# **Equipotential Surfaces**

- Draw equipotential surfaces for distributions of charges
- Equipotential surfaces are always ⊥ to electric field lines and to E



### **Equipotential Surfaces**





(b)

(c)

# **Equipotential Surfaces**



• *E* field lines are  $\perp$  to the equipotential surface

 If given equipotential surfaces can draw *E* field lines



### **Potential Difference**

 Calculate ΔV between points i and f in an electric field E

$$\Delta V = V_f - V_i = -\frac{W}{q_0}$$



Need to find *W* when *E* is not constant.

# **Potential Difference**

 Calculate differential amount of work

$$dW = \vec{F} \bullet d\vec{s}$$

Remember

$$\vec{F} = q\vec{E}$$

$$dW = q_0 \vec{E} \bullet d\vec{s}$$



## **Potential Difference**

$$W = q_0 \int_i^f \vec{E} \bullet d\vec{s}$$



• Substitute to find  $\Delta V$ 

$$\Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E} \cdot d\vec{s}$$

 Potential decreases if path is in the direction of the electric field

 Derive potential *V* around a charged particle

$$V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{s}$$

- Imagine moving a + test charge from *P* to ∞
- Path doesn't matter so choose line radially with *E*



- Sign of V is same sign as q
  - + charge produces + V
  - charge produces V



- IV gets larger as r gets smaller
  In fact V = ∞ when r = 0 (on top of charge)
- Graphical representation of V for charges in the x-y plane – plot value of V on the z-axis as a function of x-y position







# **Electric Potential of n Charges**

 Use superposition principle to find the potential due to n point charges

$$V = \sum_{i=1}^{n} V_{i} = k \sum_{i=1}^{n} \frac{q_{i}}{r_{i}}$$

- This is an algebraic sum, not a vector sum
- Include the sign of the charge

#### **Electric Potential of 2 Unequal Charges**



### Force, Field & Potential of Point Charge

 What is the force *F*, electric field *E*, and potential *V*, at a point *P* a distance *r* away from a point charge?

$$F = k \frac{|q||q_0|}{r^2}$$

$$E = k \frac{q}{r^2}$$

$$V = k \frac{q}{r}$$



# **Electric Potential (Exercise)**

 Rank a), b) and c) according to net electric potential V produced at point P by two protons. (Greatest first.)



### **Electric Potential**

Replace one of the protons by an electron.
 Rank the arrangements now.



ALL EQUAL

$$V = k \left( -\frac{q}{d} + \frac{q}{D} \right)$$

### **Electric Field**

• How do we calculate *E* from V?

$$W = -q_0 \Delta V$$
$$W = \vec{F} \bullet \vec{d} = q \vec{E} \bullet \vec{d}$$

$$-q_0 \, dV = q_0 E \cos\theta \, ds$$

$$E\cos\theta = -\frac{dV}{ds}$$



### **Electric Field**

Two

surfaces

equipotential

• How do we calculate E from V?

$$E\cos\theta = -\frac{dV}{ds}$$

Component of *E* in direction of *ds*

$$E_s = -\frac{\partial V}{\partial s}$$

 Component of *E* in any direction is negative rate of change of *V* with distance in that direction

### **Electric Field**

• Take *s* axis to be *x*, *y*, or *z* axes

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

• If *E* is uniform and *s* is  $\perp$  to equipotential surface

$$E = -\frac{\Delta V}{\Delta s}$$

# Electric Field (Exercise)

 3 pairs of parallel plates with same separation and V of each plate. *E* field is uniform between plates and ⊥ to the plates.



 A) Rank (greatest first) magnitude of E between the plates



$$E = -\frac{\Delta V}{\Delta s}$$

#### but asked for magnitude of E

$$E_1 = \frac{200}{d}$$
  $E_2 = \frac{220}{d}$   $E_3 = \frac{20}{d}$ 

2, then 1 & 3

# **Electric Field (Exercise)**

B) For which pair does E point to the right?



Accelerate to the left

 C) If an electron is released midway between plates in (3) what does it do?

