

"You've got mail."

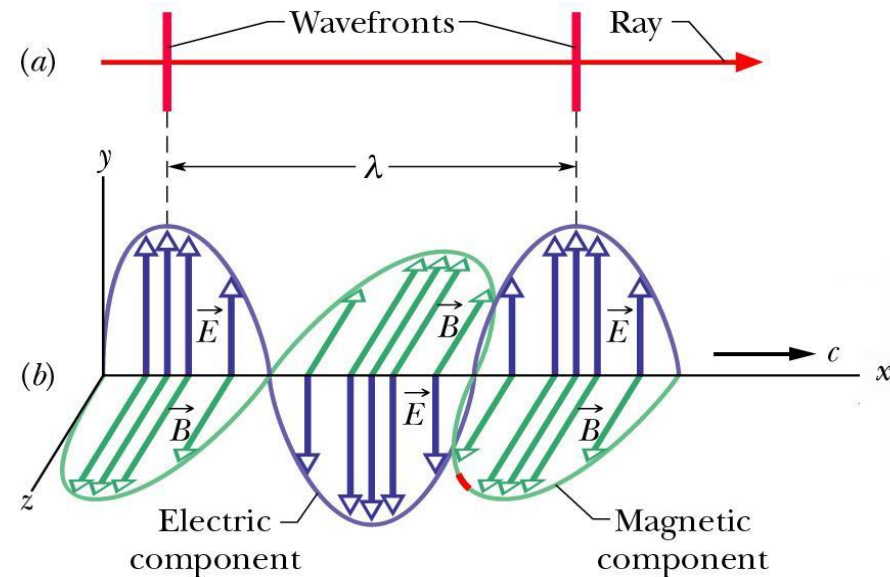
Review: EM Waves

- Write E and B fields as sinusoidal functions of position x (along path of wave) and time t

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$

- Angular frequency ω and angular wave number k
- E and B components cannot exist independently

