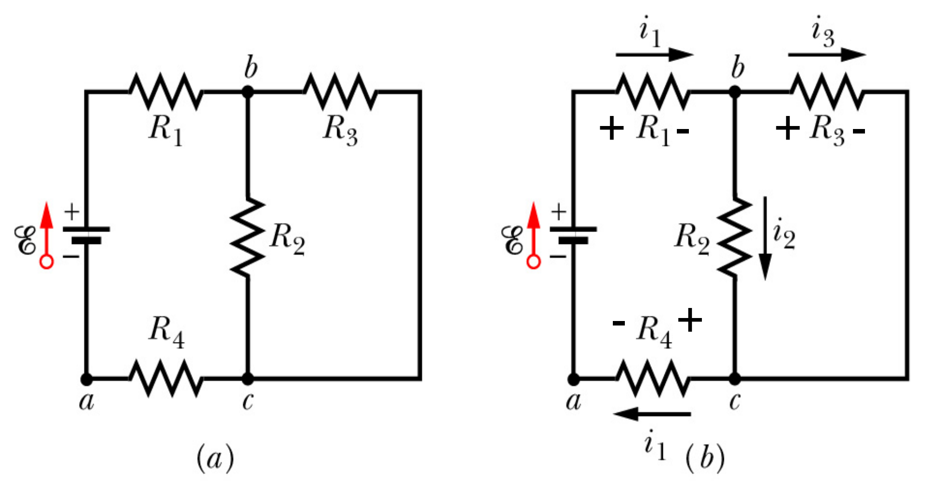
How to Analyze Complex Circuits

- Kirchhoff's junction rule (or current law) –
  - From conservation of charge
  - Sum of currents entering a junction is equal to sum of currents leaving that junction
- Kirchhoff's loop rule (or voltage law) –
  - From conservation of energy
  - Sum of changes in potential going around a complete circuit loop equals zero

Circuits

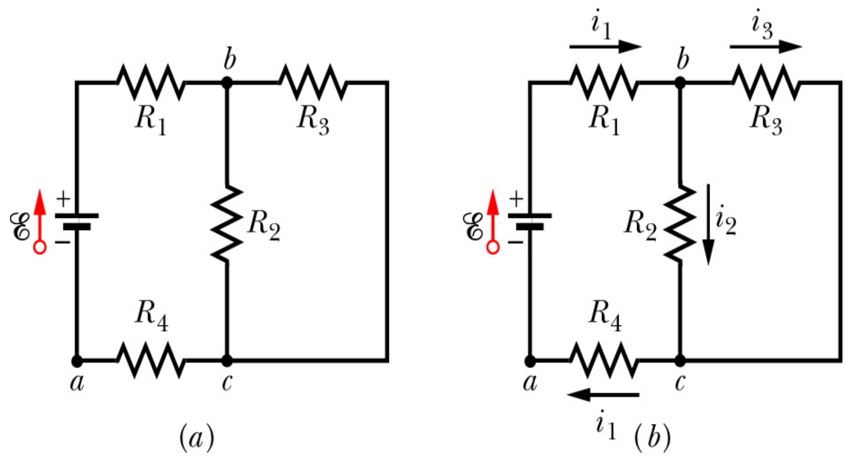
- What is *i* through the battery?
- Label currents. New label for every branch. Pick any arbitrary direction.
- *i* through R_1 or R_4 is same as for battery
- Can use loop rule



