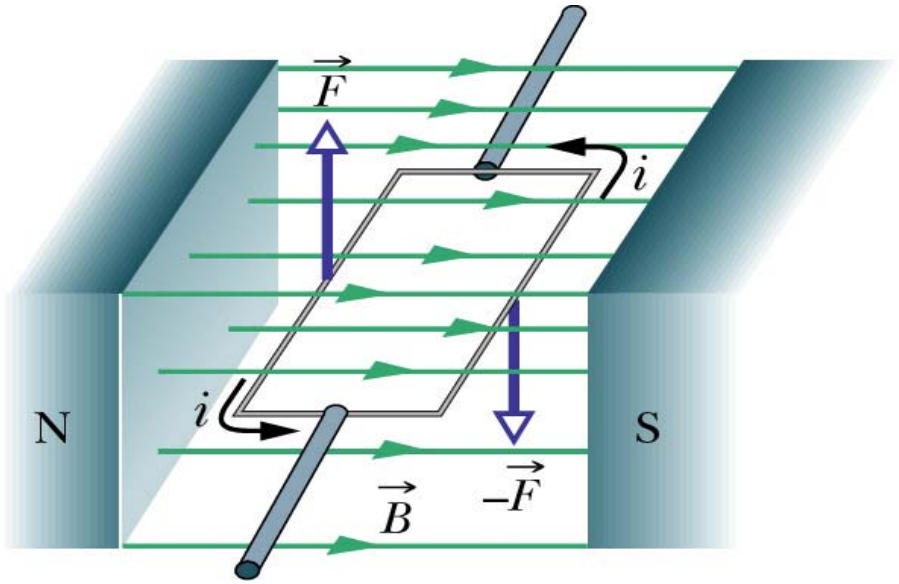
Electric Motor

- What happens if we put a loop of wire carrying a current in a B field ?



- F_B on opposite sides of the loop produce a torque on the loop causing it to rotate.

Electric motor – a commutator reverses the direction of the current every half turn to that the torque is always in the same direction.

Electric Motor

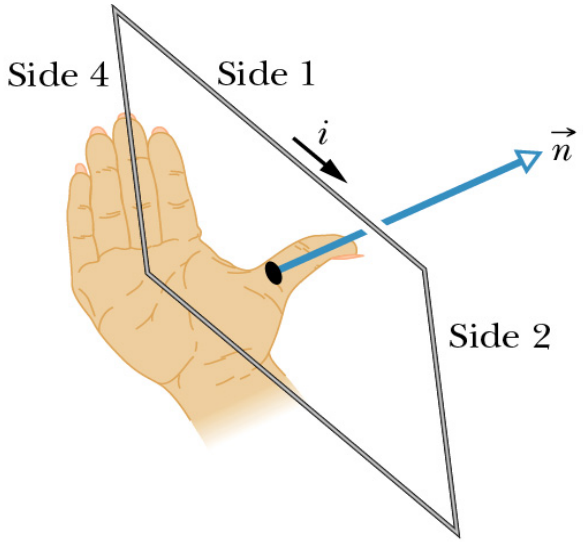
- Define normal *n* to plane using right-hand rule
- Torque tends to rotate loop to align *n* with *B* field
- Torque for single loop