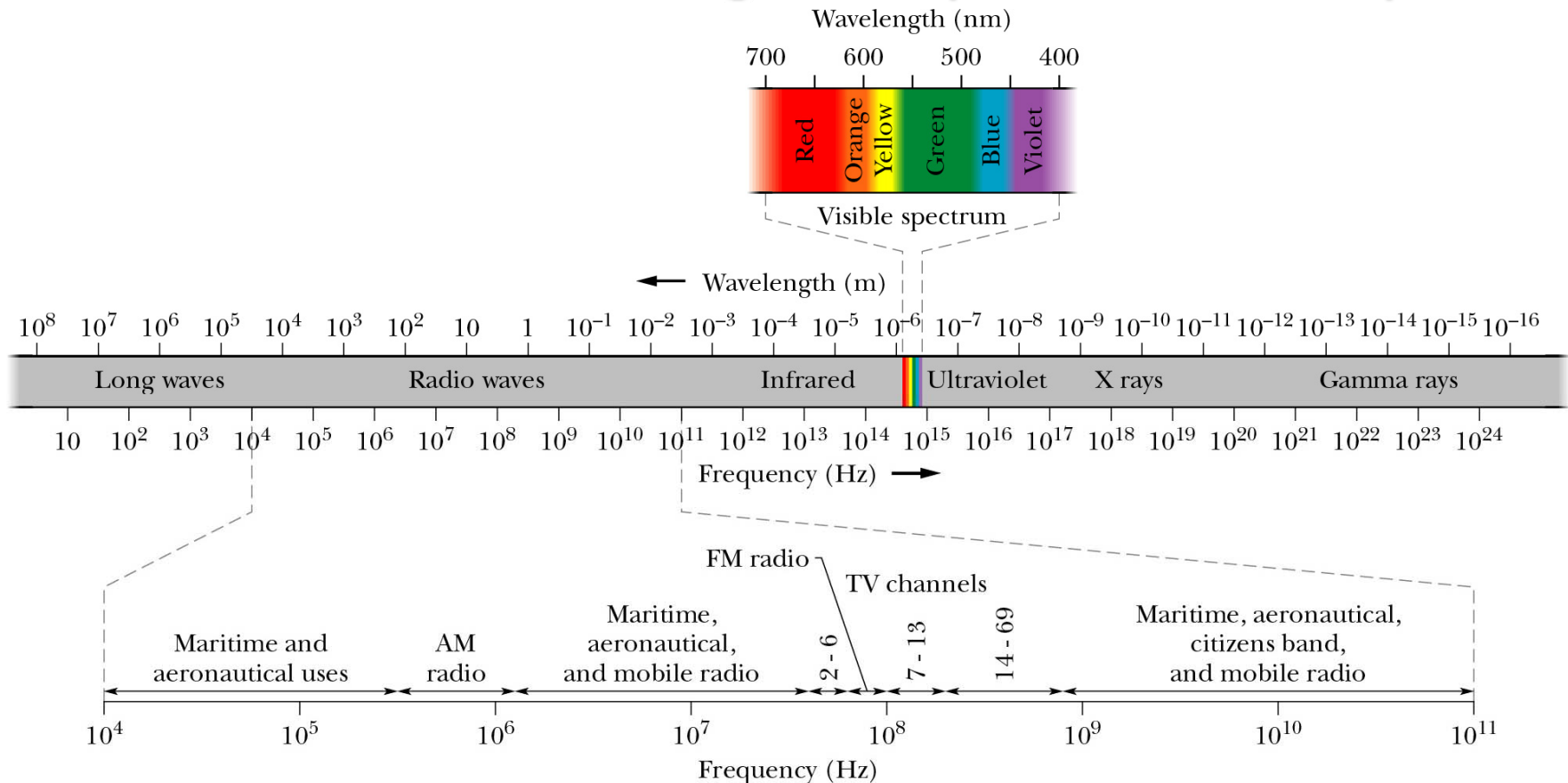


$$\omega = 2\pi f$$

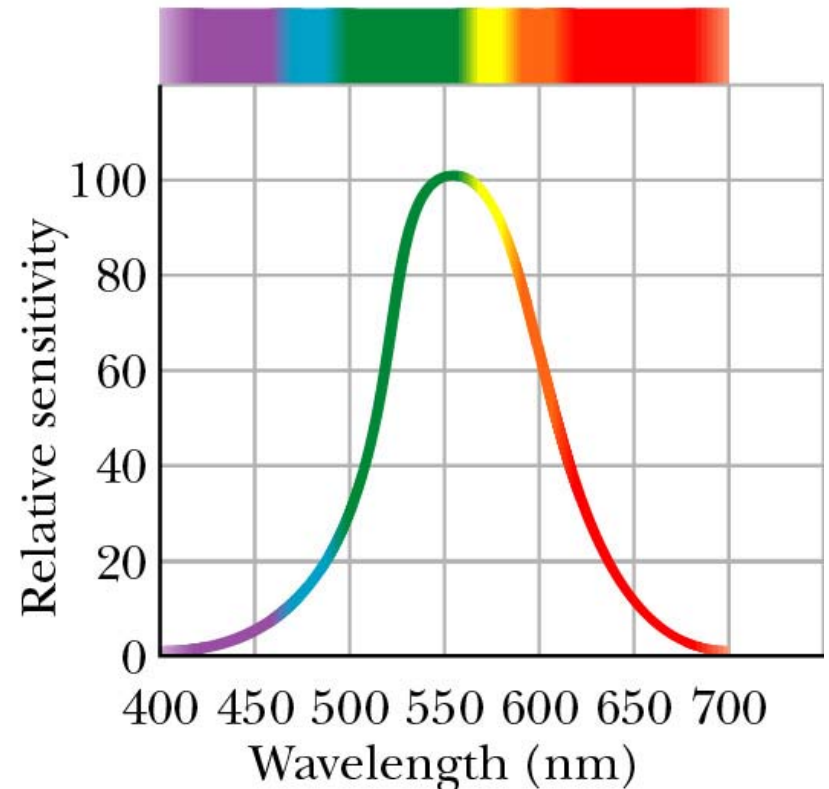
