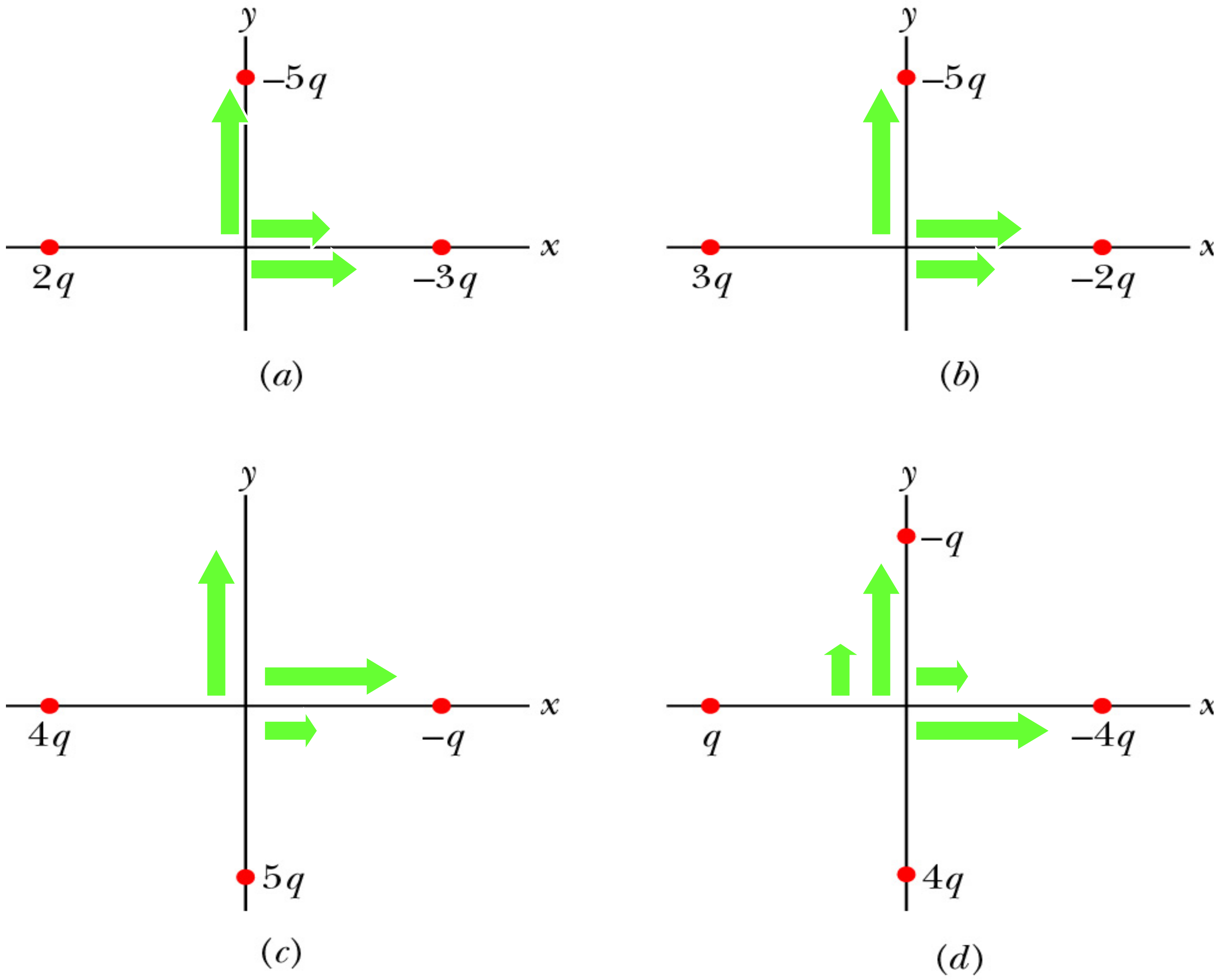
Exercise: Rank magnitude of net E



Exercise: Rank magnitude of net E

- a) $$E_x = k\frac{2q}{d^2} + k\frac{3q}{d^2} = k\frac{5q}{d^2}\hat{i}$$ $$E_y = k\frac{5q}{d^2}\hat{j}$$

