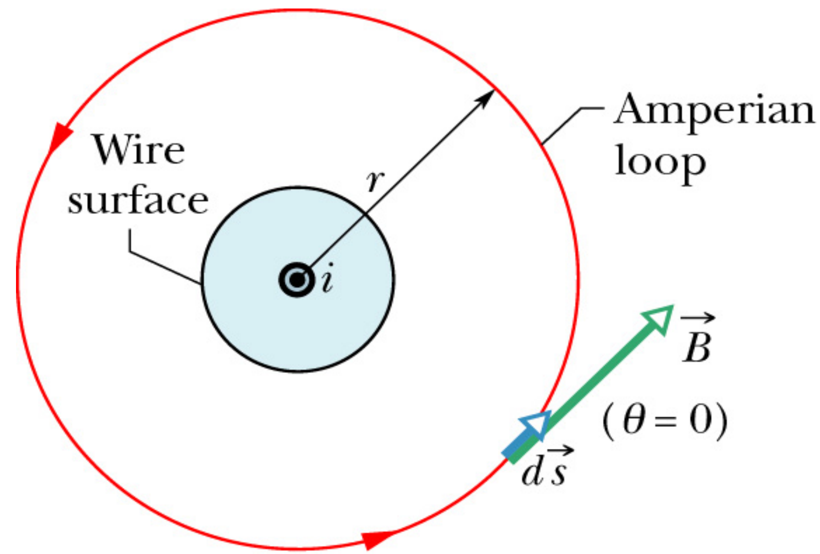
B fields from a single current

- Use Ampere's law to calculate *B* field from long straight wire

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



- Draw Amperian loop as a circle surrounding the wire (like the magnetic field lines)

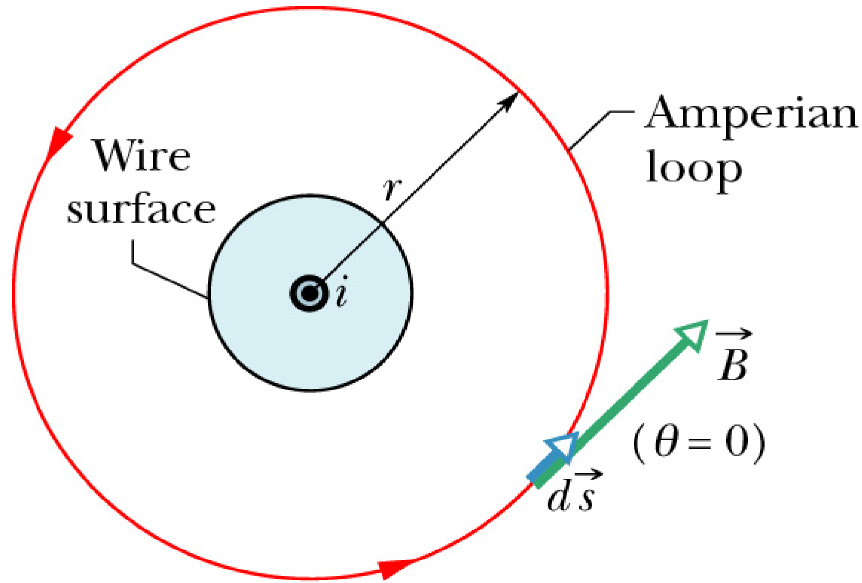
- At every point of the loop
  - Magnitude of *B* is constant
  - *B* and *ds* are || (both tangent to the circle)