$$\mathrm{E} - i_1 R_1 - i_2 R_2 - i_1 R_4 = 0$$

Circuits

$$E - i_1 R_1 - i_2 R_2 - i_1 R_4 = 0$$



- Equation has too many unknowns so need to apply loop rule again

- Take the loop through R_2 and R_3

$$-i_3 R_3 + i_2 R_2 = 0$$

$$i_1 = i_2 + i_3$$

$$-(i_1 - i_2)R_3 + i_2 R_2 = 0$$

$$i_2 = -\frac{i_1 R_3}{(R_3 + R_2)}$$

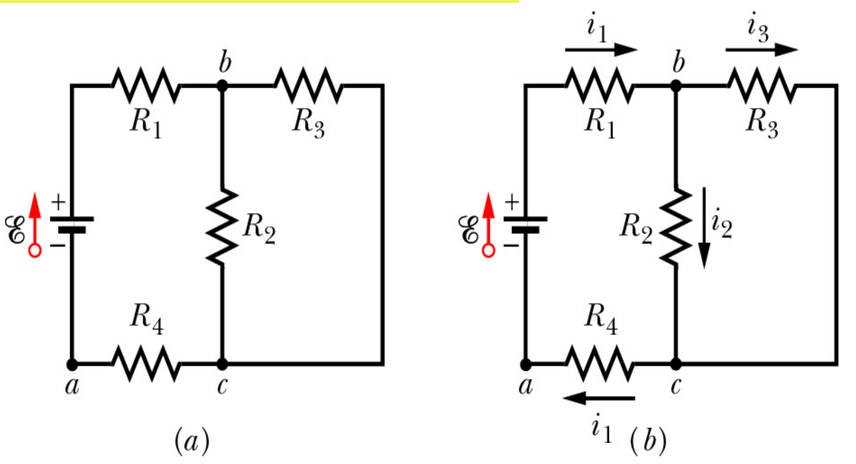
Circuits

Now solve for i_1:

$$\mathrm{E} - i_1 R_1 - i_2 R_2 - i_1 R_4 = 0$$

$$\mathrm{E} - i_1 R_1 - \frac{i_1 R_3 R_2}{(R_2 + R_3)} - i_1 R_4 = 0$$

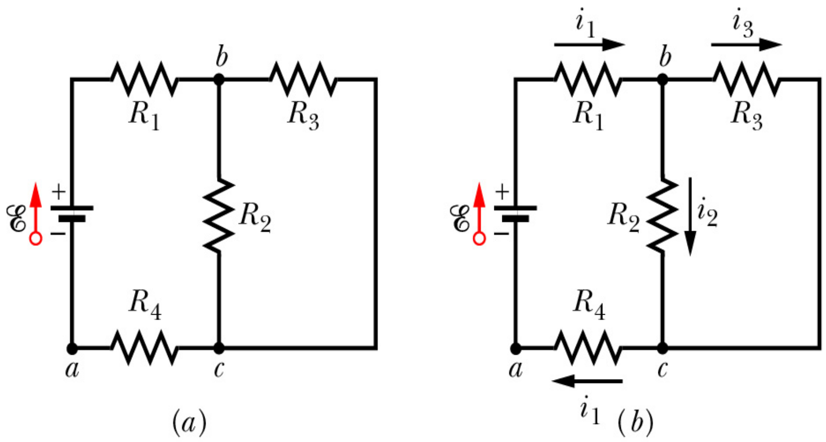
$$i_1 = \frac{\mathrm{E}}{R_1 + \frac{R_2 R_3}{(R_2 + R_3)} + R_4}$$