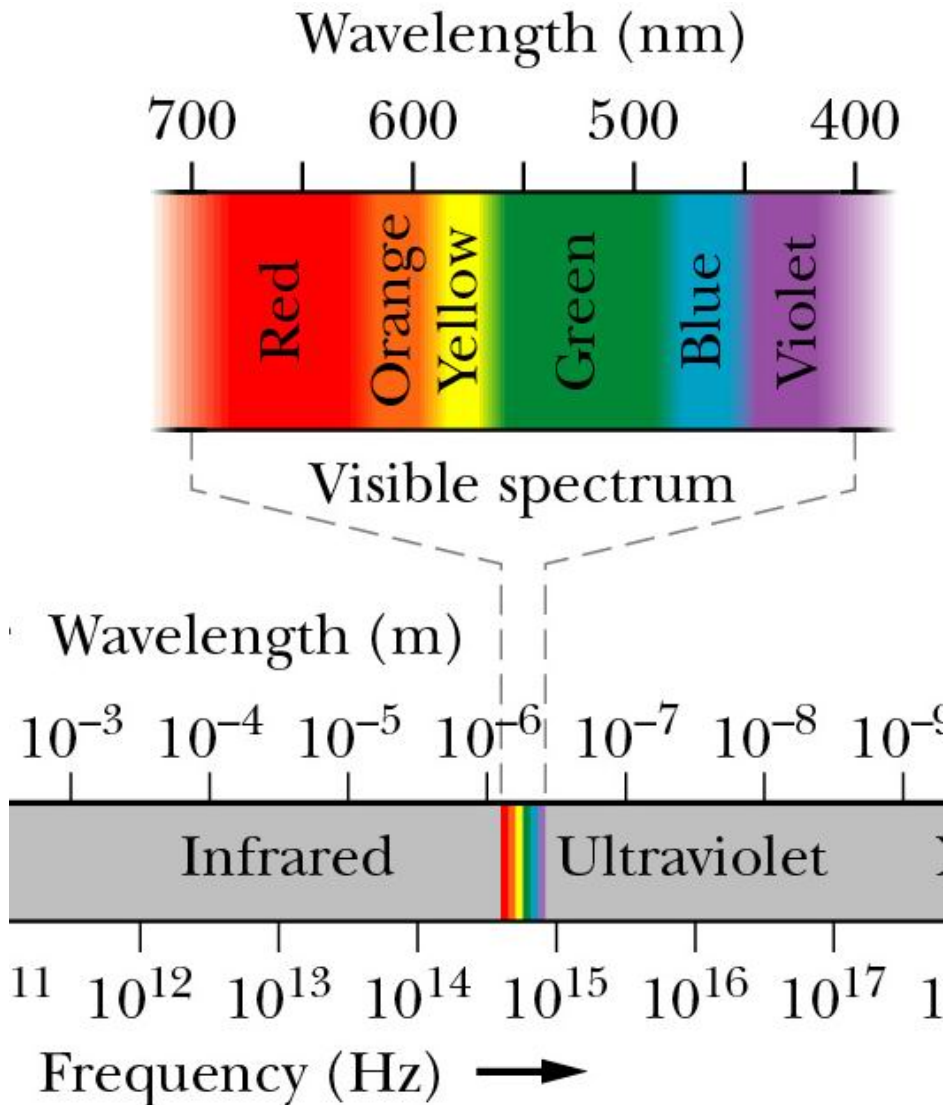
$$k = \frac{2\pi}{\lambda}$$

