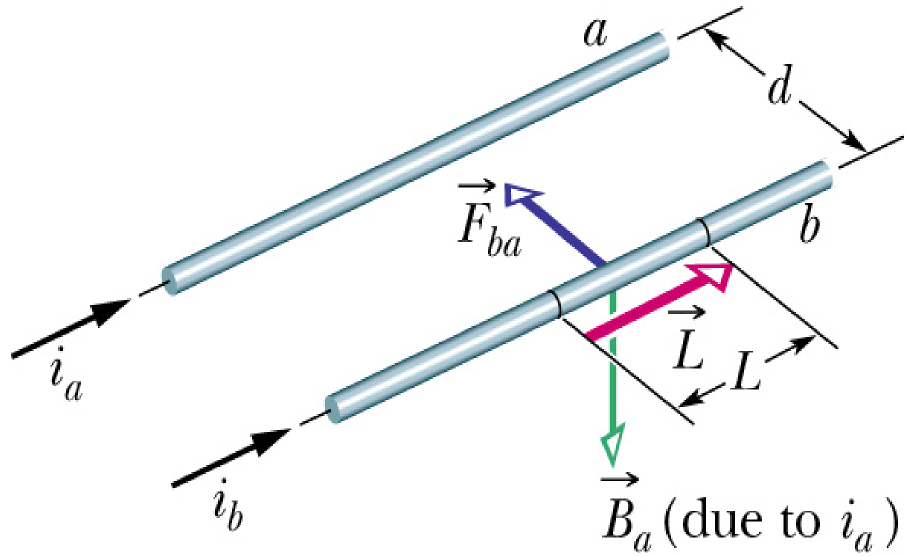
Force between two currents

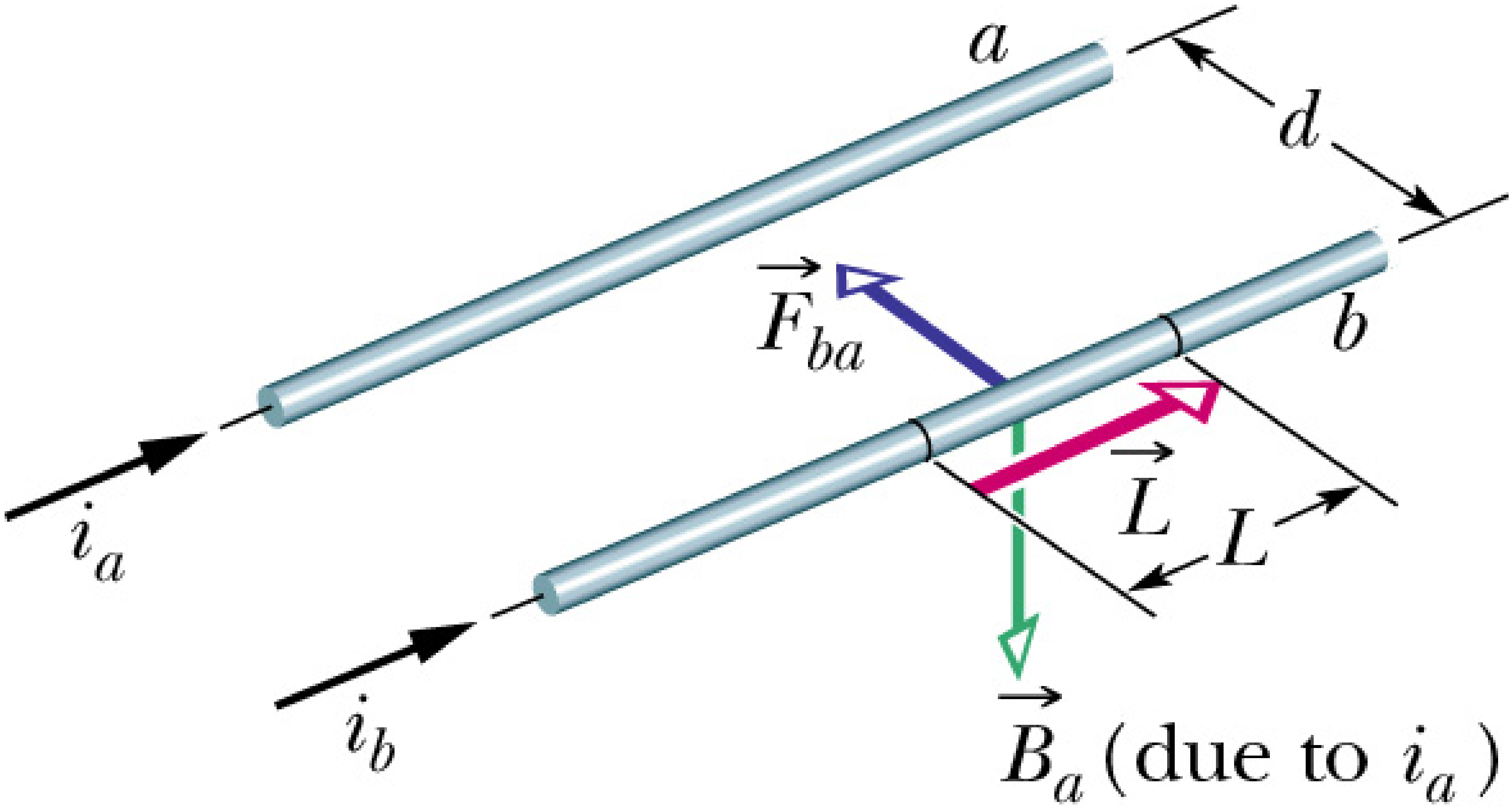
- What is the force, F_{ba} on wire b due to the current in wire a ?



