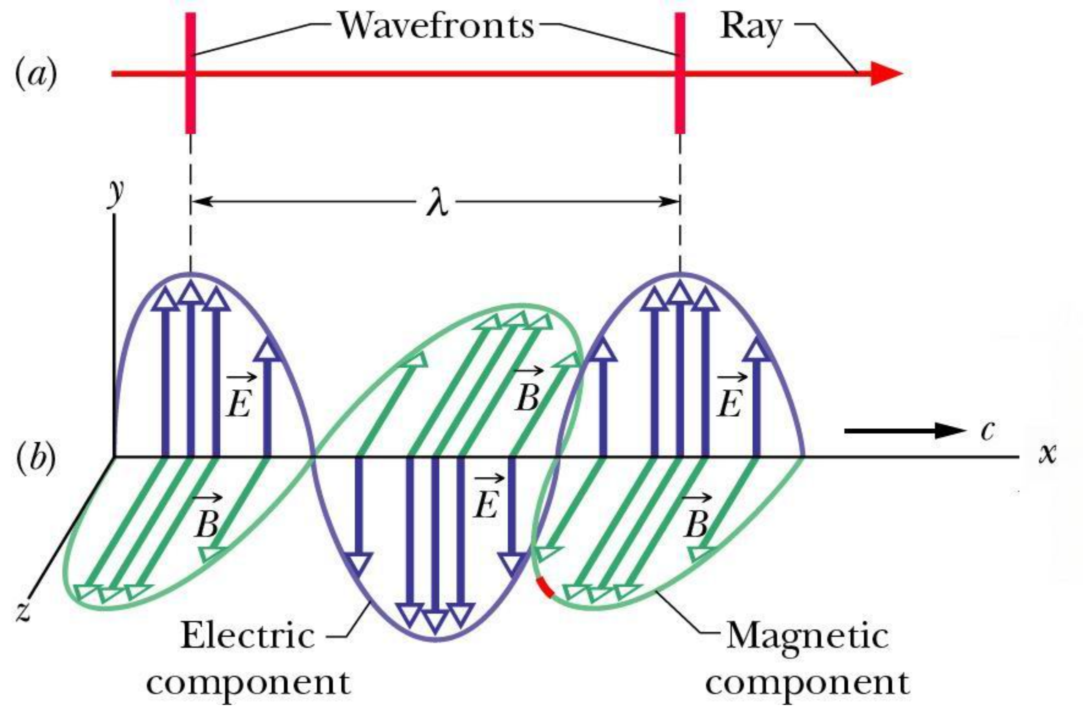
$$v = f\lambda$$

- Or

$$v = \frac{\omega}{k}$$

- In vacuum EM waves move at speed of light

$$v = c = 3\times10^{8}\, m/s$$



Maxwell's equations

- Gauss's law for *E* fields

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

- Gauss's law for *B* fields

$$\Phi_B = \oint \vec{B} \bullet d\vec{A} = 0$$

- Both cases integrate over closed Gaussian surface

Maxwell's equations

- Faraday's law of induction *E* field is induced along a closed loop by a changing magnetic flux encircled by that loop

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

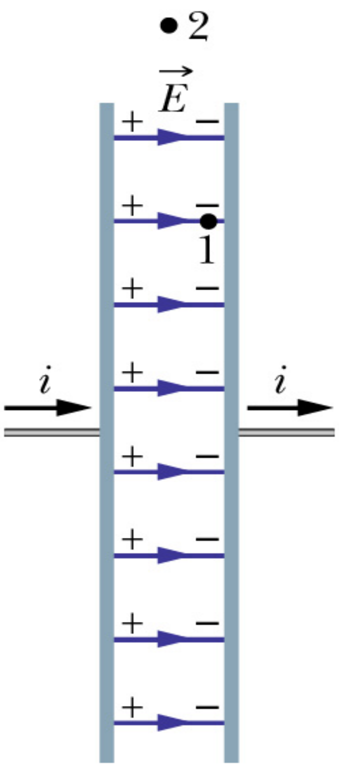
- Is the reverse true?
- Maxwell's law of induction *B* field is induced along a closed loop by a changing electric flux in region encircled by loop

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's equations

- Consider circular parallel-plate capacitor with E field increasing at a steady rate
- While E field changing, B fields are induced between plates, both inside and outside (point 1 and 2).
- If E field stops changing, B field disappears

