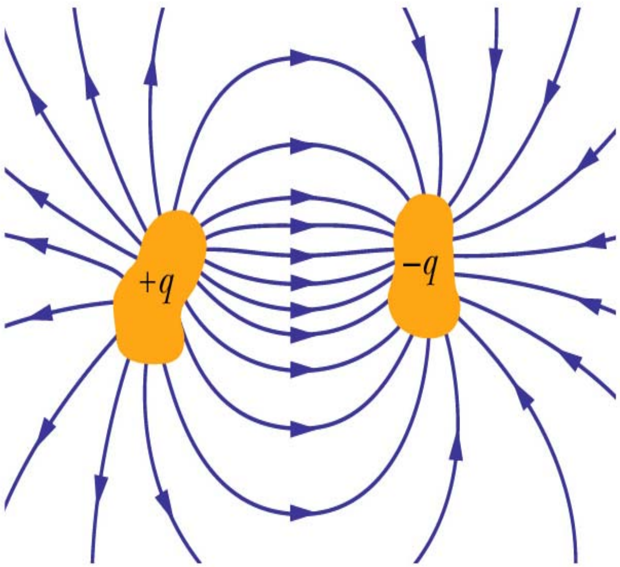
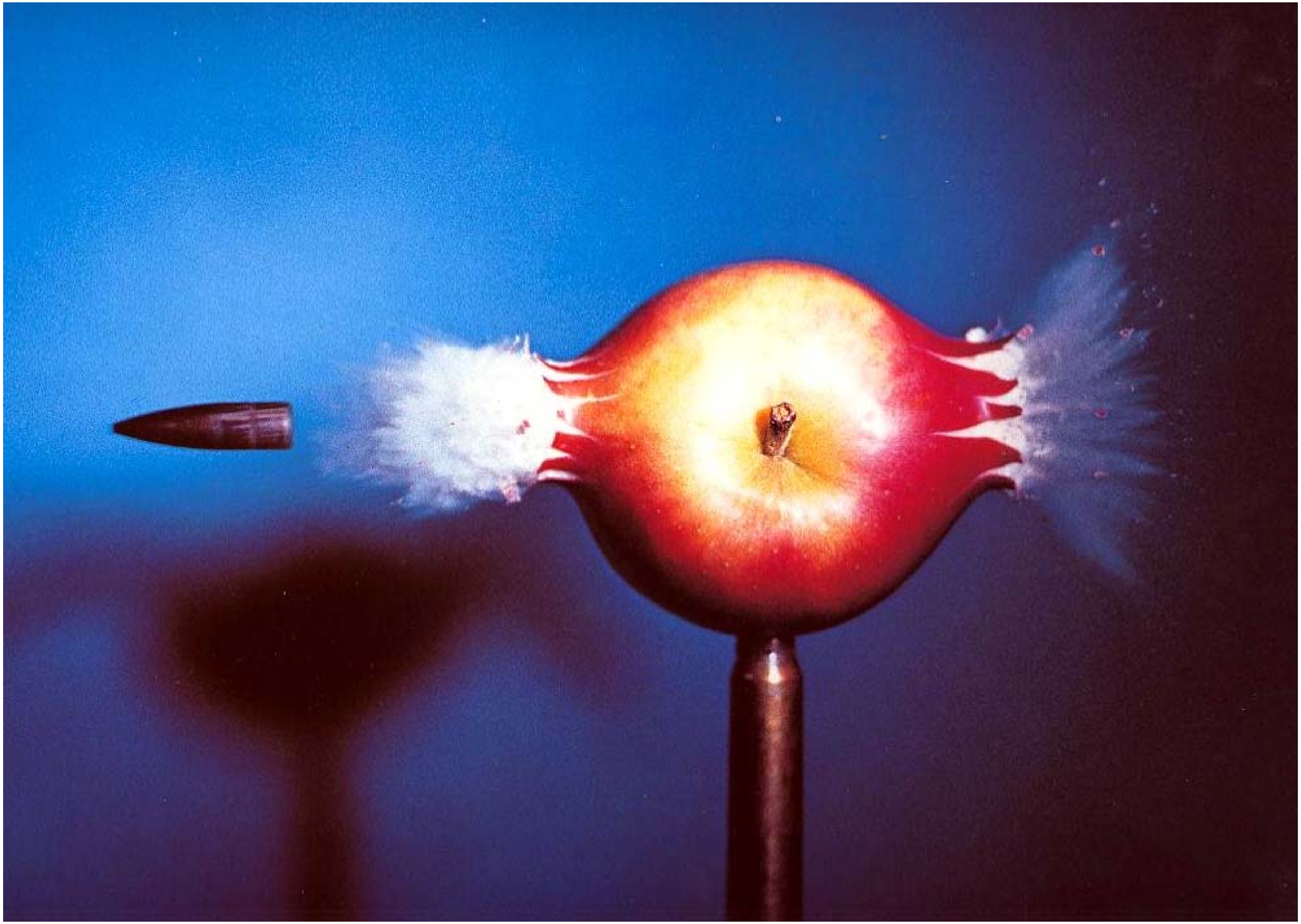
Capacitance