B fields from a single current

- *B* and *ds* are || so

$$\cos\theta = \cos 0 = 1$$

$$\vec{B} \bullet d\vec{s} = Bds$$



- *B* constant on loop so

$$\oint \vec{B} \bullet d\vec{s} = B \oint ds$$

$$\oint ds = 2\pi r$$

$$\oint \vec{B} \bullet d\vec{s} = B(2\pi r)$$

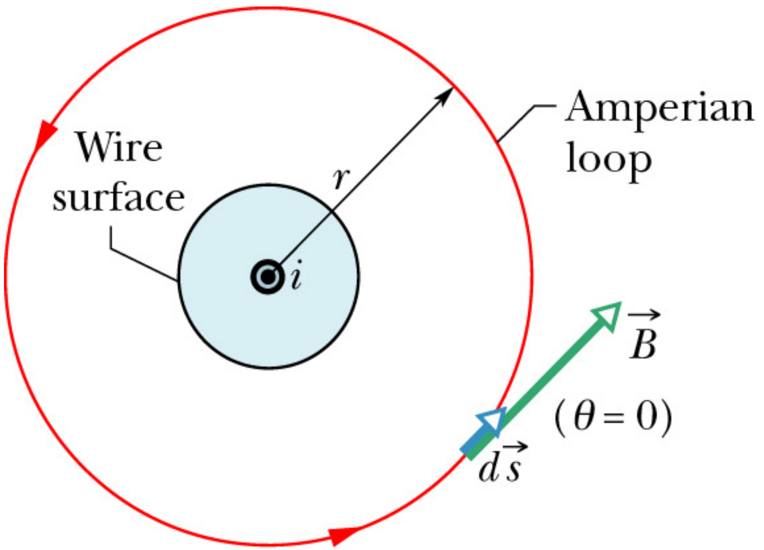
B fields from a single current

- Ampere's law becomes

$$B(2\pi r) = \mu_0 i_{enc}$$

- Current enclosed is just *i* so

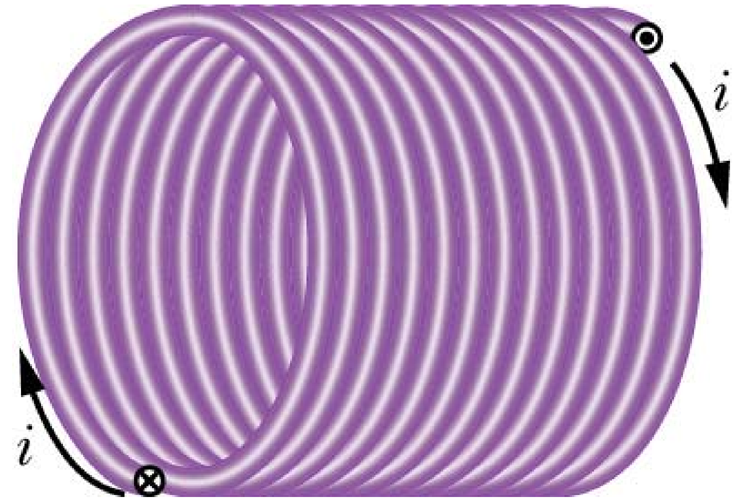
$$B = \frac{\mu_0 i}{2\pi r}$$



- Same result as with Biot-Savart law (we used *r=R* before).

B fields from solenoids

- What happens if there are several loops of wire put together?
- A long, tightly wound helical coil of wire is called a solenoid