EM Waves Spectrum

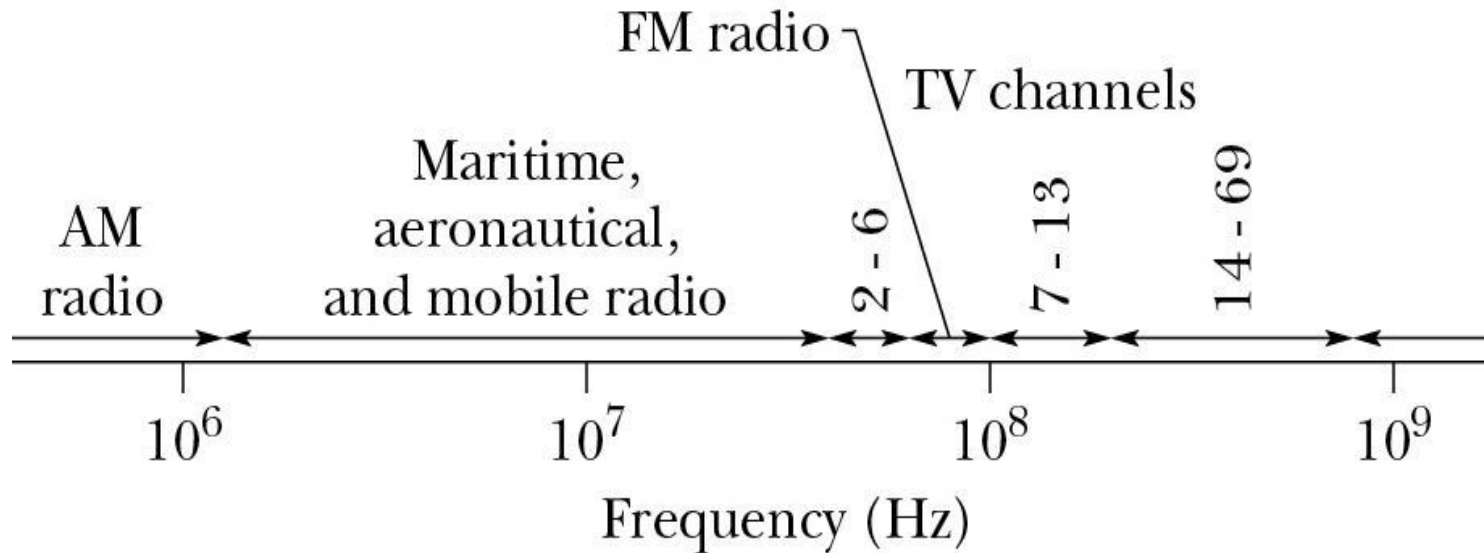
- Electromagnetic waves
 - Beam of light is a traveling wave of E and B fields
 - All waves travel through free space with same speed