- Capacitor – device used to store potential energy from an *E* field
- The *E* field comes from stored charge
- This energy might be stored slowly, but can be released quickly – photoflash, heart defibrillator
- A capacitor is formed from two isolated conductors - equipotentials
- When capacitor is charged, plates have equal but opposite charges $+q$ and $-q$

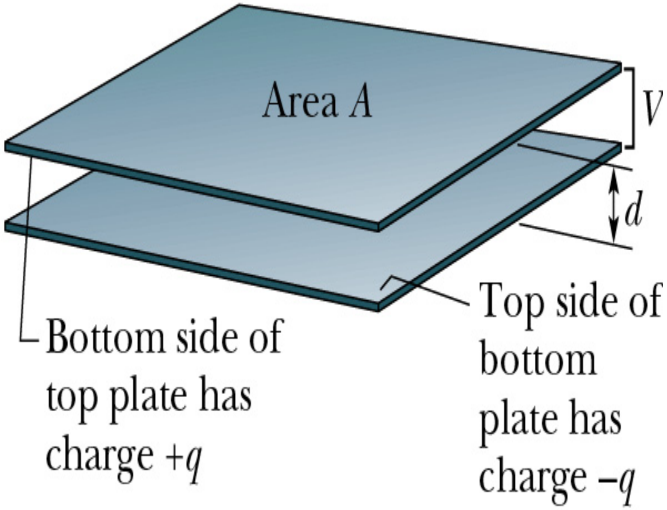


Stroboscope



Capacitance

- Capacitance is a proportionality constant relating q and V
  - q is the absolute value of the charge on one plate.
  - V is the potential difference between plates.



$$q = CV$$ or $$C = q/V$$

- C depends only on geometry of plates, not on their q or V

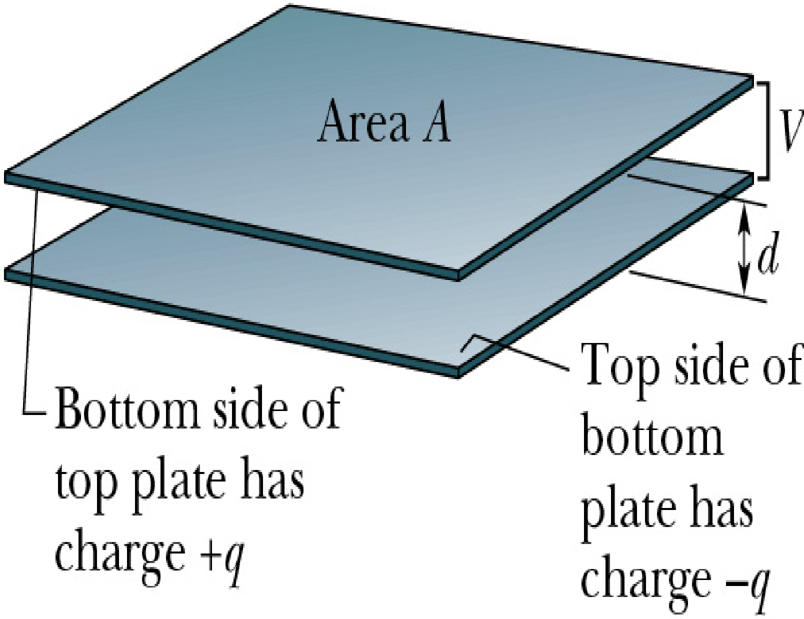
An assortment of capacitors.