$$E = \sqrt{E_x^2 + E_y^2} = k\frac{\sqrt{50}q}{d^2}$$

- Do this for the rest and find

All Equal

Charge distributions

- Calculate E field from a continuous line or region of charge - Use calculus and a charge density
- Linear charge density $\lambda = Q / Length$
- Surface charge density $\sigma = Q / Area$
- Volume charge density $\rho = Q / Volume$

Electric Field of a Charged Ring

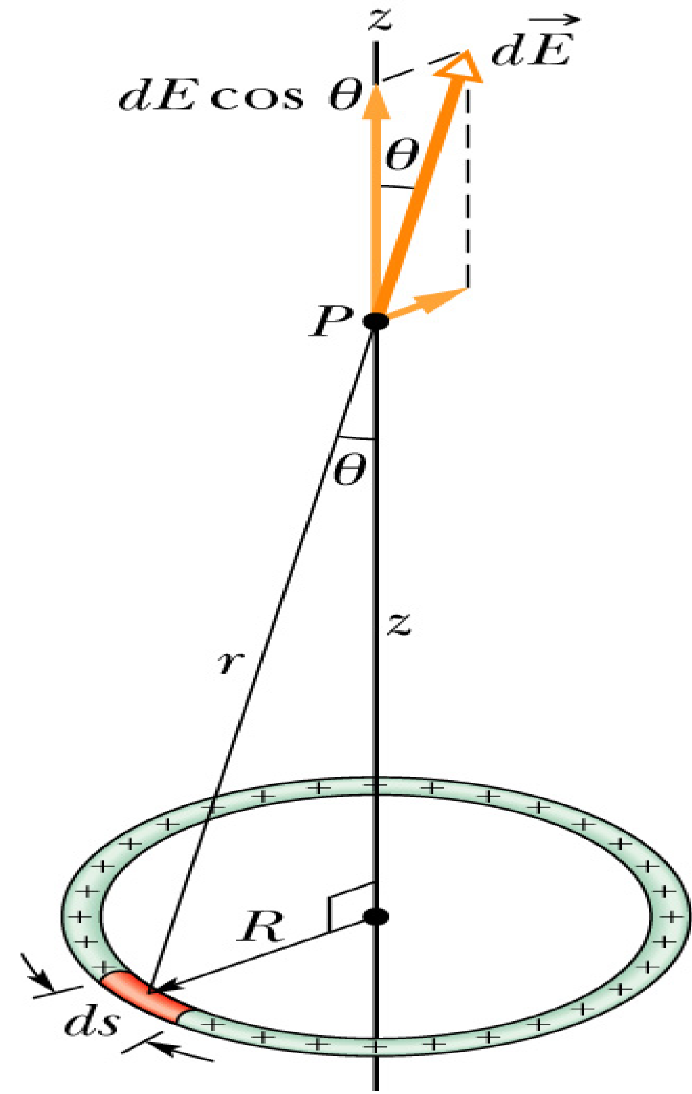
- Differential *dE* at P is