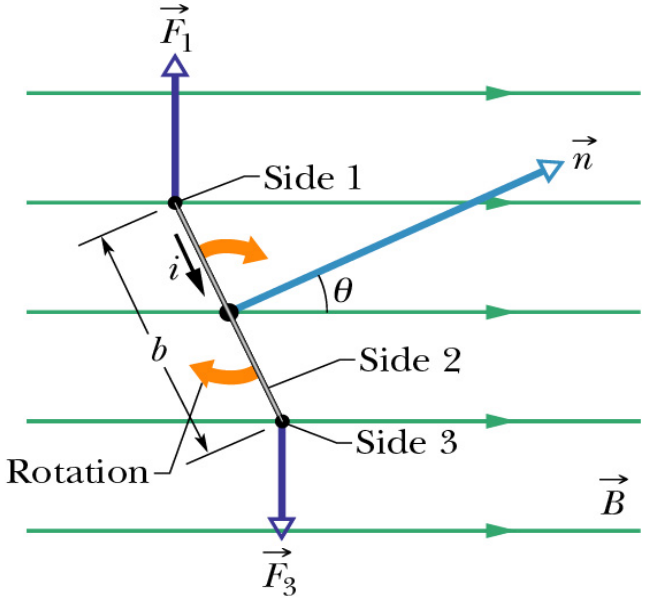
$$\tau = iAB\sin\theta$$

where A is the area of the loop and θ is between *n* and *B*

- Replace single loop with coil of *N* loops or turns

$$\tau = (NiA)B\sin\theta$$





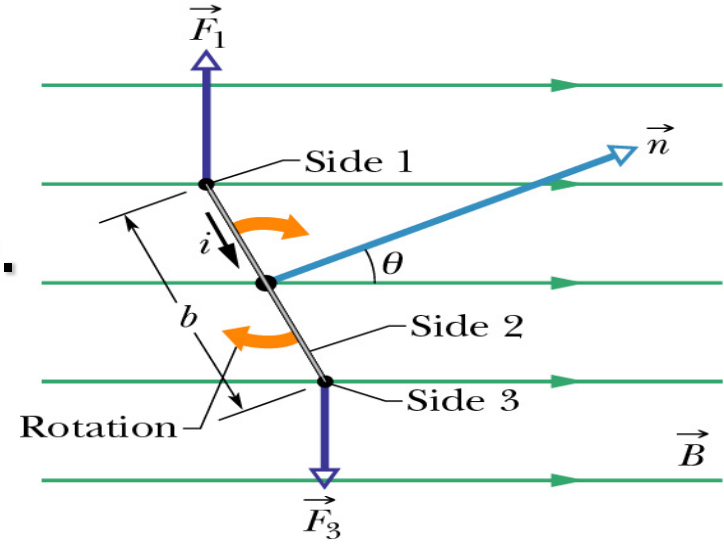
Magnetic Dipole

- Define magnetic dipole moment $\mu = NiA$

$$\tau = (NiA)B\sin\theta = \mu B\sin\theta$$

- The direction of the magnetic dipole moment is the same as the normal vector to the plane.

$$\vec{\mu} = \vec{n}$$



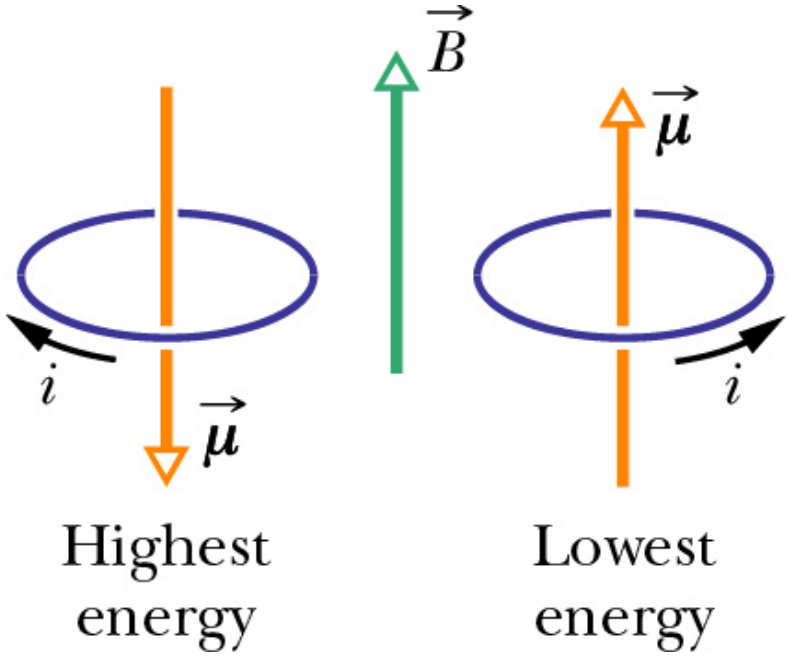
- The torque becomes

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Magnetic Dipole

- A magnetic dipole in a magnetic field has a magnetic potential energy, *U*
- Lowest energy when dipole moment lined up with *B* field
- Highest energy when dipole moment directed opposite *B* field