Capacitance

- Capacitance is a measure of how much q is needed on plates to get V between them
  - Greater C, more q required

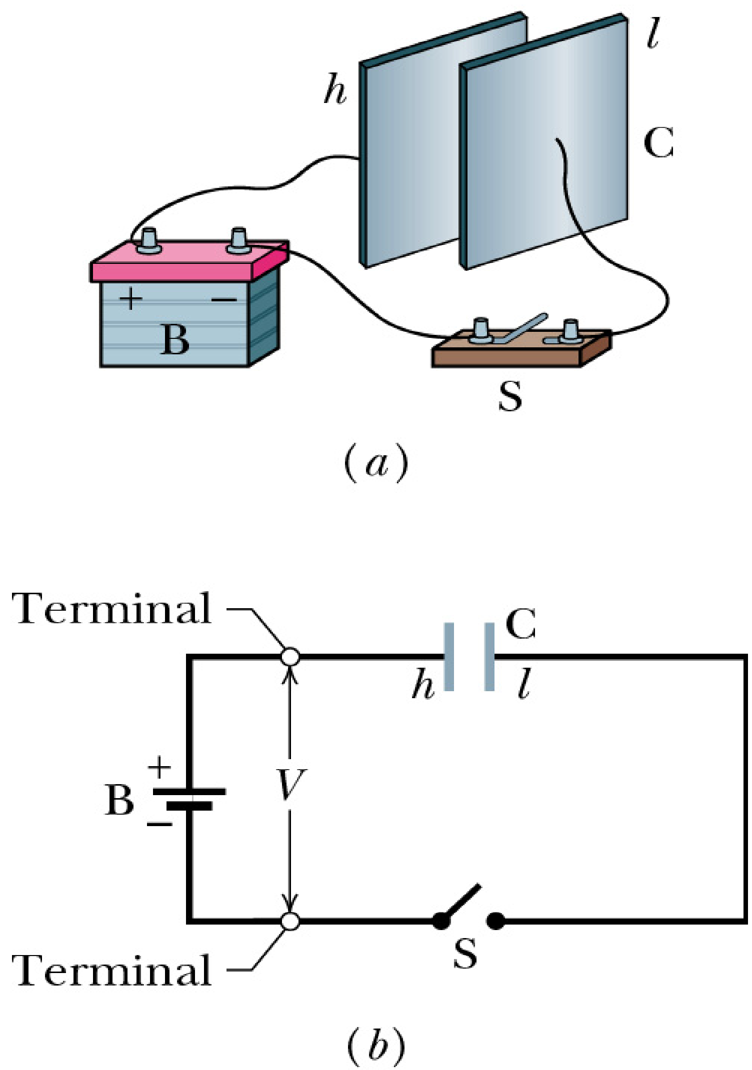
- SI unit for C is Farad

$$1F = 1C/V$$

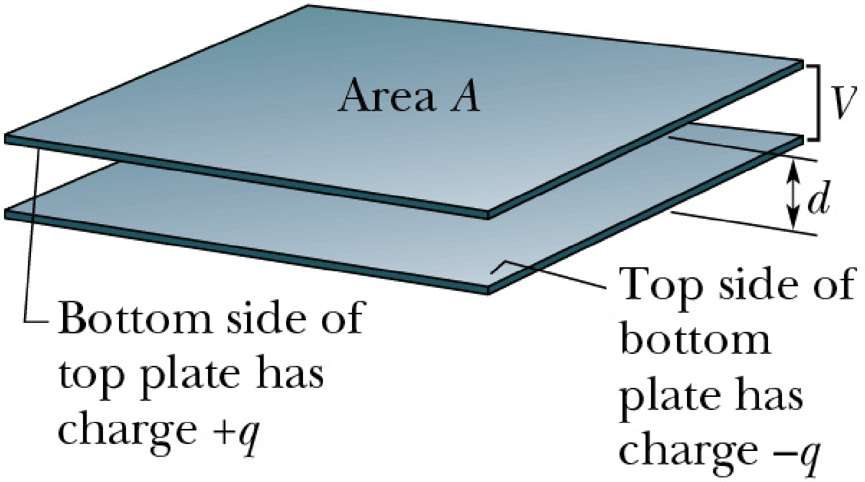


Capacitance

- Can charge a capacitor using a battery
- Battery – device maintains certain V between its terminals by internal electrochemical reactions
- Initially V on plates is 0
- Close switch, plates gradually charge up to V of battery through flow of electrons