$$dE = k\frac{dq}{r^2} = k\frac{\lambda ds}{r^2}$$

- From trig

$$r^2 = z^2 + R^2$$

- Look for symmetry
  - All $\perp$ cancel and $||$ point upward



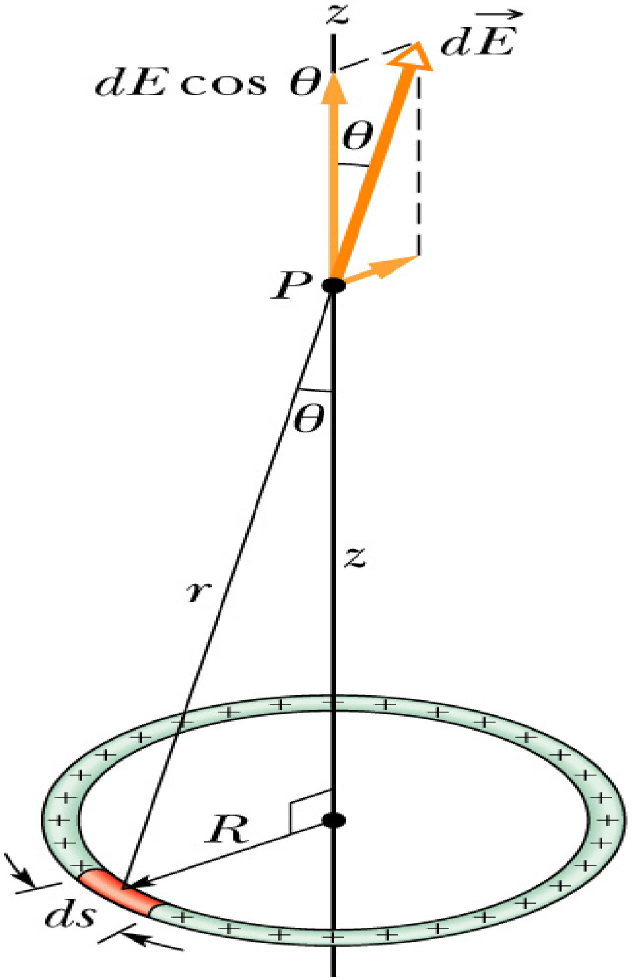
Electric Field of a Charged Ring

- Parallel component *dE* is

$$dE\cos\theta = k\frac{\lambda ds}{\left(z^2+R^2\right)}\cos\theta$$

- Use trig to rewrite cos θ

$$\cos\theta = \frac{z}{r} = \frac{z}{\left(z^2+R^2\right)^{1/2}}$$



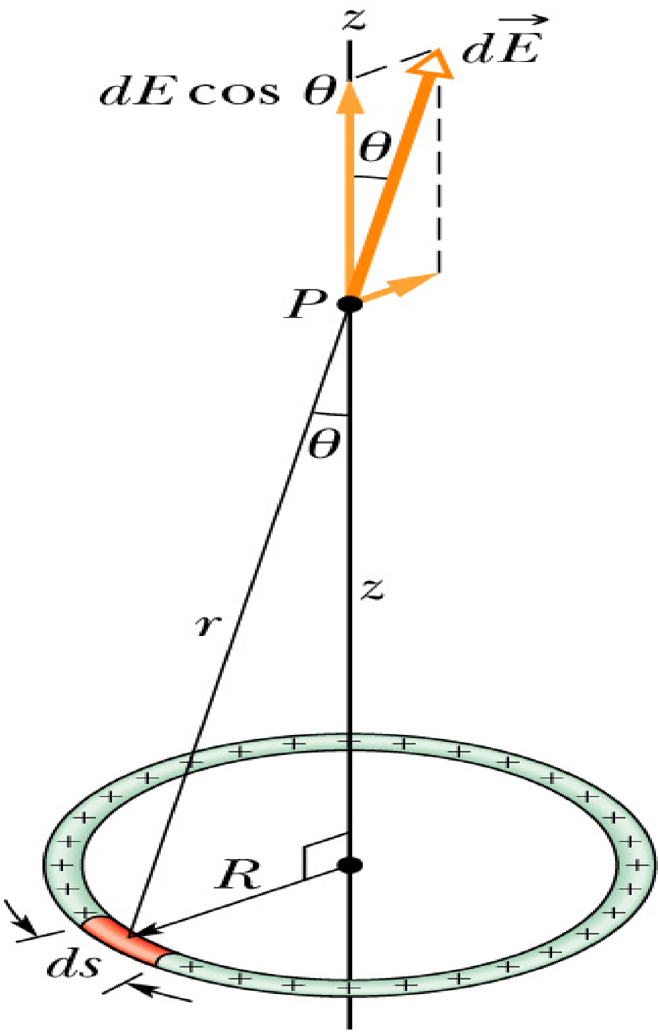