EM Waves Spectrum



EM Waves Spectrum

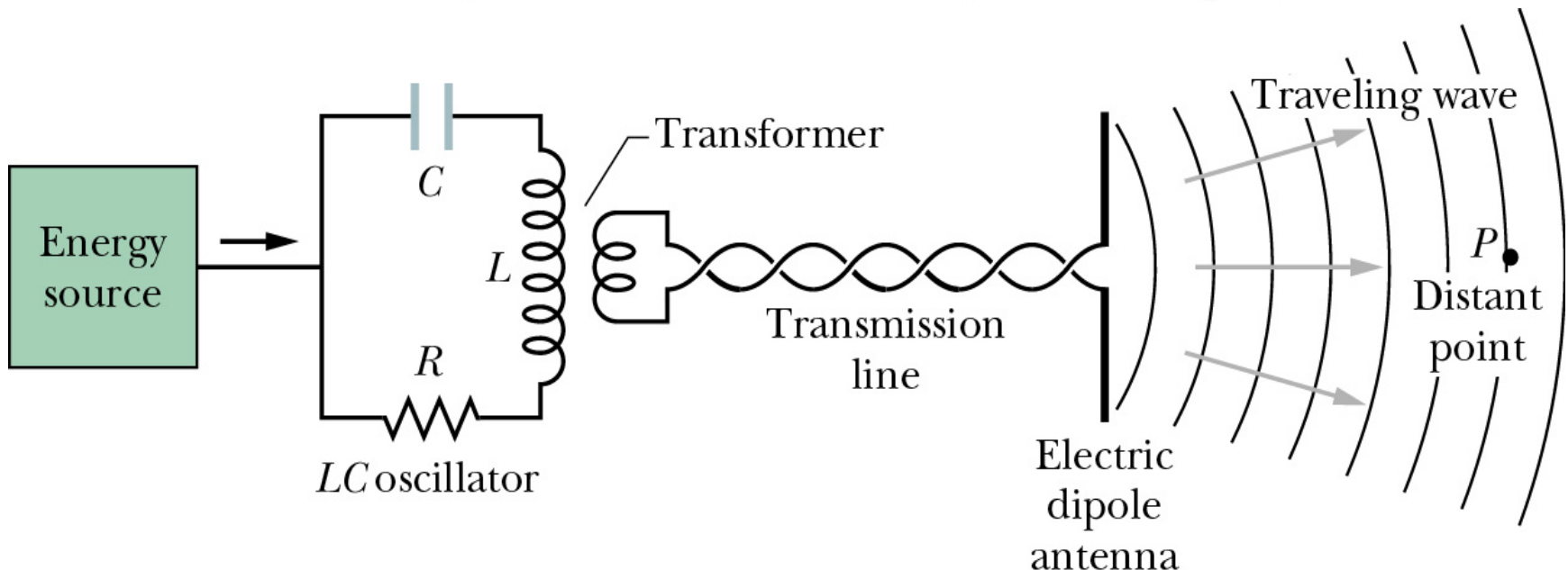


- AM radio 100-1000 meters
- FM radio 1-10 meters
- TV channels 0.1-10 meters

Traveling EM Waves

- **Generating electromagnetic (EM) waves**
 - Sinusoidal current in RLC causes charge and current to oscillate along rods of antenna with angular frequency ω
 - Changing E and B fields form EM wave that travels away from antenna at speed of light, c

$$\omega = \frac{1}{\sqrt{LC}}$$



Energy transport in EM Waves

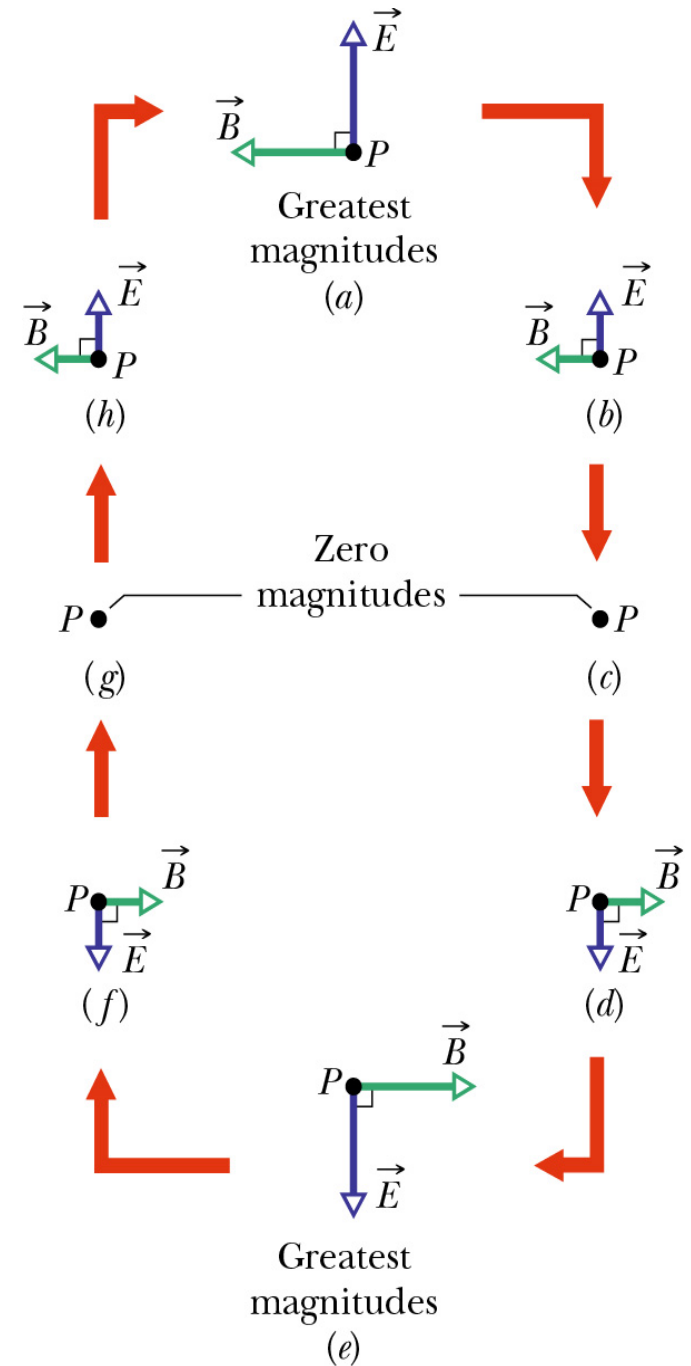
- EM waves can transport energy and deliver it to an object it falls on
- Rate of energy transported per unit area is given by **Poynting vector**, S , and defined as