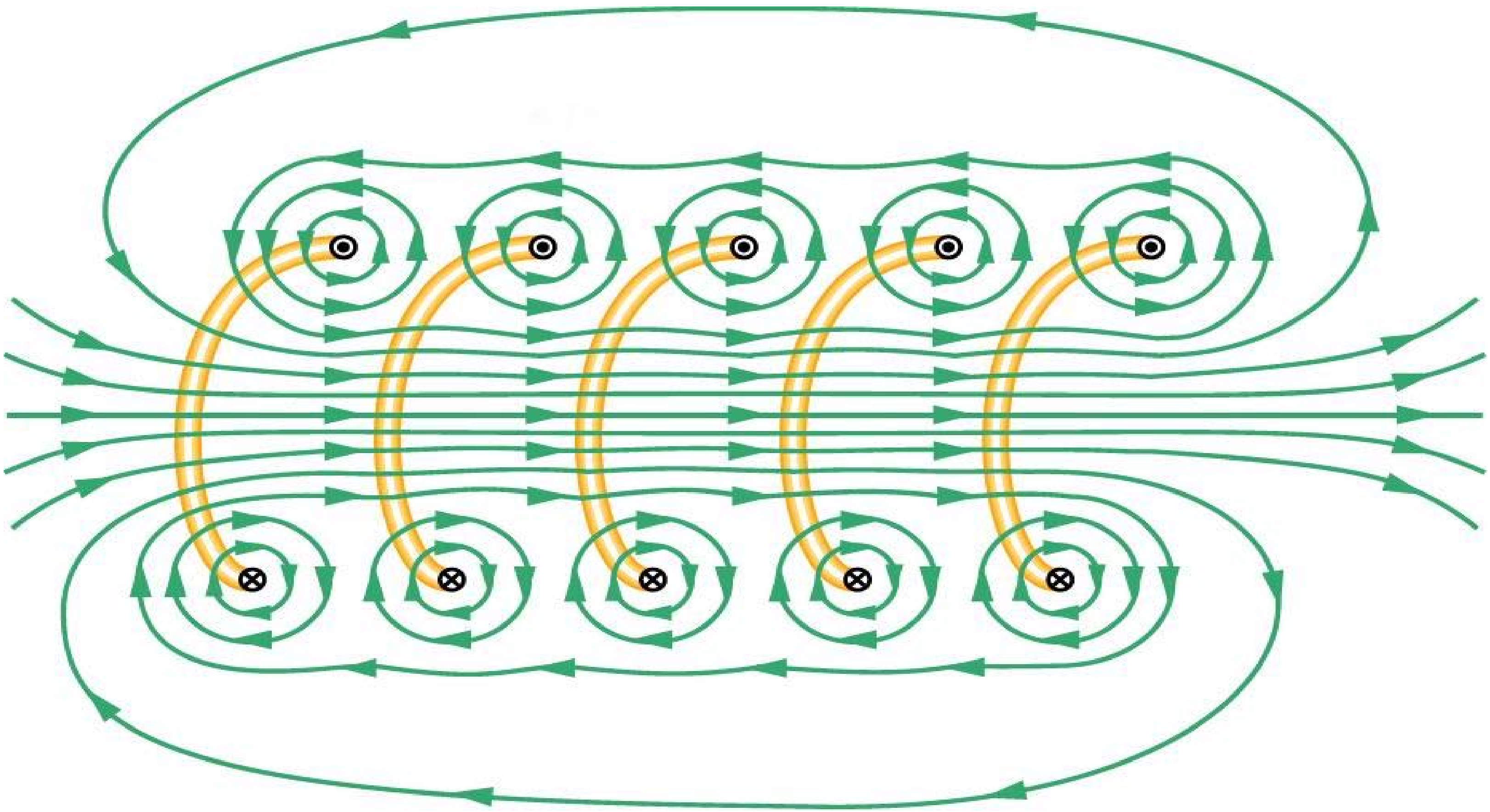


- Bend solenoid so ends meet to make a hollow donut gives a toroid

- Use Ampere's law to calculate *B* field for a solenoid and a toroid

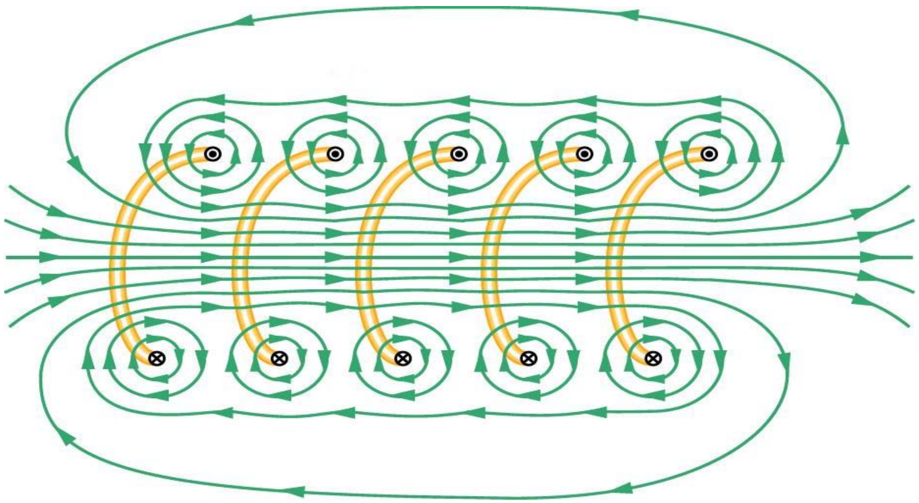
B fields from solenoids

Solenoid's B field is vector sum of fields produced by each turn (loop) in solenoid



B fields from solenoids

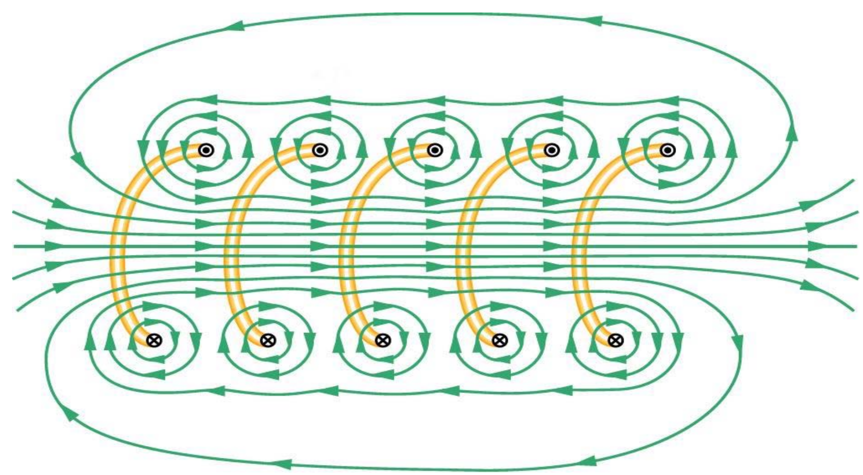
- Near each loop it looks like an infinite straight wire
- Between the loops fields tend to cancel
- Inside the solenoid, far from the wire, *B* field is parallel to axis