Electric Field of a Charged Ring

- Substituting

$$dE\cos\theta = k\frac{z\lambda}{\left(z^2+R^2\right)^{3/2}}ds$$

- Integrate around the ring

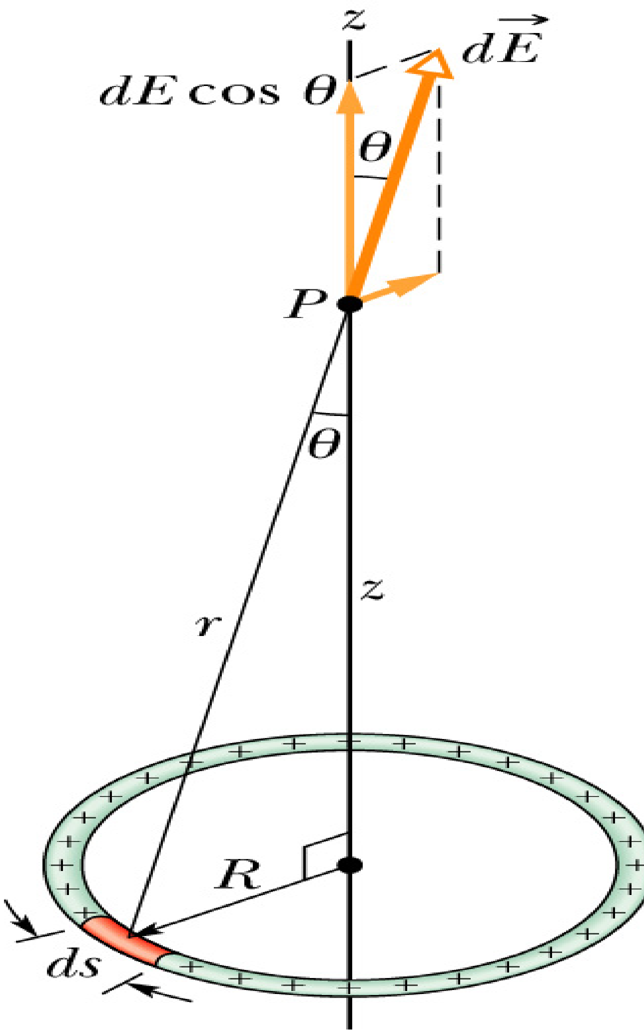
$$E = \int dE\cos\theta = \frac{kz\lambda}{\left(z^2+R^2\right)^{3/2}}\int_0^{2\pi r} ds$$



Electric Field of a Charged Ring

- Finally get $$E = \frac{kz\lambda(2\pi R)}{\left(z^2 + R^2\right)^{3/2}}$$
- Replace λ with $$\lambda = q/(2\pi r)$$
- Charge ring has *E* of