$$U = -\vec{\mu} \bullet \vec{B}$$



Magnetic Dipole

- Magnetic dipole moment μ has

Torque

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Potential Energy

$$U = -\vec{\mu} \bullet \vec{B}$$

- Remember electric dipole moment p

Torque

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Potential Energy

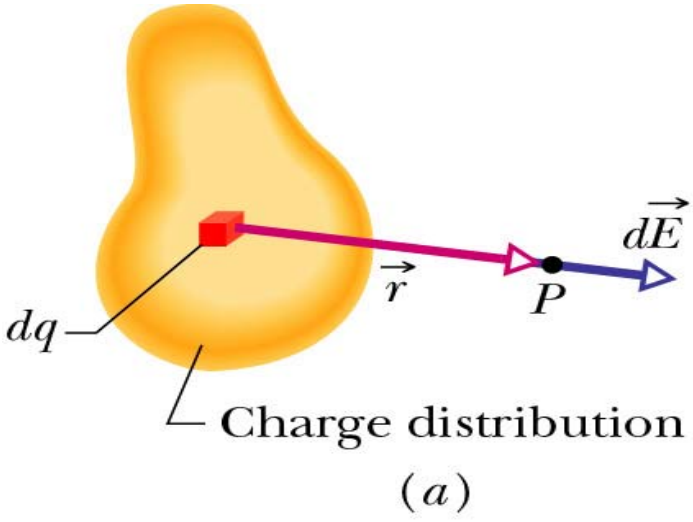
$$U = -\vec{p} \bullet \vec{E}$$

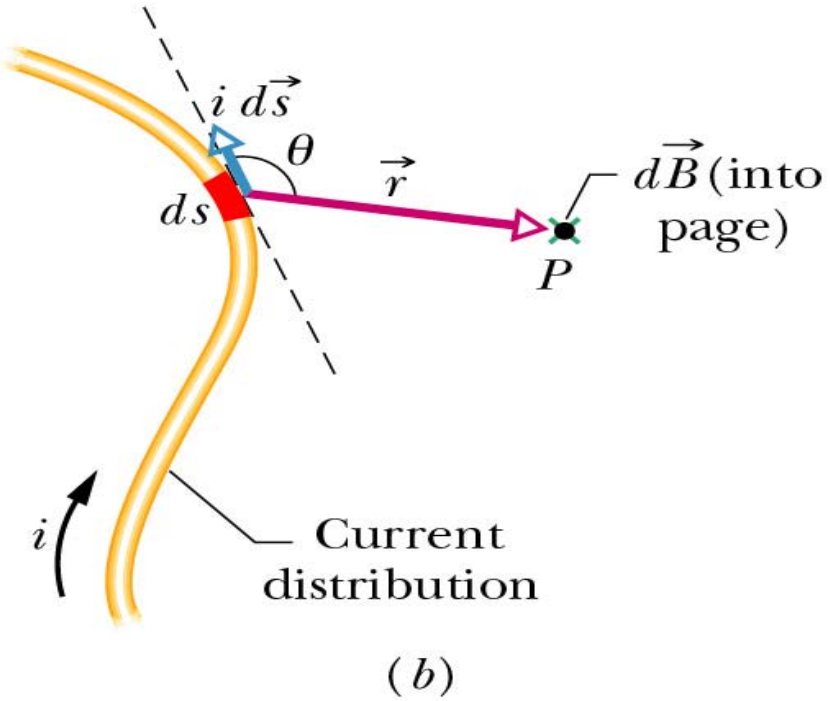
B Fields from Currents

- Calculate *B* field produced by distribution of currents
- Similar to finding *E* from distribution of charges