- Wires are separated by a distance, d, and have currents, i_a and i_b

- First calculate B field from wire a at the site of wire b

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

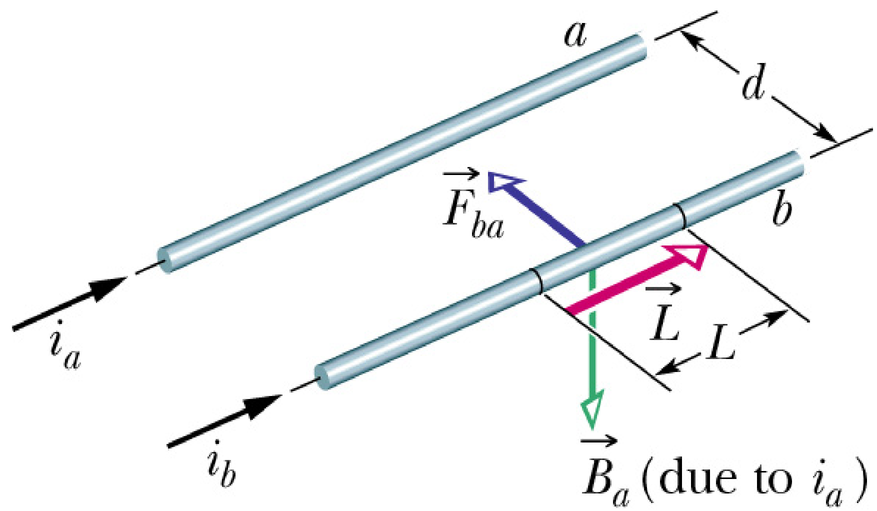


a
d
$\vec{F}_{ba}$
b
$\vec{L}$
L
i_a
i_b
$\vec{B}_a$ (due to i_a)

Force between two currents

- Using right-hand rule find that the B_a field is directed down

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$



- Now calculate force F_{ba} on wire b using

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

- Where the current i_b is current in wire b
- B field is from wire a
- L is length of wire b

Force between two currents

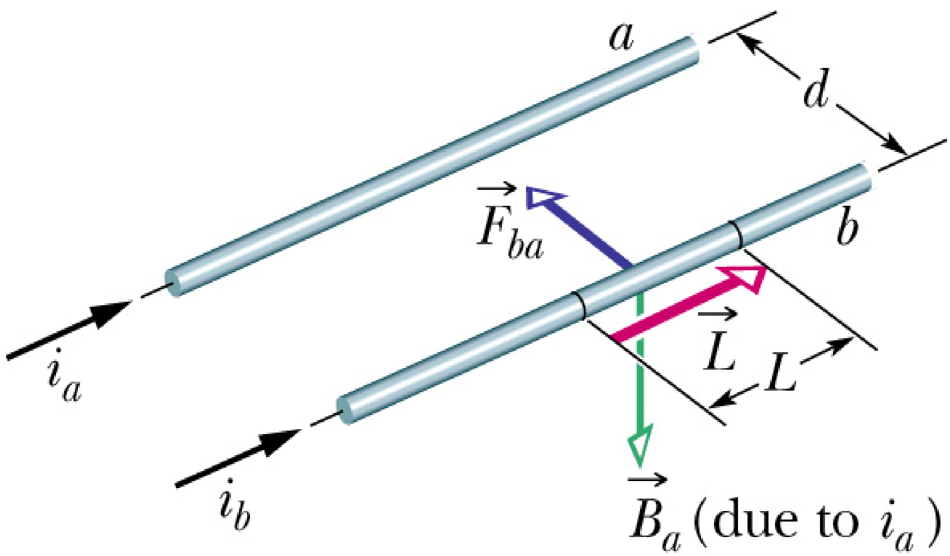
- Current i_b and B_a are $\perp$ to each other