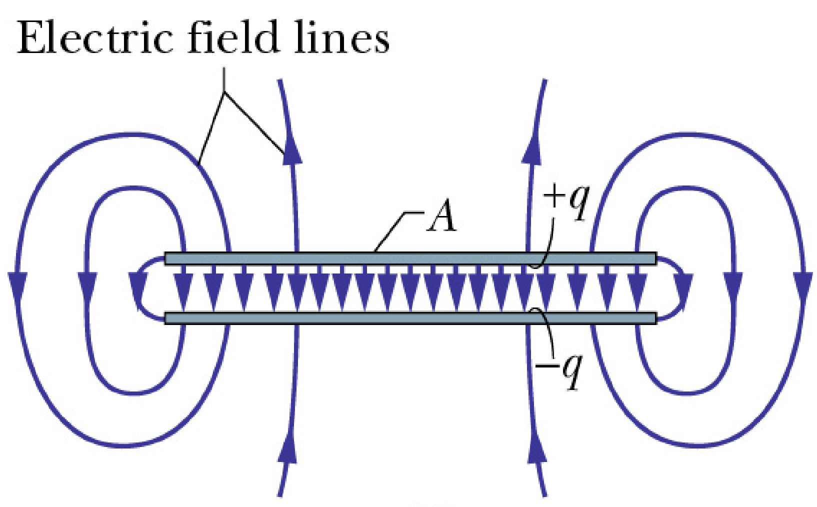


Capacitance of Parallel Plates



(a)

We ignore these (edge) fringe fields



(b)

Capacitance (Exercise)

- Does the C of a capacitor increase, decrease or remain the same when
  - A) charge, q, on it is doubled
  - B) V across it is tripled

Remember C of capacitor only depends on its geometry so C is the same for A and B

Capacitance – Method of Calculation

- Calculate *C* of a capacitor from its geometry using steps:
- 1) Assume charge, *q*, on the plates
- 2) Find *E* between plates using *q* and Gauss' law

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

- 3) Find *V* from *E* using

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{s}$$

- 4) Get *C* using

$$C = \frac{q}{V}$$

Capacitance of Parallel Plates

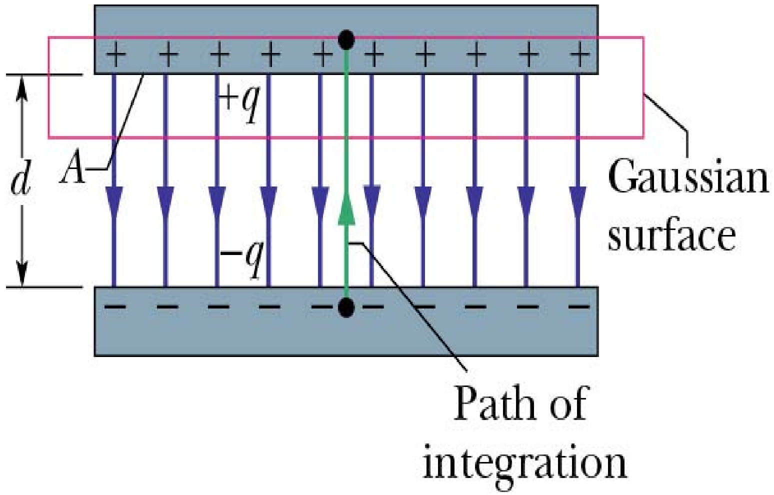
- Simplify Gauss' law

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

- 1) Pick Gaussian surface to enclose charge on + plate and E and dA to be parallel