- An ideal solenoid
  - is infinity long with closely packed turns of wire
  - has uniform *B* field which is parallel to solenoid axis

B fields from solenoids

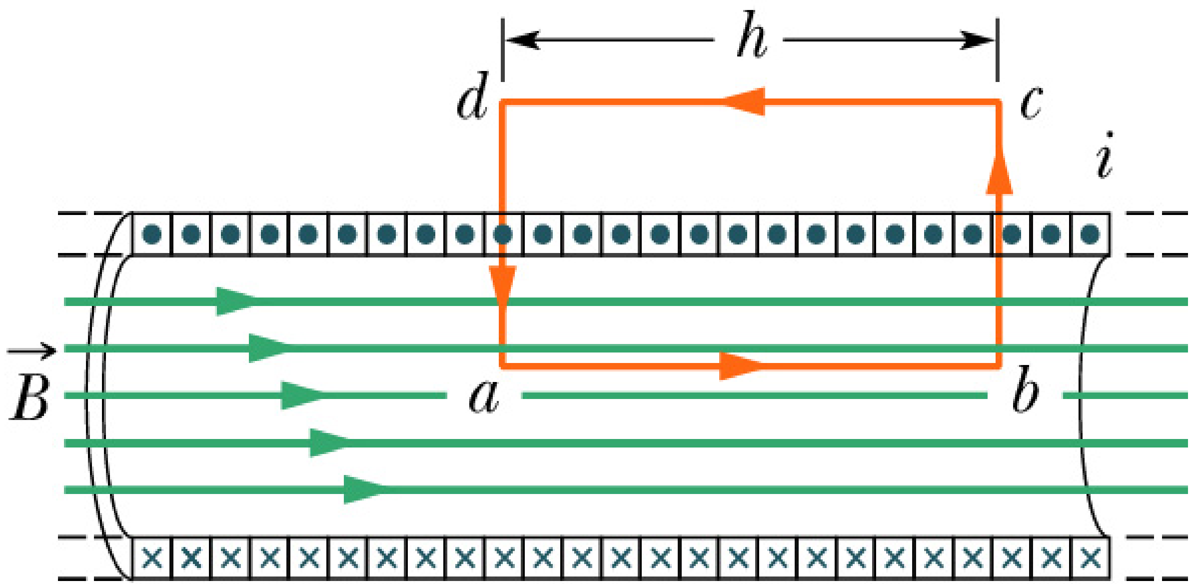
- For points outside the solenoid *B* fields from the upper parts of the turns tend to cancel the lower
- Ideal solenoid $B_{outside}=0$
- For a real solenoid can assume $B_{outside}=0$ if
  - length >> diameter
  - Only consider points not near ends of solenoid



- Use right-hand rule to find direction of *B* field
  - Grasp solenoid so fingers follow direction of *i* in loops, thumb points in *B*

B fields from solenoids

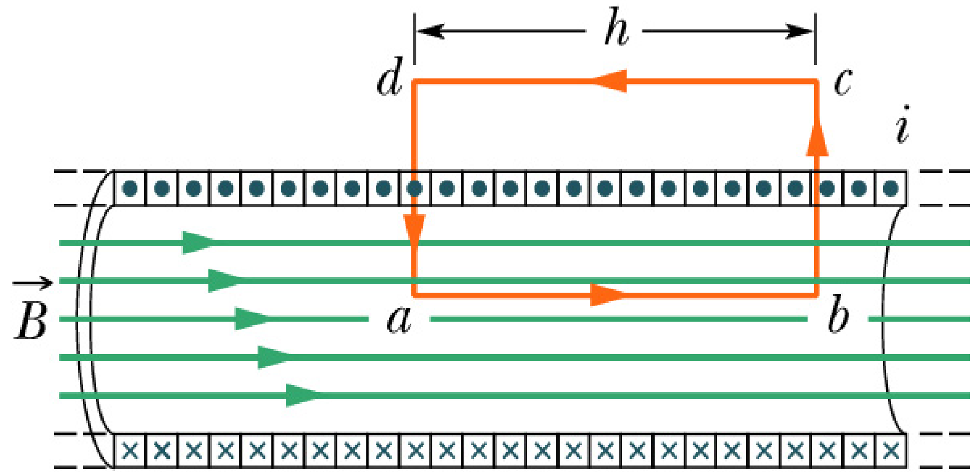
- Use Ampere's law to calculate *B* field of ideal solenoid