$$F_{ba} = i_b L B_a \sin 90$$

- Substituting B_a

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

$$\boxed{F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d}}$$



- Applying right-hand rule, direction of F_{ba} is towards wire *a*

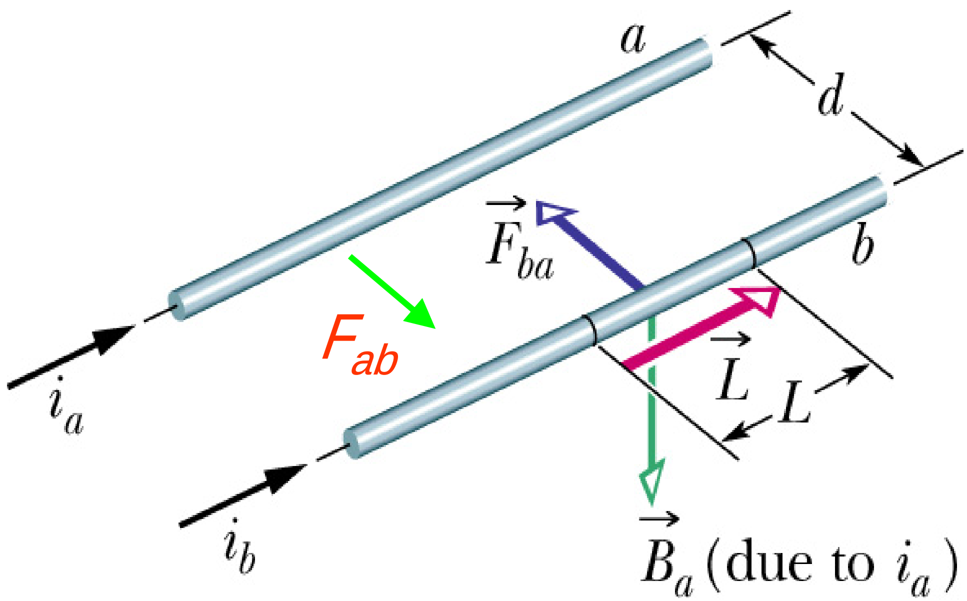
Force between two currents

- What is the force, F_{ab} on wire a due to the current in wire b?
- Calculate B field from wire b at site of wire a

$$B_b = \frac{\mu_0 i_b}{2\pi d}$$

- Force on a from b is

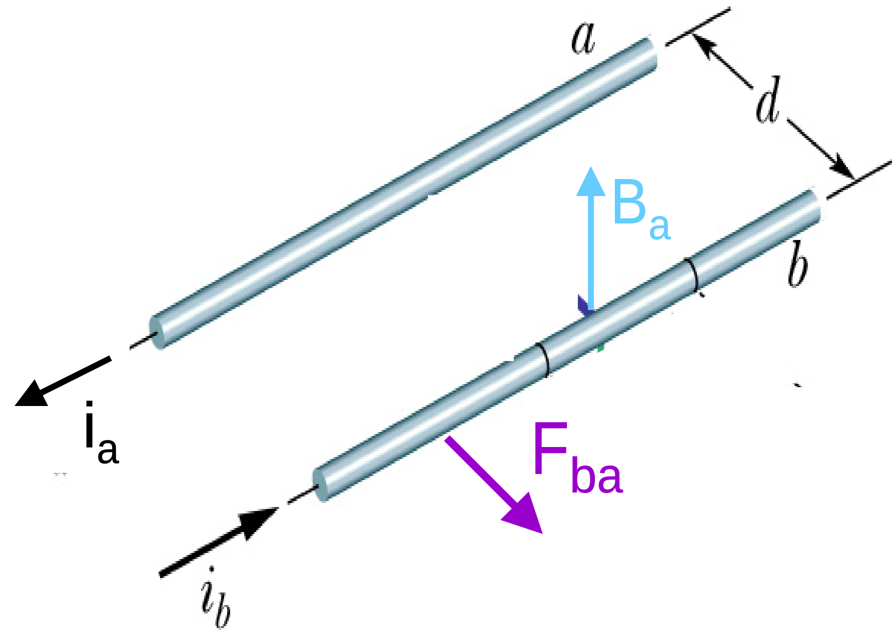
$$F_{ab} = i_a L B_b = \frac{\mu_0 L i_a i_b}{2\pi d}$$



- Apply right-hand rule and find F_{ab} has same magnitude as F_{ba} but opposite direction