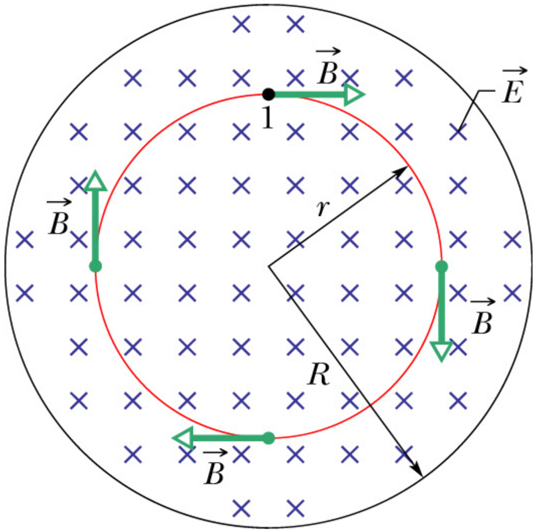
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$



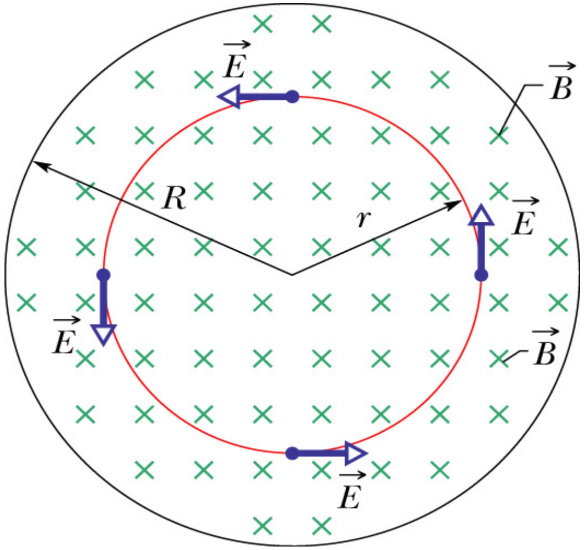
Maxwell's equations

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$



- Two differences
  - Extra symbols, μ_0 and ε_0, to preserve SI units
  - Minus sign – means induced E field and induced B field have opposite directions when produced in similar situations



Maxwell's equations

- Ampere's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Combine Ampere's and Maxwell's law

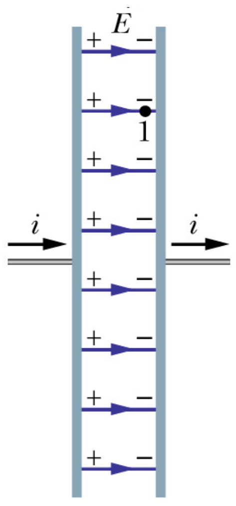
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- *B* field can be produced by a current and/or a changing *E* field
  - Wire carrying constant current, $d\Phi_E/dt = 0$
  - Charging a capacitor, no current so $i_{enc} = 0$

Maxwell's equations

- What is the induced B field inside a circular capacitor which is being charged?

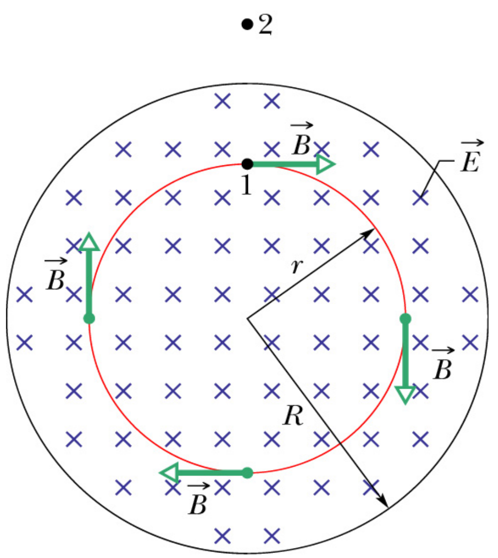
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$



(a)

- No current between capacitor plates so $i_{enc} = 0$ and equation becomes

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$



Maxwell's 4 equations

- Gauss' Law $$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$
- Gauss' Law for magnetism $$\oint \vec{B} \bullet d\vec{A} = 0$$
- Faraday's Law $$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$
- Ampere-Maxwell Law $$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

Traveling EM Waves

- Use Faraday's and Maxwell's laws of induction

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Can prove that speed of light c is given by (proof done in class and in textbook – long story)

$$c = \frac{E_m}{B_m} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$c = 3 \times 10^8 m/s$$

- Light travels at same speed regardless of what reference frame its measured in