Circuits

- What is current Voltage lost (V) in R_2?
- Recall that V=iR

$$V_2 = i_2 R_2$$



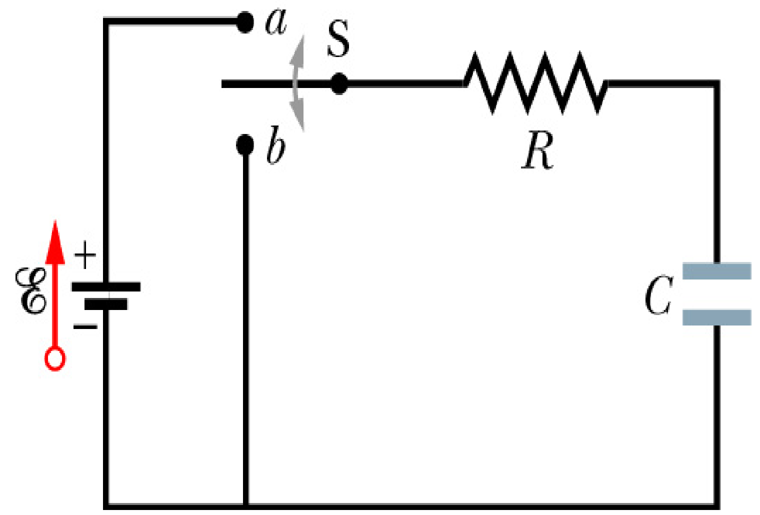
$$E - i_1 R_1 - i_2 R_2 - i_1 R_4 = 0$$

$$i_2 = \frac{i_1 R_1 + i_1 R_4 - E}{R_2}$$

$$i_1 = \frac{E}{R_1 + \frac{R_2 R_3}{(R_2 + R_3)} + R_4}$$

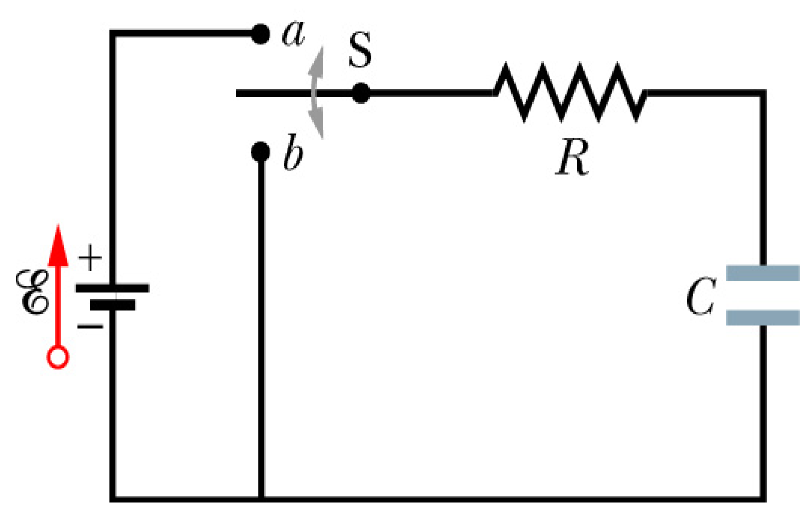
RC - circuit

- Circuits where current varies with time
- RC series circuit – a resistor and capacitor are in series with a battery and a switch
- At $t = 0$ switch is open and capacitor is uncharged so $q = 0$



RC - circuit

- Close the switch at point a
- Charge flows (current) from battery to capacitor, increasing *q* on plates and *V* across plates

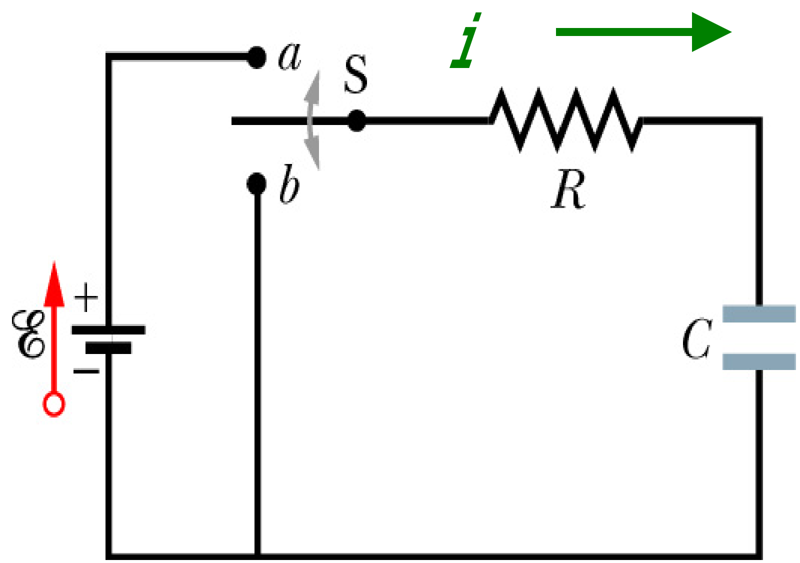


- When V_C equal $V_{battery}$ flow of charge stops (current is zero) and charge on capacitor is

$$q = CV = C\mathcal{E}$$

RC - circuit

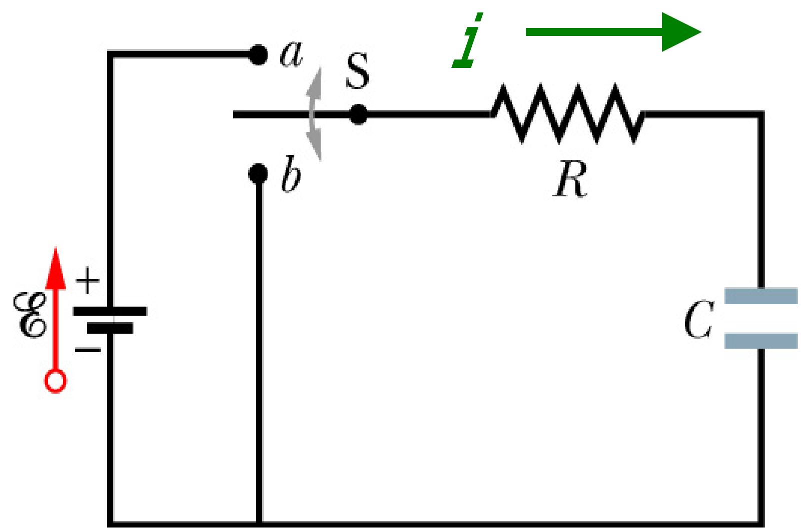
- Want to know how q and V of capacitor and i of the circuit change with time when charging the capacitor
- Apply loop rule, traversing clockwise from battery