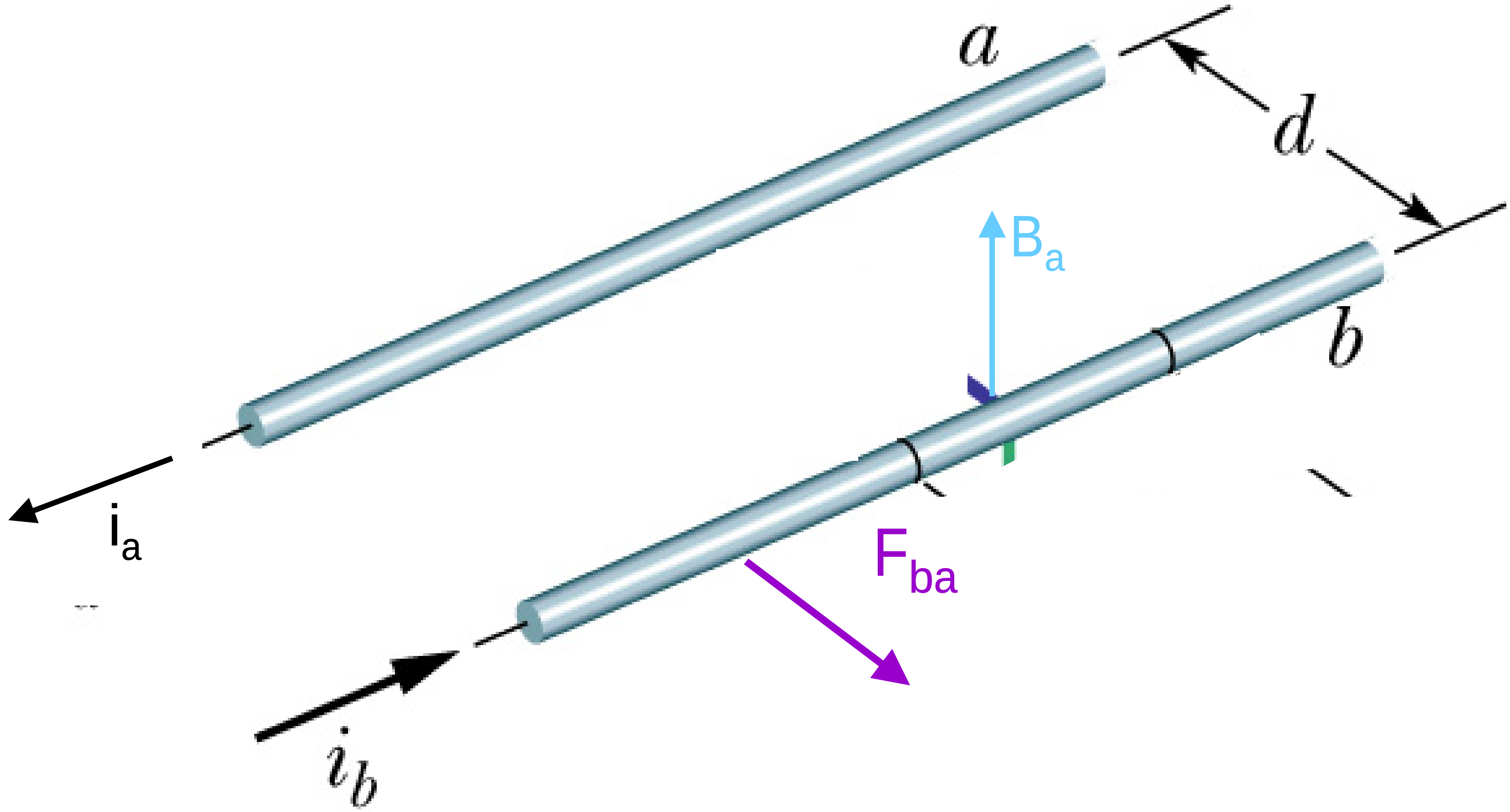
Force between two currents

- What happens if we flip current in wire *a* so that its moving opposite to the current in wire *b* ?



- Use right-hand rules
- B_a points up
- F_{ba} points away from wire *a*

- Parallel currents attract, anti-parallel currents repel



a
d
B_a
b
i_a
F_ba
i_b

Exercise

- Three long, straight, parallel wires are equally spaced with identical currents, either into or out of page. Rank the wires according to the magnitude of the force on each wire due to the currents in the other two wires, greatest first.

Exercise

- What's the force on wire *a* due to wires *b* and *c* ?
- First find net *B* field at *a* from wires *b* and *c*
  - Calculate magnitude for B_b and B_c
  - Use right-hand rule to find direction B_b and B_c
  - Add B_b and B_c as vectors to get B_{bc}

$$B = \frac{\mu_0 i}{2\pi d}$$

- Find force on wire a with $\vec{F}_{abc} = i_a \vec{L} \times \vec{B}_{bc}$

Exercise

- Forces on each wire due to other two are:

$$F_{abc} = i_a L B_{bc} = \frac{\mu_0 i^2 L}{4\pi d}$$

$$F_{bac} = i_b L B_{ac} = \frac{\mu_0 i^2 L}{\pi d}$$

$$F_{cab} = i_c L B_{ab} = \frac{3\mu_0 i^2 L}{4\pi d}$$

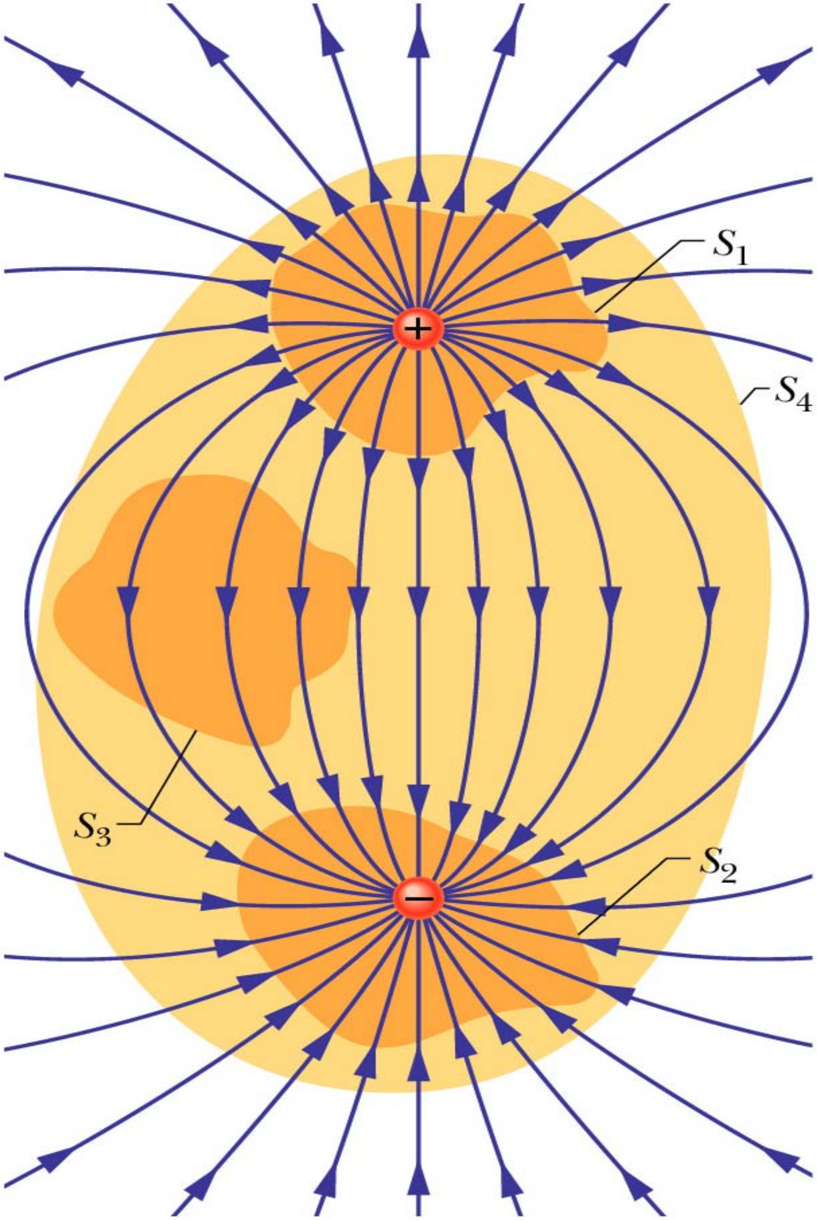
b, c, a

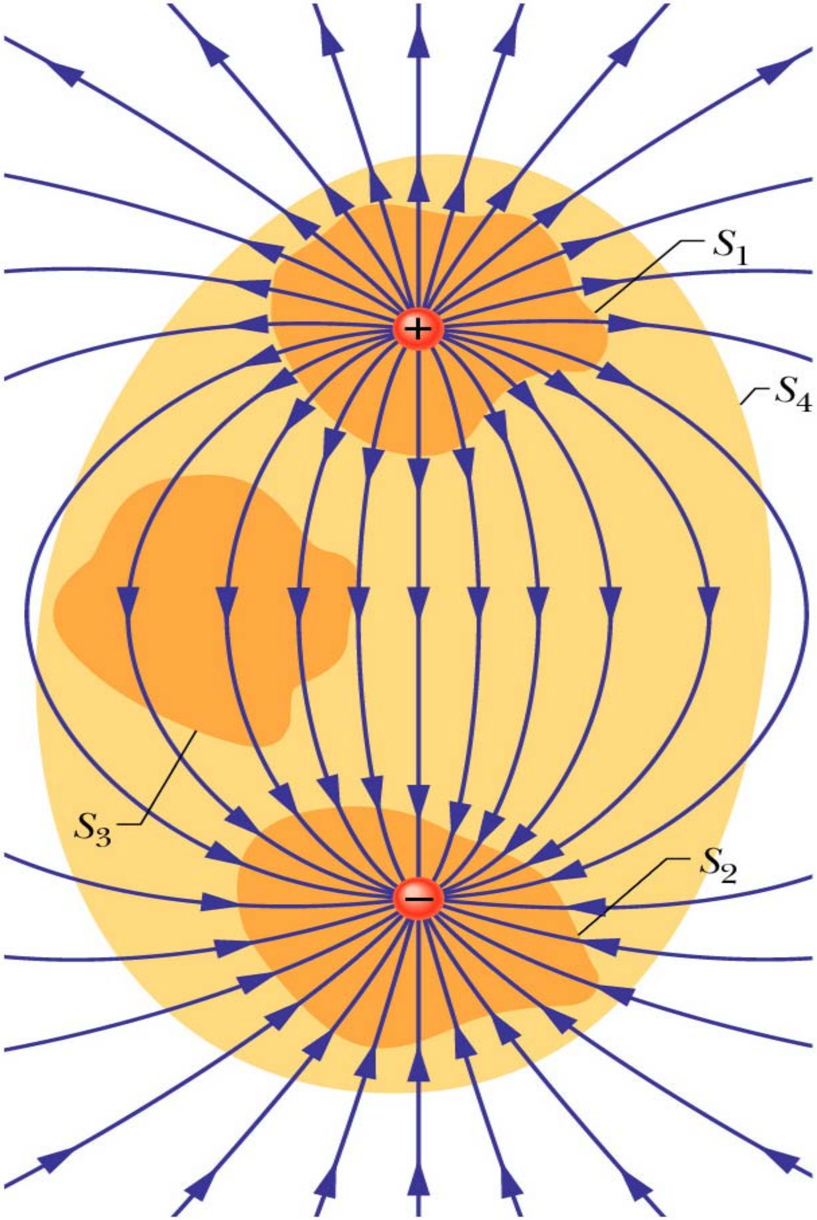
Review

- For certain symmetric distributions of charge w used Gauss' law to calculate the *E* field