$$E = \frac{kqz}{\left(z^2 + R^2\right)^{3/2}}$$



Electric Field of a Charged Ring

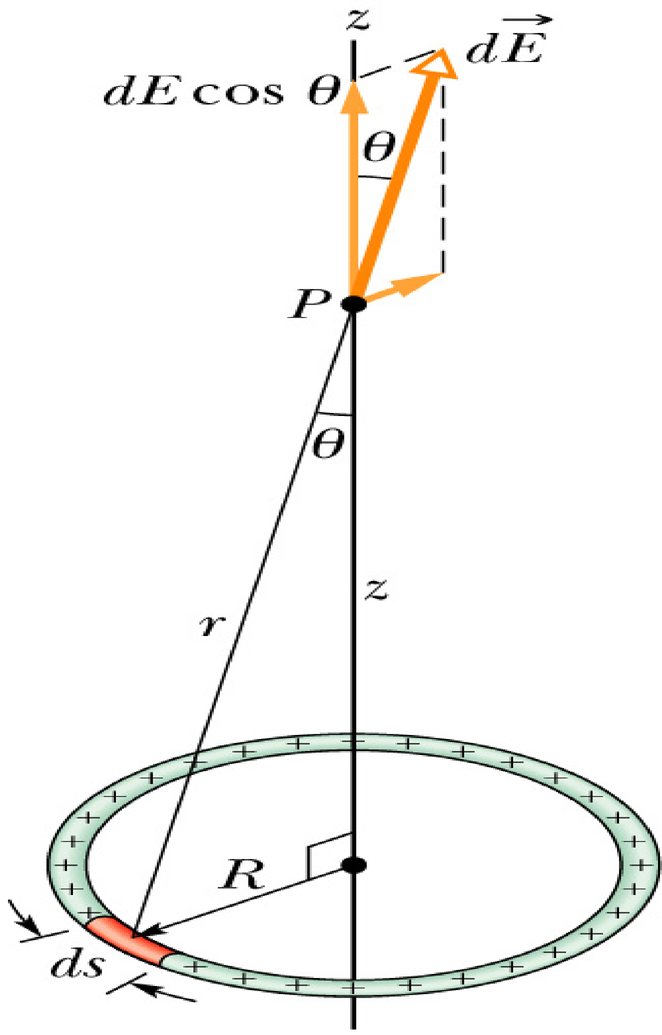
- Charge ring has E of

$$E = \frac{kqz}{\left(z^2 + R^2\right)^{3/2}}$$

- Check $z >> R$ then

$$E = \frac{kq}{z^2}$$

- From far away ring looks like point charge



Electric Field of a Charged Disk

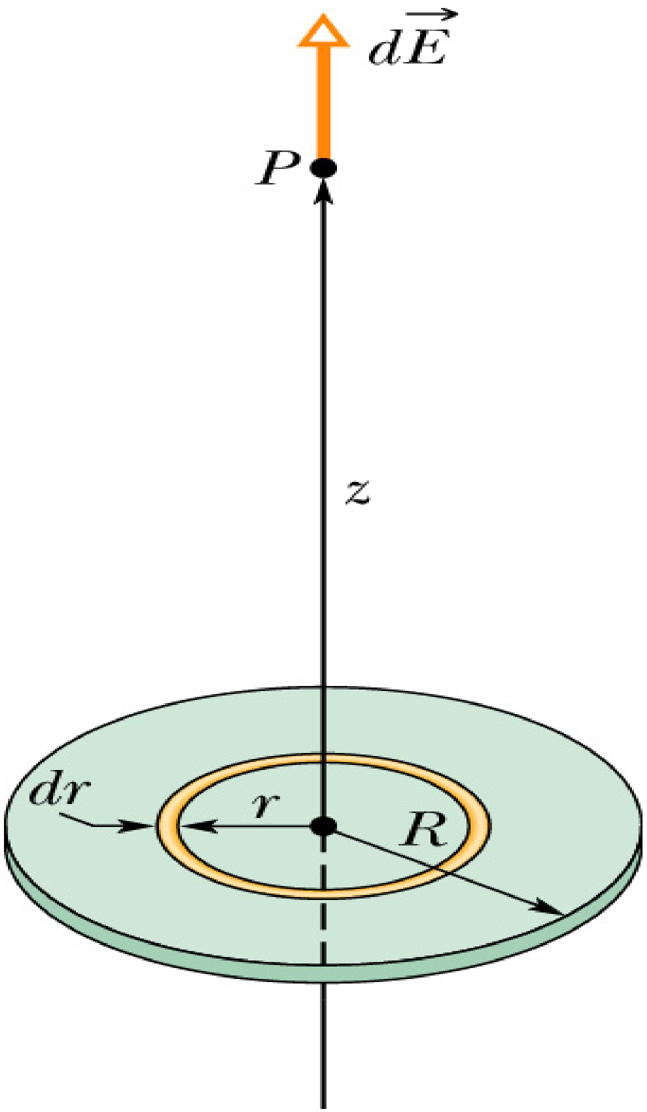
- Also do this for charged disk

- But now surface charge

$$dq = \sigma A = \sigma\left(2\pi r dr\right)$$

- Use ring field, with q→dq. Then integrate

$$E = kz\pi\sigma\int_0^R \frac{2rdr}{\left(z^2 + r^2\right)^{3/2}}$$



Electric Field of a Charged Disk

- Integrating and using $k = 1/(4\pi\varepsilon_0)$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