$$\mathrm{E} - iR - \frac{q}{C} = 0$$

RC - circuit

$$\mathcal{E} - iR - \frac{q}{C} = 0$$



- Contains 2 of the variables we want *i* and *q*
- Remember

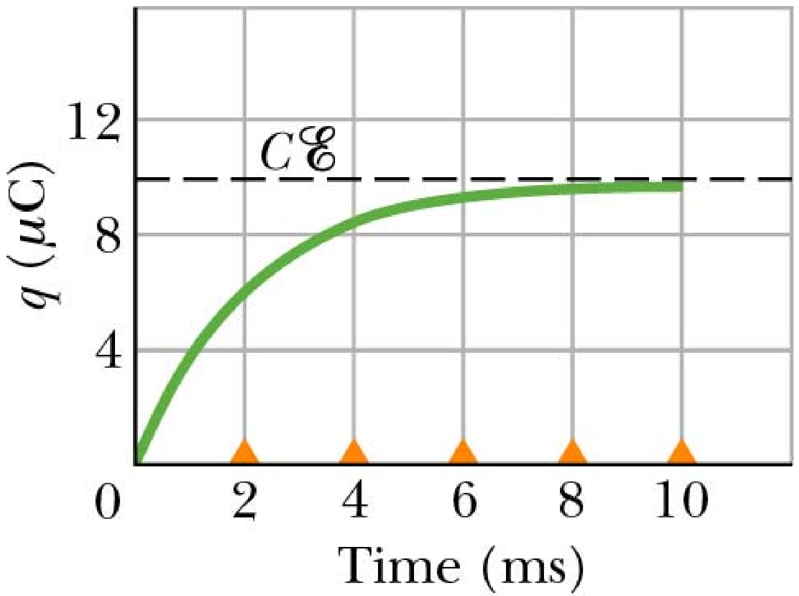
$$i = \frac{dq}{dt}$$

- Substituting gives

$$R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}$$

RC - circuit

$$R\frac{dq}{dt}+\frac{q}{C}=\mathcal{E}$$

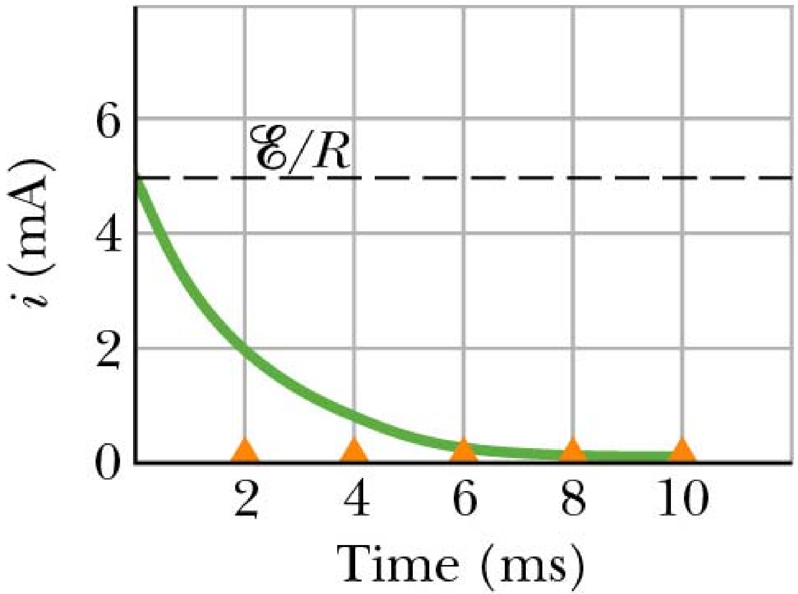


- Need a function which satisfies initial condition $q = 0$ at $t = 0$ and final condition of $q = C\mathcal{E}$ at $t = \infty$
- For charging a capacitor