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

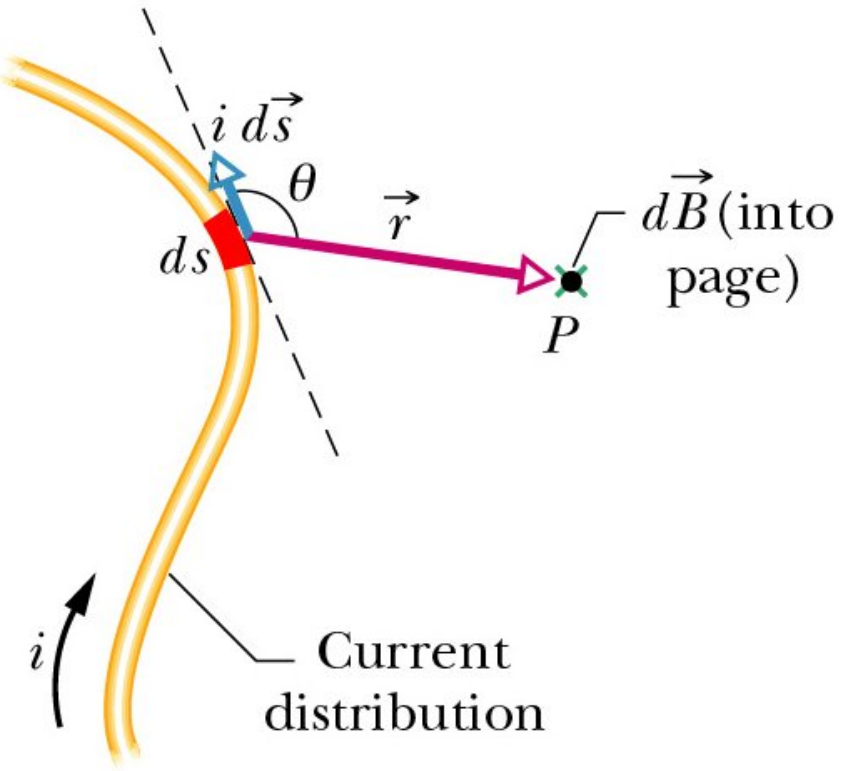
- *B* fields, like *E* fields, can be superimposed to find net field





Biot-Savart Law

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$



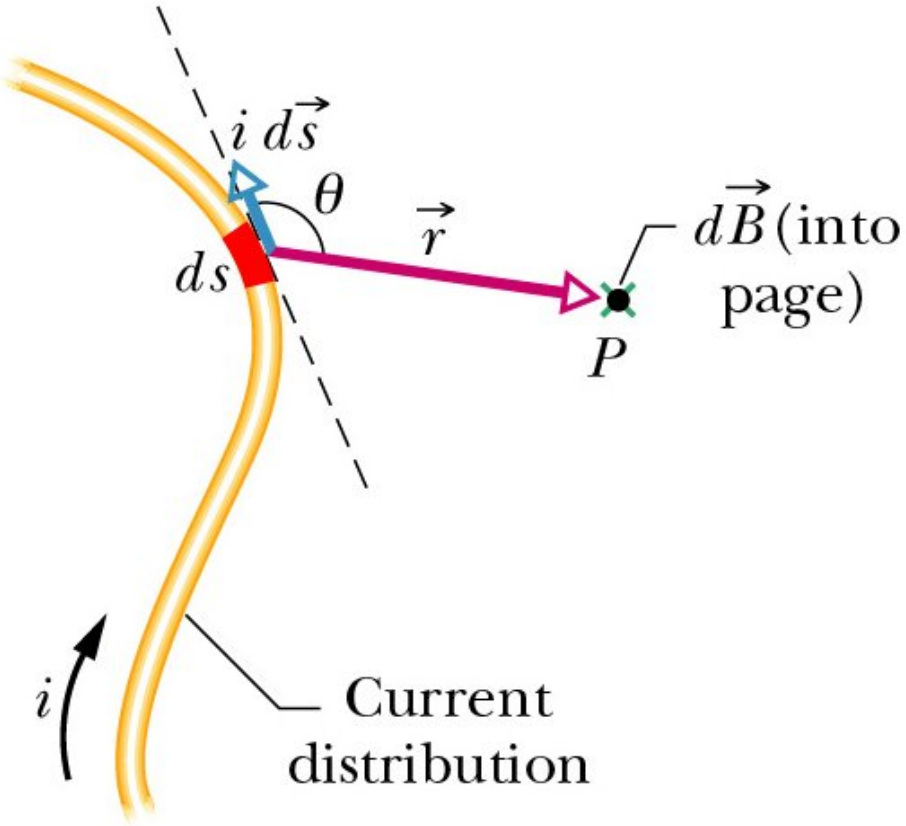
- Current-length element, *i ds*, is product of a scalar and a vector
- Find net *B* field by integrating
- BUT remember it is a vector sum

- Permeability constant, μ_0

$$\mu_0 = 4\pi \times 10^{-7}\,T\cdot m/A$$

Biot-Savart Law

$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2}$$



- Rewrite in vector form

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\vec{r}}{r^3}$$

- Known as Biot-Savart Law