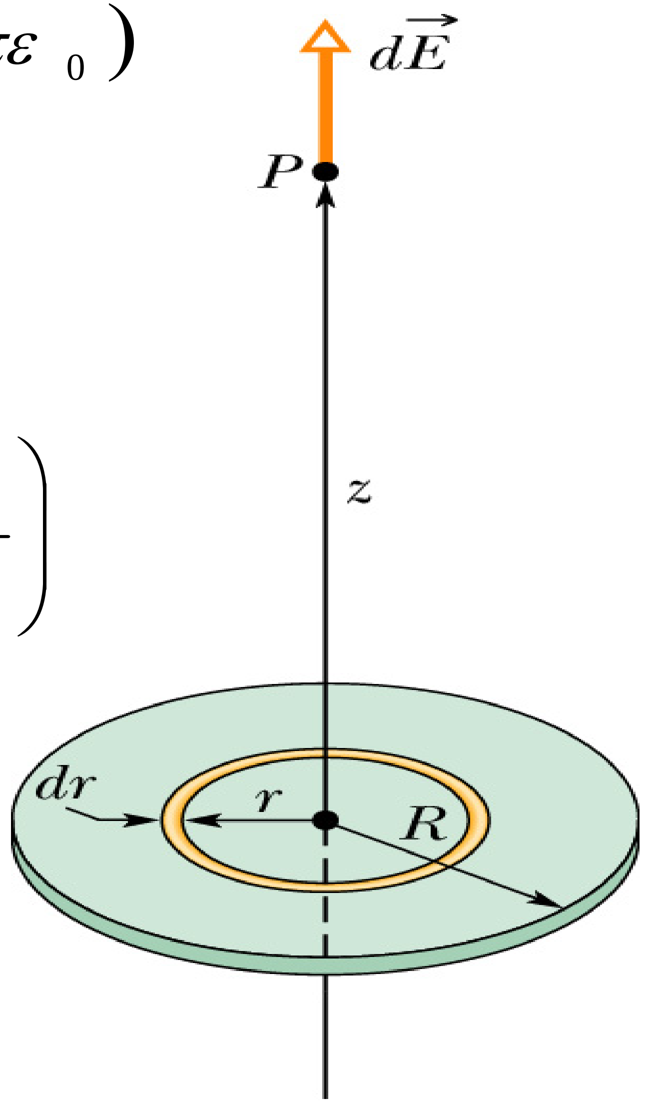
- Let $z \to \infty$ and use

$$(z^2 + R^2)^{-1/2} = \frac{1}{z}\left(1 - \frac{1}{2}\frac{R^2}{z^2}\right)$$

to get

$$E = 2\pi k\sigma\left(\frac{1}{2}\frac{R^2}{z^2}\right) = k\frac{q}{z^2}$$

- From far away disk looks like point charge



Electric Field of a Charged Disk

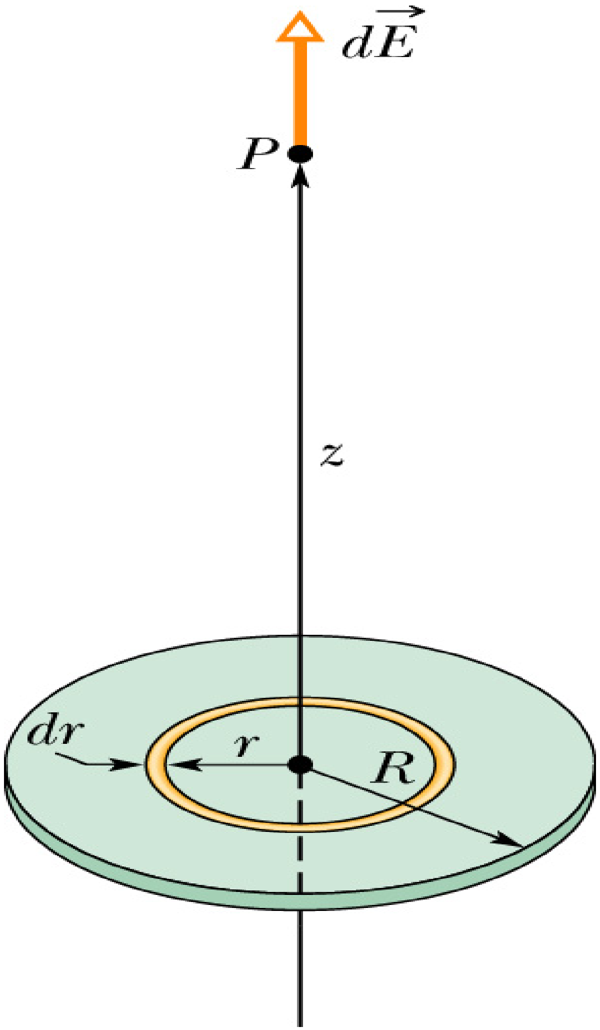
- Charge disk of radius *R*

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