$$q=C\mathcal{E}\left(1-e^{-t/RC}\right)$$

RC - circuit

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

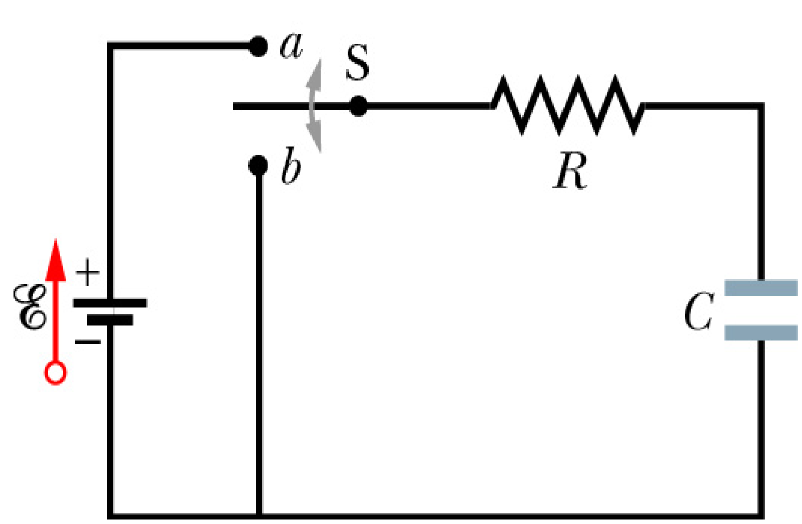


- Want current as a function of time
- For charging a capacitor

$$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R}\right)e^{-t/RC}$$

RC - circuit

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

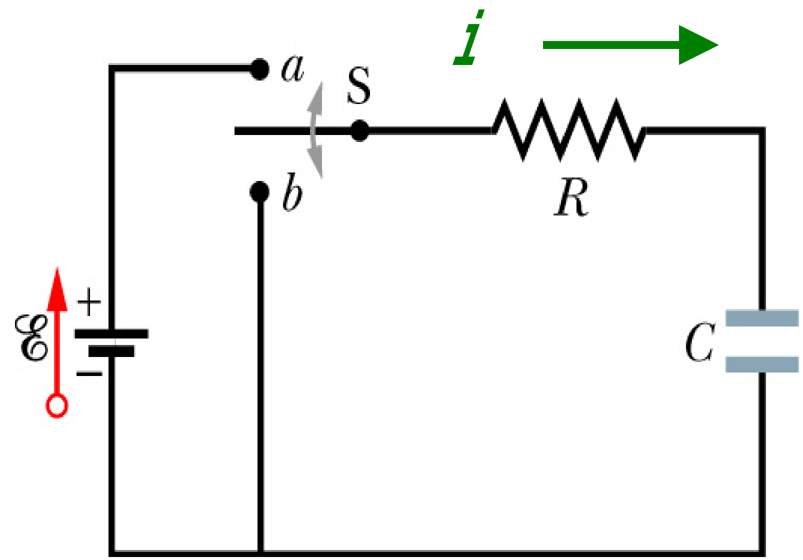


- Want V across the capacitor as function of time
- For charging a capacitor

$$V_C = \frac{q}{C} = \mathcal{E}\left(1 - e^{-t/RC}\right)$$

RC - circuit

- Want to know how *q* of capacitor and *i* of the circuit change with time when discharging the capacitor
- At new time *t* = 0, throw switch to point b and discharge capacitor through resistor *R*