$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

- This is an integral over t Gaussian surface *A*



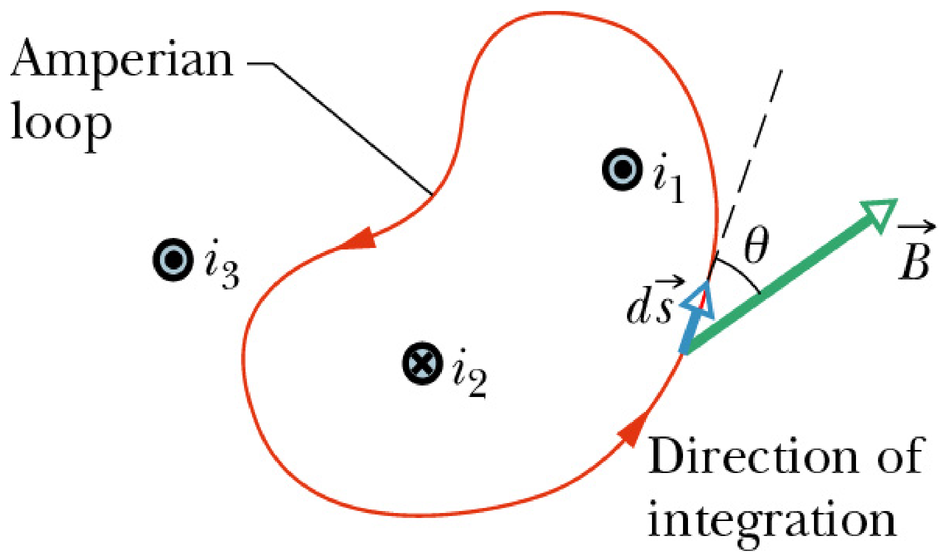

S_1
S_4
S_3
S_2

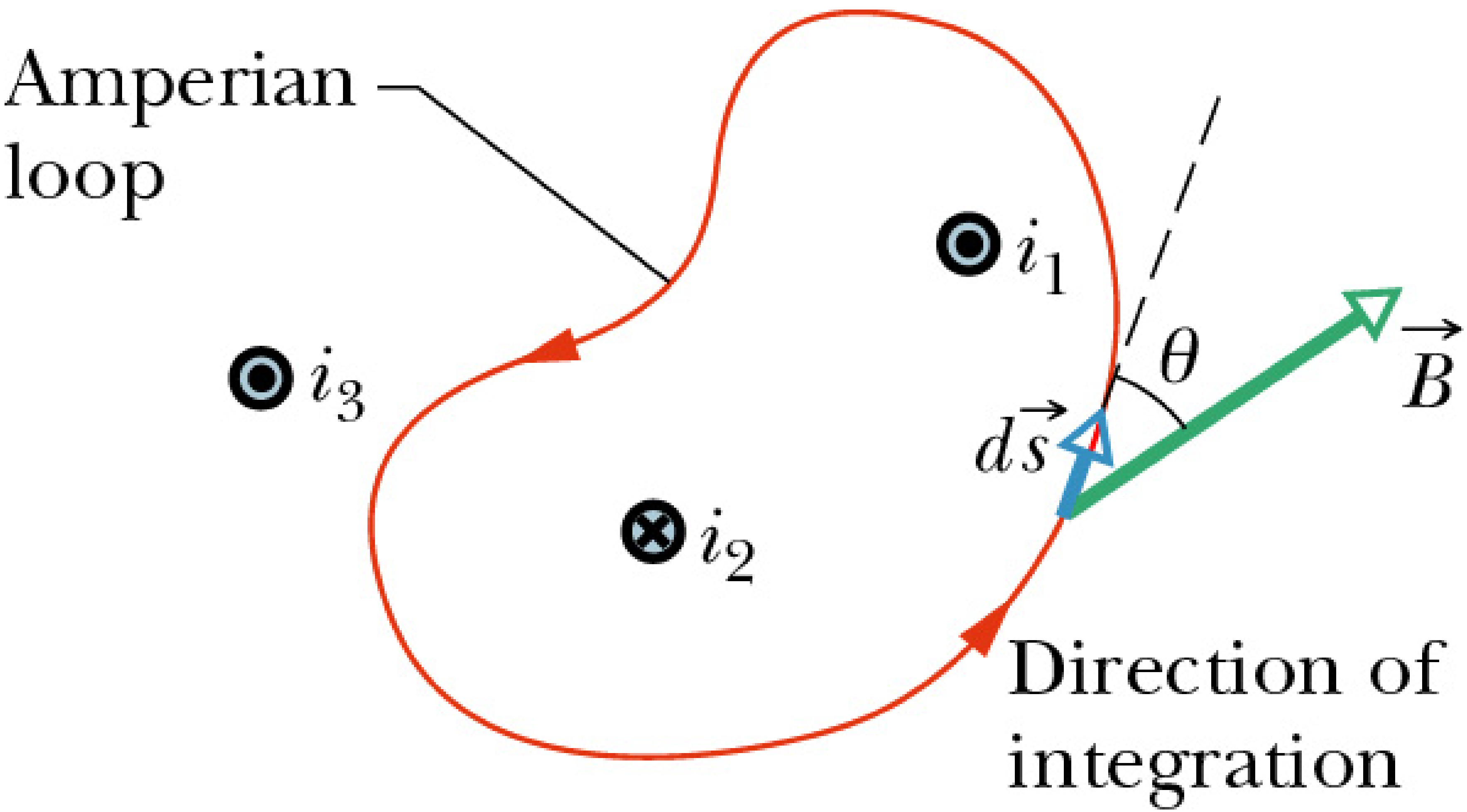
Ampere's Law