$$\vec{E} \bullet d\vec{A} = EA$$

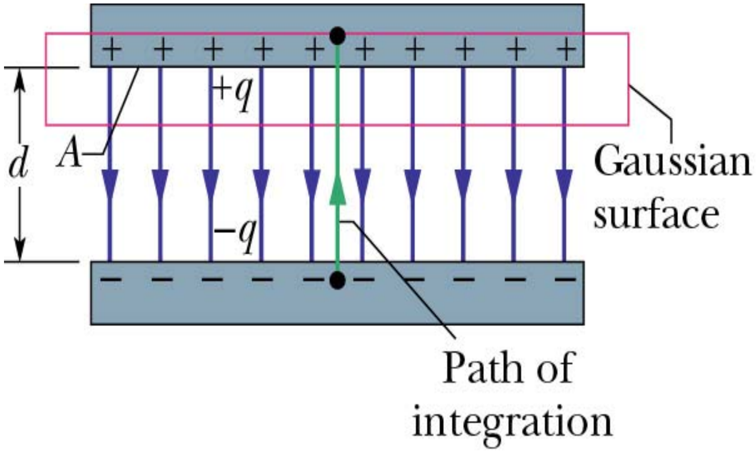
$$q = \varepsilon_0 EA$$



Capacitance of Parallel Plates

$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{s}$$

- 2) For *V* choose path that follows the *E* field line from – plate to + plate then *E* and *ds* are in opposite directions

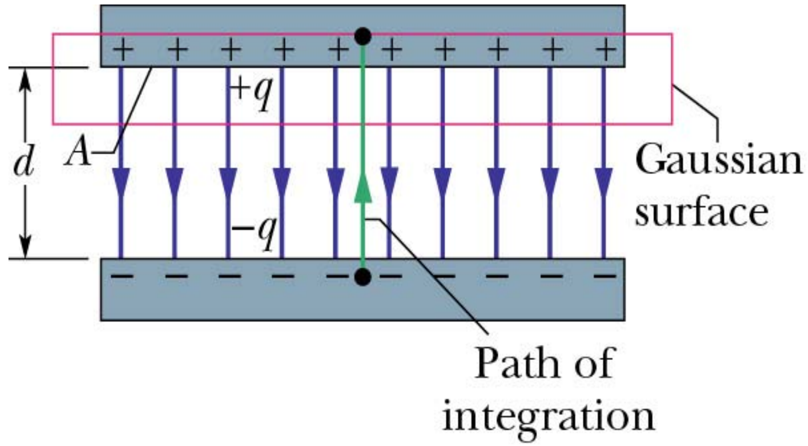


$$\vec{E} \bullet d\vec{s} = -Eds$$

$$V = V_f - V_i = \int_-^+ Eds$$

Capacitance of Parallel Plates

- Find *C* for parallel plate capacitor separated by *d*
  - E is constant between plates



$$V = \int_{-}^{+} E ds = E \int_{0}^{d} ds = Ed$$

- *A* is area of plates

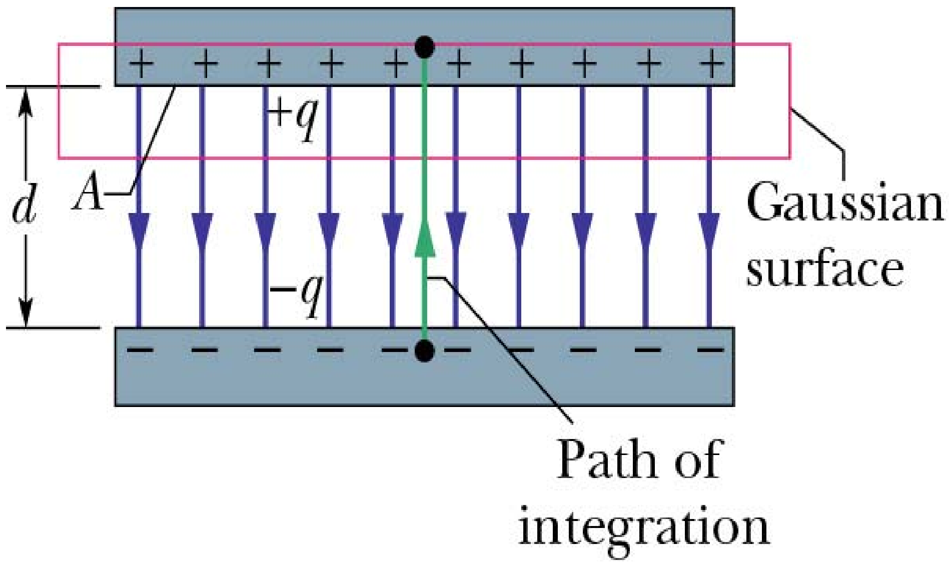
$$q = \varepsilon_0 EA$$

$$C = \frac{q}{V} = \frac{\varepsilon_0 EA}{Ed}$$

Capacitance of Parallel Plates

- Parallel-plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$



- Only depends on area *A* of plates and separation *d*
- *C* increases if we increase *A* or decrease *d*