- Draw Amperian loop a-b-c-d-a intersecting solenoid

B fields from solenoids

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



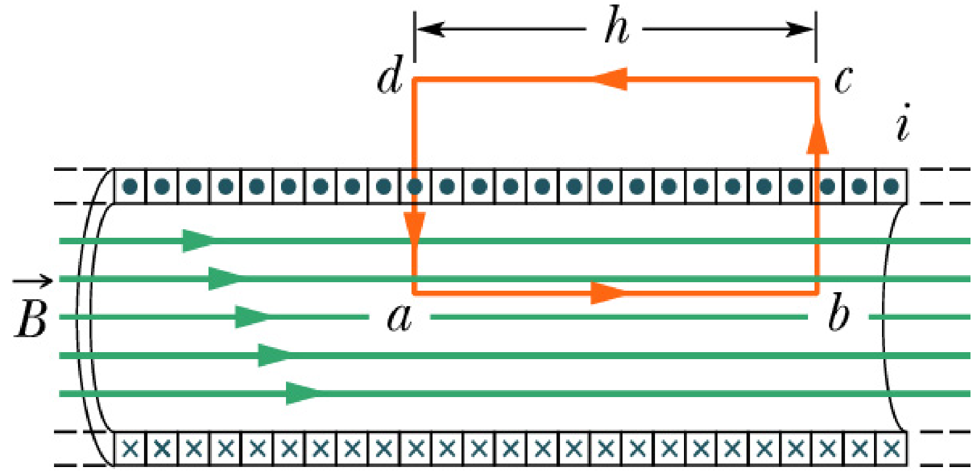
- Integral can be written as sum of four integrals, one for each side

$$\oint \vec{B} \bullet d\vec{s} = \int_a^b \vec{B} \bullet d\vec{s} + \int_b^c \vec{B} \bullet d\vec{s} + \int_c^d \vec{B} \bullet d\vec{s} + \int_d^a \vec{B} \bullet d\vec{s}$$

B fields from solenoids

- First integral B field is || to ds

$$\int_a^b \vec{B} \bullet d\vec{s} = B\left[s\right]_a^b = Bh$$



- For sides bc and da B is ⊥ to ds so

$$\int_b^c \vec{B} \bullet d\vec{s} = \int_d^a \vec{B} \bullet d\vec{s} = 0$$

- For the length outside the solenoid B = 0

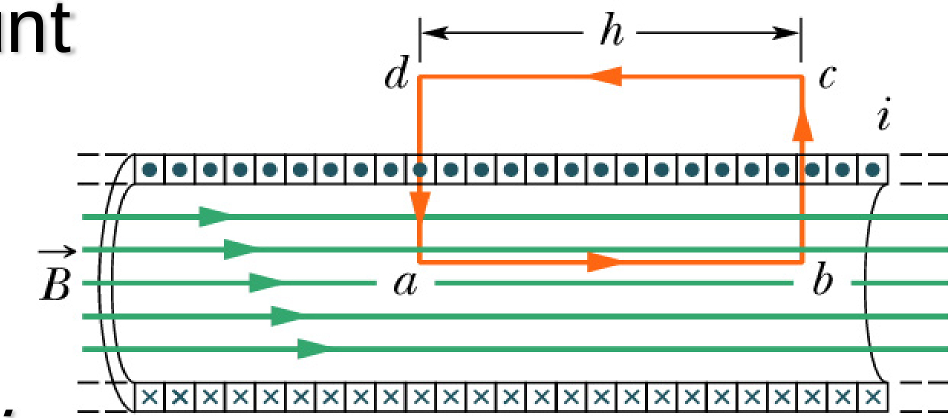
$$\int_c^d \vec{B} \bullet d\vec{s} = 0$$

$$\oint \vec{B} \bullet d\vec{s} = Bh$$

B fields from solenoids

- Now need to find amount of current enclosed

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



- Single coil has current *i*
- But Amperian loop encloses several coils so total current is