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

- SI unit is W/m^2
- Direction of S gives wave's direction of travel

Traveling EM Waves

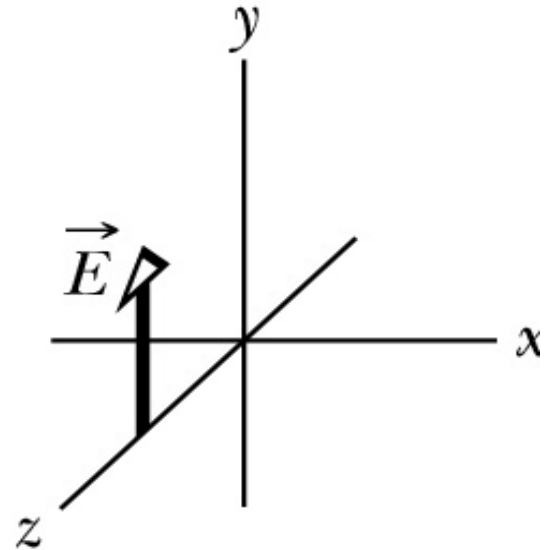
$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$



Exercise

- Have an E field shown in picture. A wave is transporting energy in the negative z direction. What is the direction of the B field of the wave?
- Poynting vector gives

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$



- Use right-hand rule to find B field

Positive x direction

Energy Transport in EM Waves

- Magnitude of S is given by

$$S = \frac{1}{\mu_0} EB$$

- Found relation

$$c = \frac{E}{B}$$

- Rewrite S in terms of E since most instruments measure E component rather than B

$$S = \frac{1}{\mu_0} E \frac{E}{c}$$

- **Instantaneous energy flow rate** is

$$S = \frac{1}{c\mu_0} E^2$$

Energy Transport in EM Waves

- Usually want time-averaged value of S also called intensity I

$$I = S_{avg} = \left(\frac{\text{energy / time}}{\text{area}} \right)_{ave} = \left(\frac{\text{power}}{\text{area}} \right)_{ave}$$

$$I = \frac{1}{\mu_0 c} [E^2]_{avg} = \frac{1}{\mu_0 c} [E_m^2 \sin^2(kx - \omega t)]_{avg}$$

- Average value over full cycle of $\sin^2 \theta = 1/2$

- Use the rms value

$$E_{rms} = \frac{E_m}{\sqrt{2}}$$

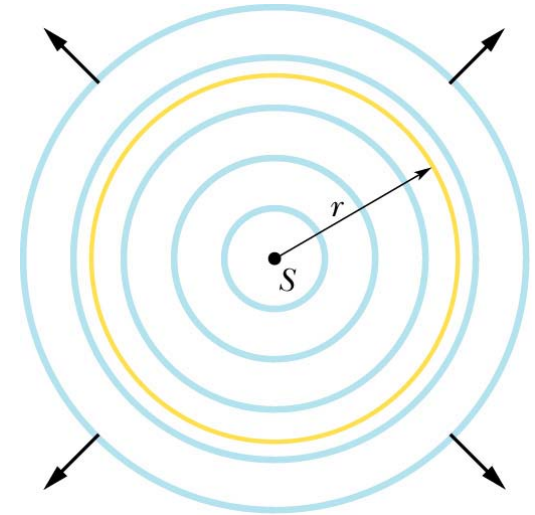
- Rewrite average S or intensity as

$$I = \frac{1}{\mu_0 c} E_{rms}^2$$

Energy Transport

- Find intensity, I , of point source which emits light isotropically – equal in all directions

$$I = S_{avg} = \left(\frac{\text{energy / time}}{\text{area}} \right)_{ave} = \left(\frac{\text{power}}{\text{area}} \right)_{ave}$$