Hollow Conductor

- Cylindrical capacitor

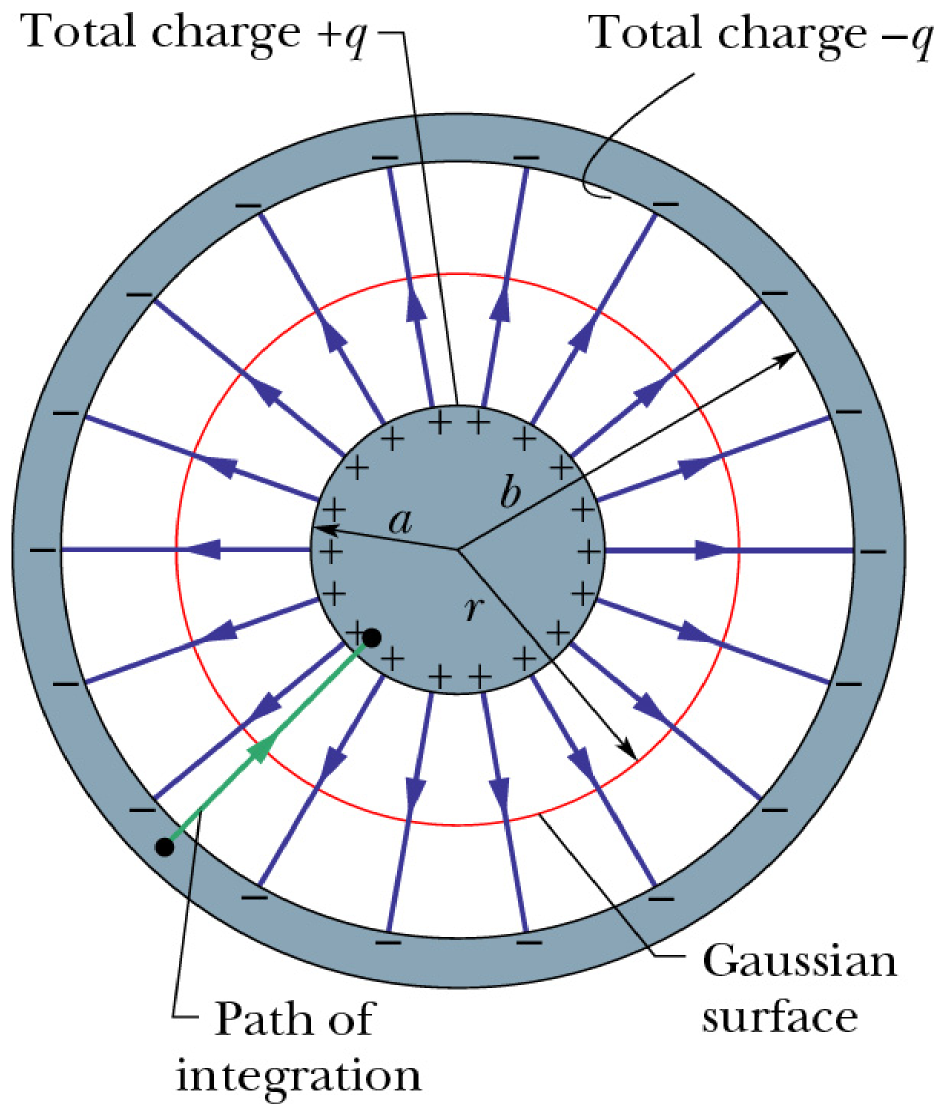
$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

- Spherical capacitor

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

- Isolated Sphere

$$C = 4\pi\varepsilon_0 r$$



Capacitance (Exercise)

- For capacitors charged by same battery, does q stored by capacitor increase, decrease or remain same if plate separation of parallel-plate capacitor is increased.

$$\boxed{q = CV}$$

- All capacitors have same potential V from battery and so q increases (decreases) with C

Capacitance

- If plate separation (*d*) of parallel plate capacitor is increased,

$$C = \frac{\varepsilon_0 A}{d}$$

- *d* increases so *C* decreases
- *C* decreases so *q* decreases

$$q = CV$$