- Let R→∞ then get

$$E = \frac{\sigma}{2\varepsilon_0}$$

- Acts as infinite sheet of a non-conductor with uniform charge



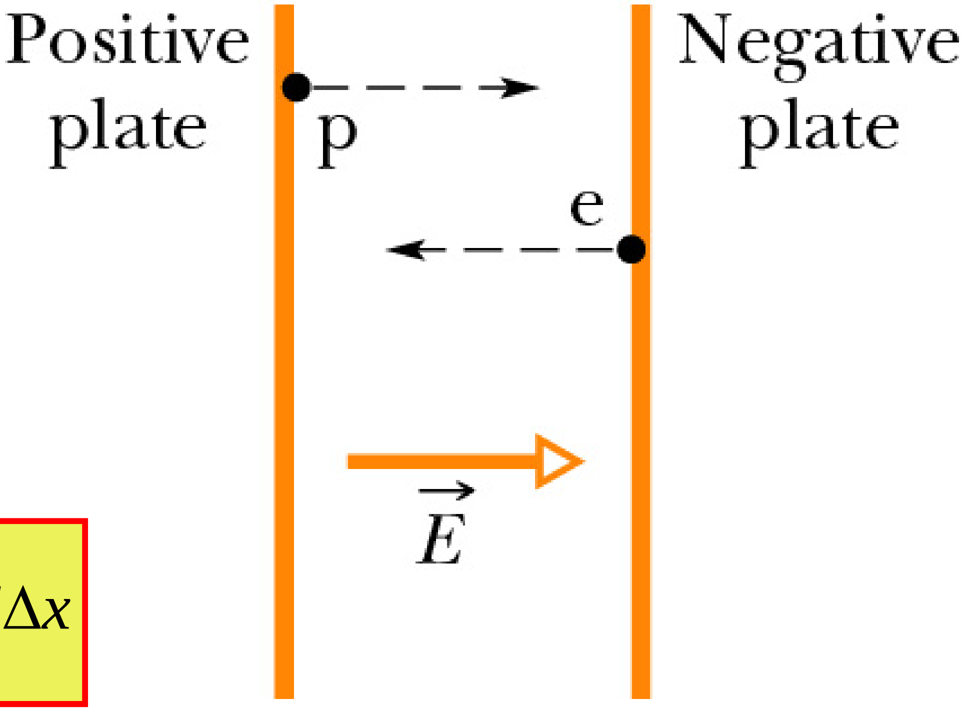
Kinetic Energy Gain in an E-Field

$$v_f = at = \frac{qE}{m}t$$

$$x_f = \frac{1}{2}at^2 = \frac{qE}{2m}t^2$$

$$v_f^2 = 2ax_f = \left(\frac{2qE}{m}\right)x_f$$

$$K = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(\frac{2qE}{m}\right)\Delta x = qE\Delta x$$