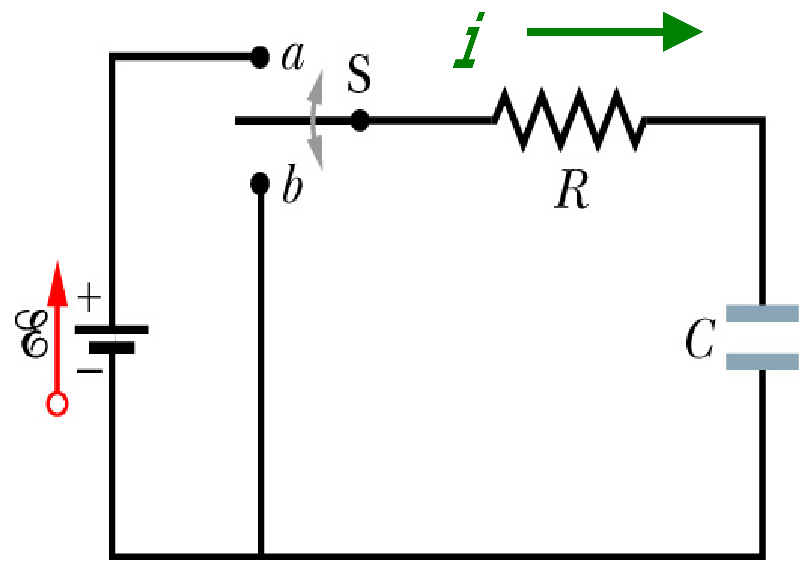
RC - circuit

- Apply the loop rule again but this time no battery

$$-\ iR\ -\ \frac{q}{C} = 0$$

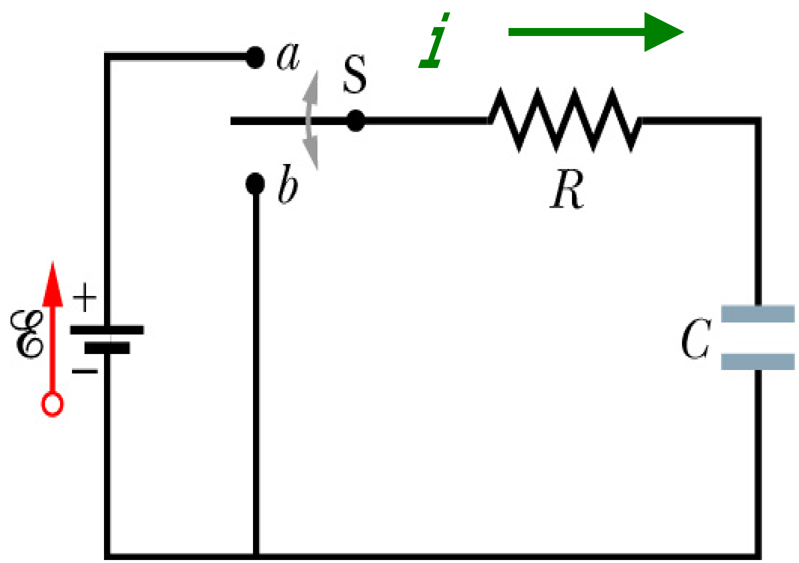
- Substituting for *i* again gives differential equation

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$



RC - circuit

$$R\frac{dq}{dt}+\frac{q}{C}=0$$



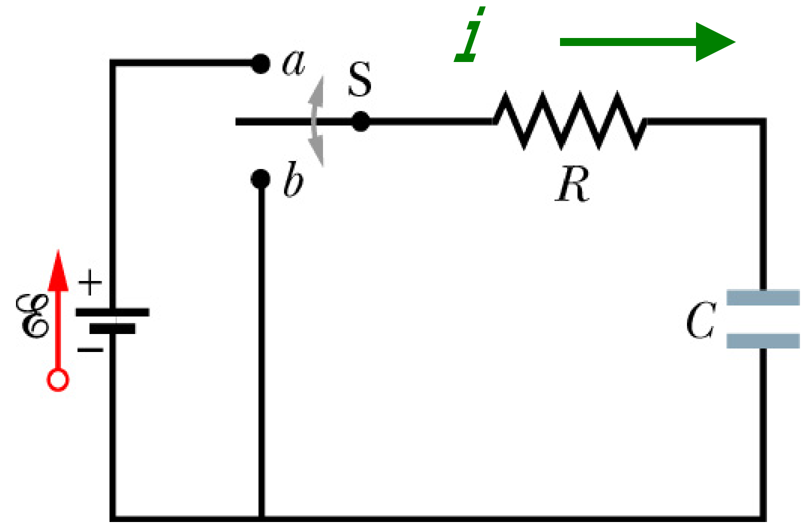
- Solution must satisfy initial condition that $q_0 = CV_0$

- For discharging a capacitor

$$q=q_0e^{-t/RC}$$

RC - circuit

$$q = q_0 e^{-t/RC}$$



- Find i for discharging capacitor with initial condition at $i_0 = V_0/R = q_0/RC$ at $t = 0$

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$$

Negative sign means charge is decreasing

Circuits

- Charging capacitor

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$i = \left(\frac{\mathcal{E}}{R}\right)e^{-t/RC}$$

- Discharging capacitor

$$q = q_0 e^{-t/RC}$$

$$i = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$$

- Define capacitive time constant – greater τ, greater (dis)charging time

$$\tau = RC$$