$$i_{enc} = inh$$

- *where n* is the number of turns per unit length

$$n = \frac{N}{L}$$

- *N* = total # of turns
- *L* = length

B fields from solenoids

- Substituting into Ampere's law

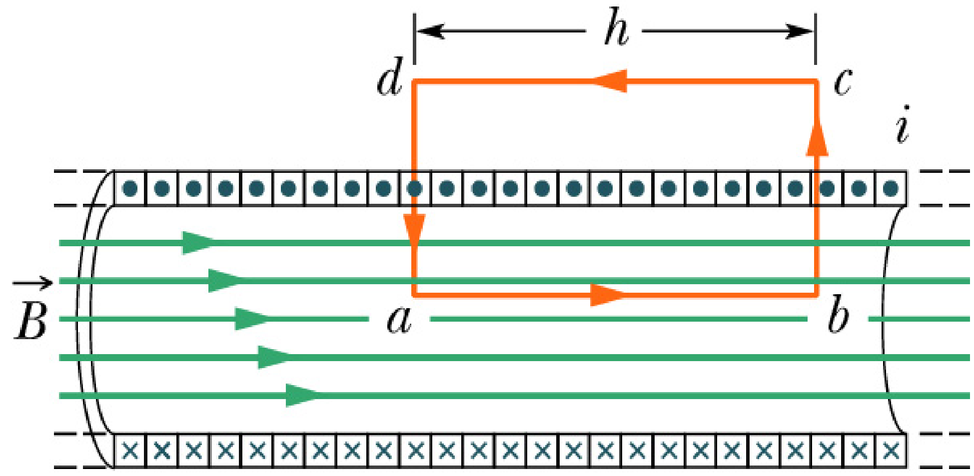
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

$$Bh = inh$$

- For ideal solenoid:

$$B = \mu_0 in$$

- n is # turns/length



- *B* field of solenoid
  - does not depend on diameter or length of solenoid
  - is uniform over its cross section

B fields from toroids

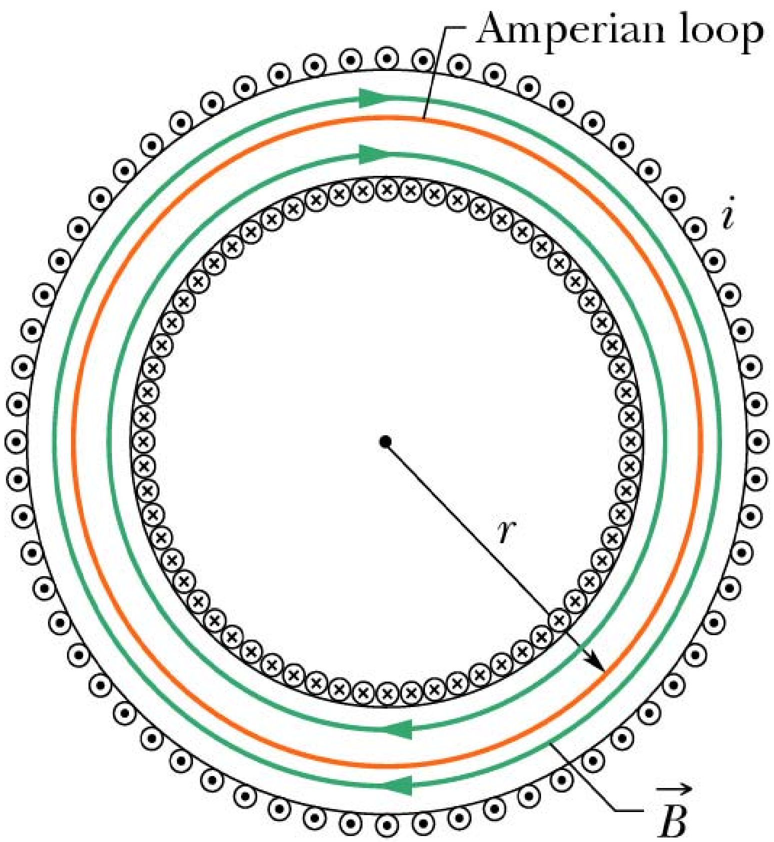
- Calculate *B* field for a toroid using Ampere's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Choose Amperian loop to be a concentric circle inside toroid
- *B* and *ds* are parallel along entire loop so

$$\oint \vec{B} \bullet d\vec{s} = B\int ds = B(2\pi r)$$

(a)



(b)