Or using work-Kinetic energy theorem:

$$W = F_e\Delta x = qE\Delta x$$

$$W = K$$

2-D motion in an E-Field

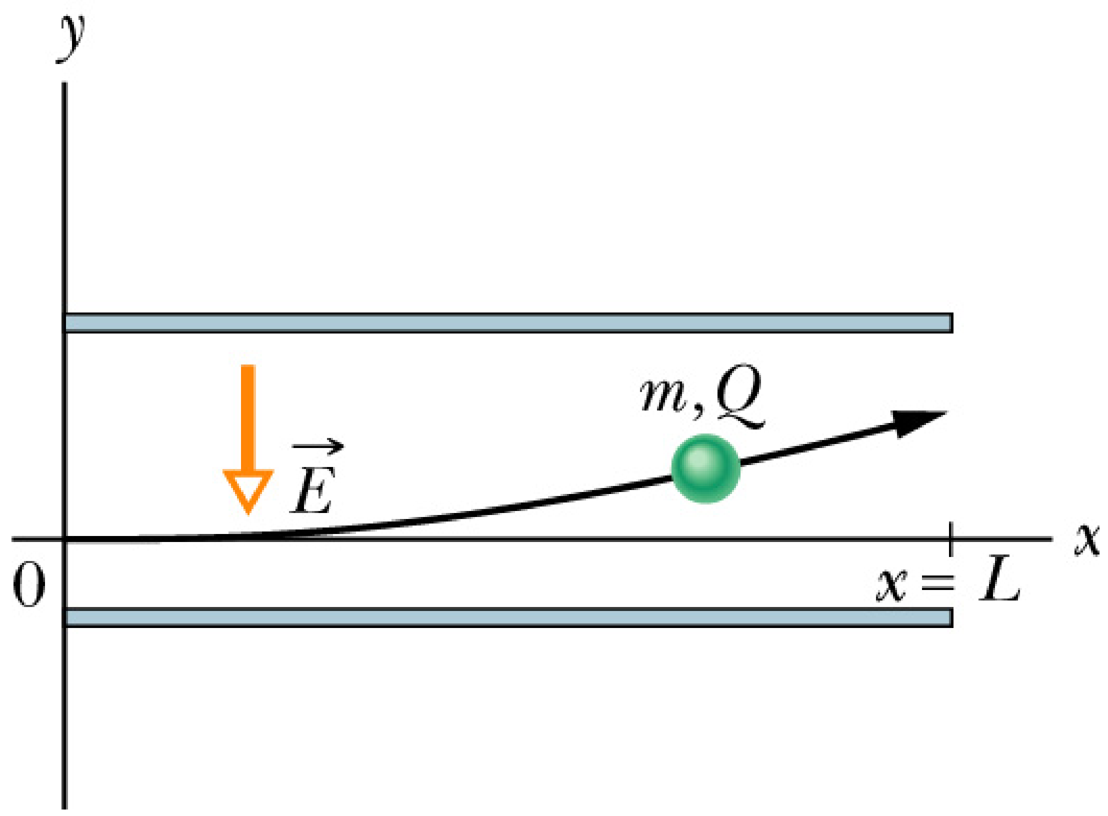
$$v_x = v_i = \text{constant}$$

$$v_y = a_y t = \frac{QE}{m_e} t$$

$$x_f = v_i t$$

$$y_f = \frac{1}{2} a_y t^2 = \frac{1}{2} \frac{QE}{m} t^2$$

$$= \frac{1}{2} \frac{QE}{m v_i^2} x_f^2$$



Trajectory is a parabola