- For symmetric distributions of charge use Ampere's law to calculate *B* field

$$\boxed{\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}}$$

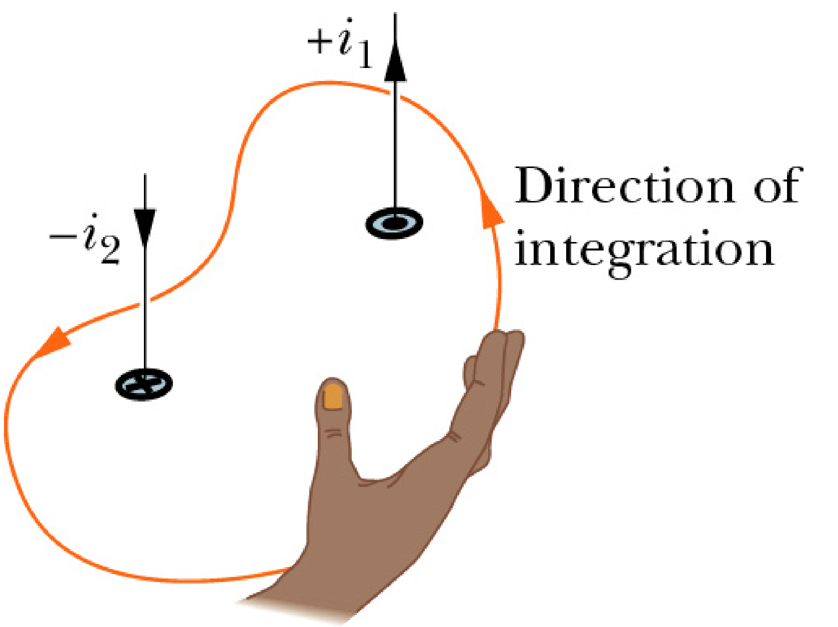
- Integral around closed loop called Amperian loop