B fields from toroids

- Current enclosed by loop is

$$i_{enc} = iN$$

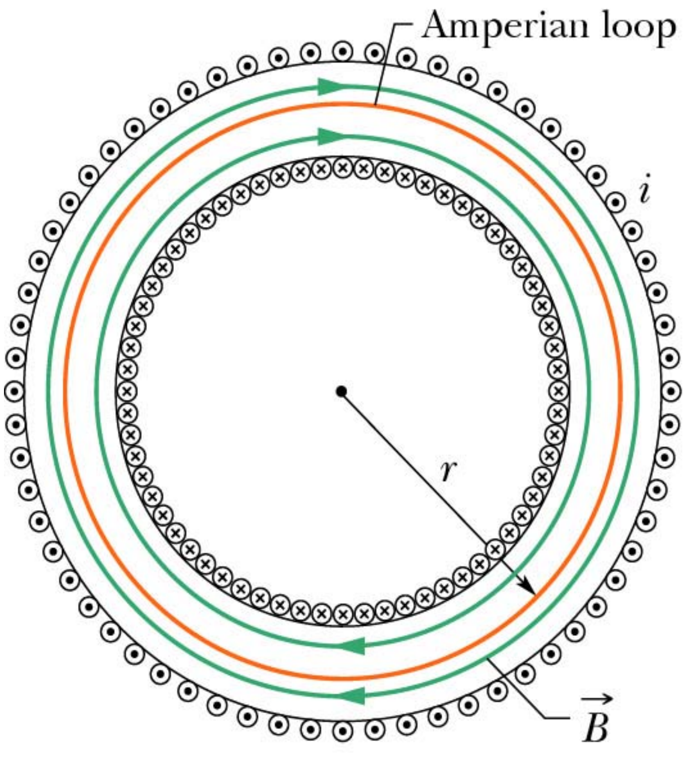
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

$$B(2\pi r) = \mu_0 iN$$

- *B* field for toroid is

$$B = \frac{\mu_0 iN}{2\pi r}$$

(a)



(b)

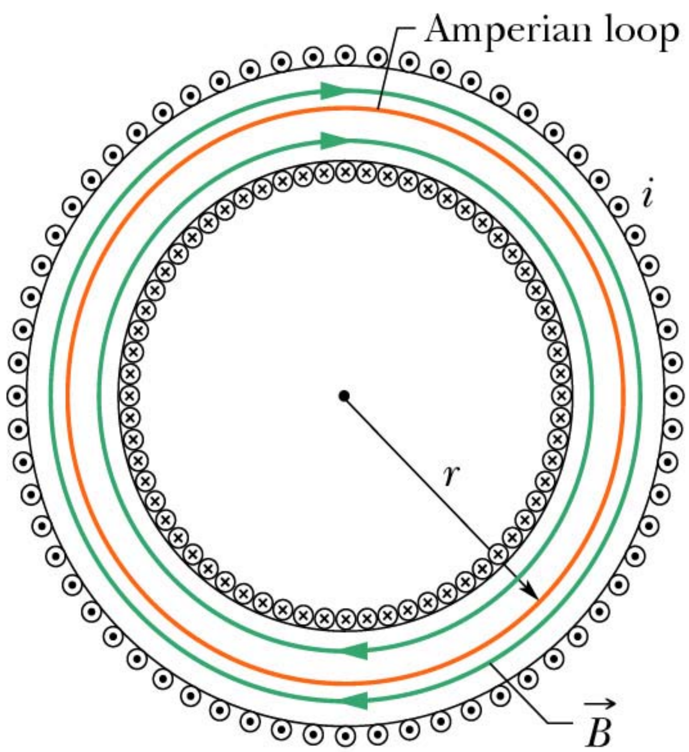
B fields from toroids

- Toroid – *B* field is not constant over its cross section

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r}$$

- N = total # of turns
- Use right-hand rule to find direction of *B* field
  - Grasp toroid with fingers in direction of current in windings, thumb points in B
- *B* = 0 outside toroid

(a)

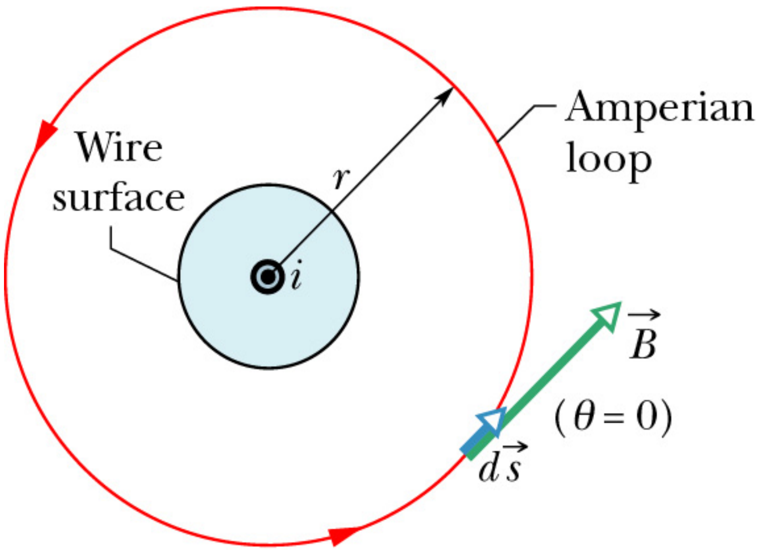


(b)