- Find I at distance r from source
- Imagine sphere of radius r and area

$$A = 4\pi r^2$$

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P_s}{4\pi r^2}$$

- I decreases with square of distance

Exercise

- Isotropic point light source has power of 250 W. You are 1.8 meters away. Calculate the rms values of the E and B fields.

- To find E_{rms} need

$$I = \frac{1}{c\mu_0} E_{rms}^2$$

$$I = \frac{P_s}{4\pi r^2}$$

- Find intensity I from

$$E_{rms} = \sqrt{Ic\mu_0} = \sqrt{\frac{P_s c \mu_0}{4\pi r^2}}$$

$$E_{rms} = \sqrt{\frac{(250)(3 \times 10^8)(1.26 \times 10^{-8})}{(4\pi)(1.8)^2}} = 48.1 \text{ V / m}$$

Exercise

- Isotropic point light source has power of 250 W. You are 1.8 meters away. Calculate the rms values of the E and B fields.
- To find B_{rms} need

$$c = \frac{E_{rms}}{B_{rms}}$$

$$B_{rms} = \frac{E_{rms}}{c}$$

$$B_{rms} = \frac{48.1 \text{ V} / \text{m}}{3 \times 10^8 \text{ m} / \text{s}} = 1.6 \times 10^{-7} \text{ T}$$

Exercise

- Look at sizes of E_{rms} and B_{rms}

$$E_{rms} = 48.1 \text{ V} / \text{m}$$

$$B_{rms} = 1.6 \times 10^{-7} \text{ T}$$

- This is why most instruments measure E
- Does not mean that E component is stronger than B component in EM wave
 - Can't compare different units
- Average energies are equal for E and B

EM Waves: Energy Density

- The energy density of electric field, u_E is equal to energy density of magnetic field, u_B

$$u_E = \frac{1}{2} \epsilon_0 E^2 \quad E = Bc$$

$$u_E = \frac{1}{2} \epsilon_0 (cB)^2 = \frac{1}{2} \epsilon_0 c^2 B^2$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$u_E = \frac{1}{2} \epsilon_0 \frac{1}{\mu_0 \epsilon_0} B^2 = \frac{B^2}{2\mu_0}$$

$$u_B = \frac{B^2}{2\mu_0}$$

$$u_E = u_B$$

EM Waves: Radiation Pressure

- EM waves linear have momentum momentum as well as energy
- Light shining on object exerts a pressure – radiation pressure
- Object's change in momentum is related to its change in energy
 - If object absorbs all radiation from EM wave (total absorption)
 - If object reflects all radiation back in original direction (total reflection)

$$\Delta p = \frac{\Delta U}{c}$$

$$\Delta p = \frac{2\Delta U}{c}$$

EM Waves: Radiation Pressure

- Just defined intensity, I as power per unit area A so power is

$$P = IA$$

- Change in energy is amount of power P in time t

$$\Delta U = P \Delta t = IA \Delta t$$

- Want force of radiation on object

$$F = \frac{\Delta p}{\Delta t}$$

- For total absorption

$$\Delta p = \frac{\Delta U}{c}$$

- Find force is

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta U}{c \Delta t} = \frac{IA \Delta t}{c \Delta t} = \frac{IA}{c}$$

EM Waves: Radiation Pressure

- For total reflection back along original path

- We have $\Delta p = \frac{2\Delta U}{c}$ and $F = \frac{2IA}{c}$

EM Waves: Radiation Pressure

- Express in terms of radiation pressure p_r which is force/area

$$p_r = \frac{F}{A}$$

- SI unit is N/m² called **pascal** *Pa*

- Total absorption

$$p_r = \frac{I}{c}$$

- Total reflection

$$p_r = \frac{2I}{c}$$