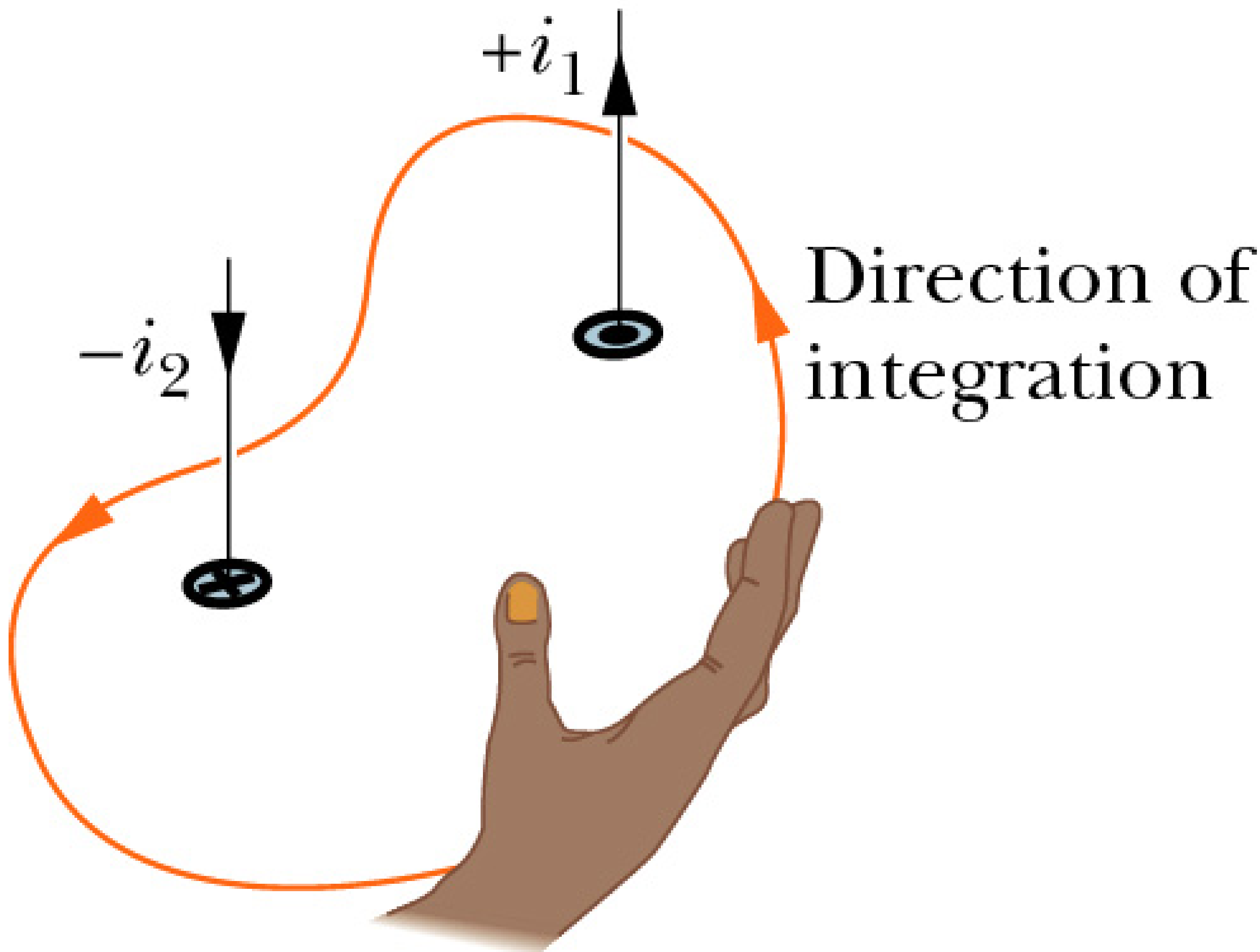



Amperian loop
i_1
i_2
i_3
θ
$\vec{B}$
$d\vec{s}$
Direction of integration

Ampere's Law

- Use the right-hand rule to determine the signs for the currents encircled by the Amperian loop
- Curl right hand around Amperian loop with fingers pointing in direction of integration
- Current going through loop in the same direction as thumb is positive.
- Current going in the opposite direction is negative.

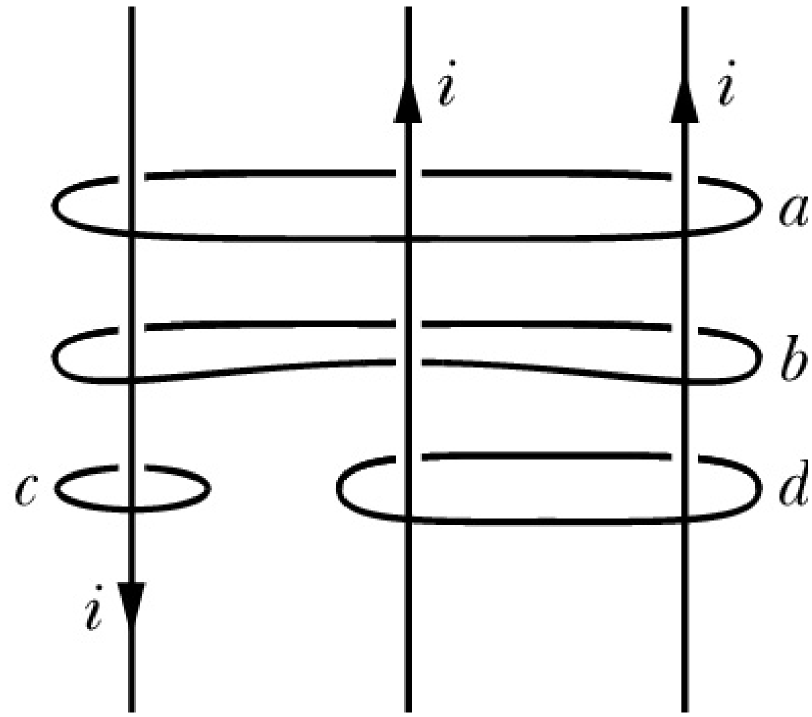




$+i_1$
$-i_2$
Direction of integration

Exercise

- Rank the loops according to the magnitude of $\oint \vec{B} \bullet d\vec{s}$



Assume loops are counterclockwise:

a: i_{enc} = i + i – i = i

b: i_{enc} = i – i = 0

c: i_{enc} = - i

d: i_{enc} = i + i = 2 i

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

d, a & c, b

(a & c tie: same magnitude)