B fields from finite wires

- Ampere's law becomes

$$B(2\pi r) = \mu_0 i_{enc}$$



- Current enclosed is just *i* so

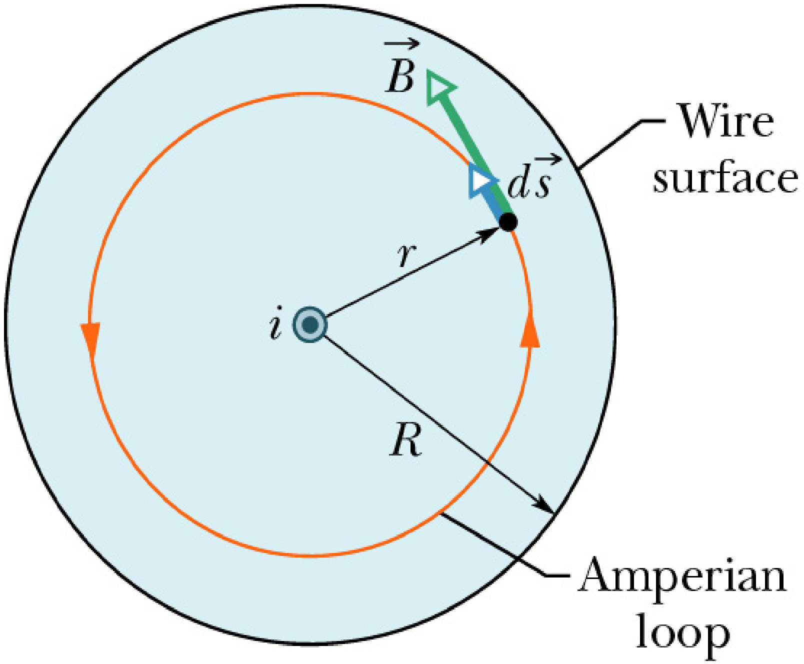
$$B = \frac{\mu_0 i}{2\pi r}$$

- Same result as with Biot-Savart law (we used *r=R* before).

B fields from finite wires

- Calculate *B* field inside a long straight wire

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



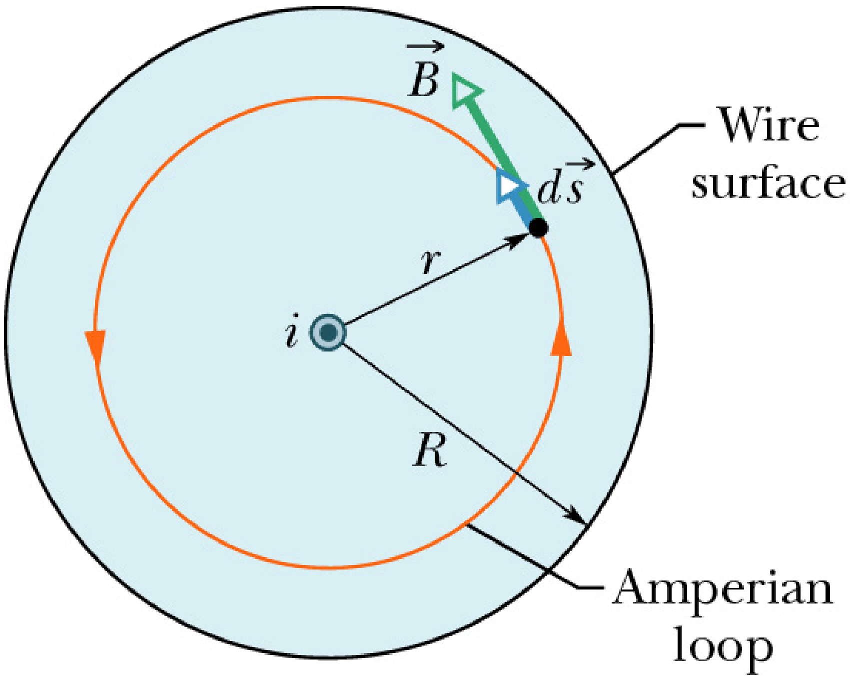
- Again *B* and *ds* are || and *B* is a constant so

$$\oint \vec{B} \bullet d\vec{s} = B\oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i_{enc}$$

B fields from finite wires

- Need to find i_{enc}
- Current is uniformly distributed so i enclosed by loop is α to area enclosed



$$i_{enc} = i\frac{\pi r^2}{\pi R^2}$$

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$