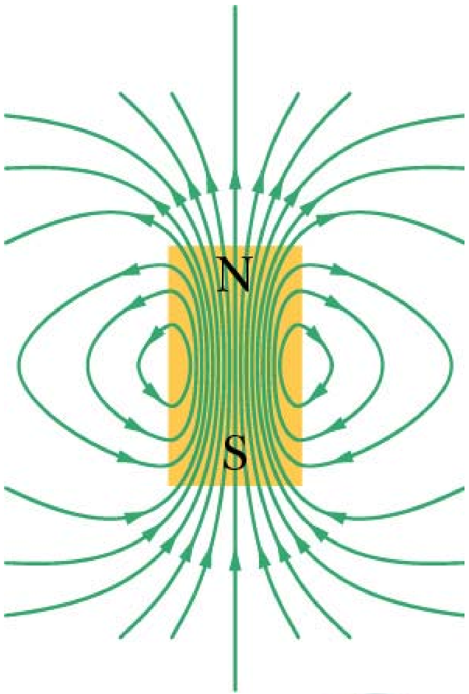
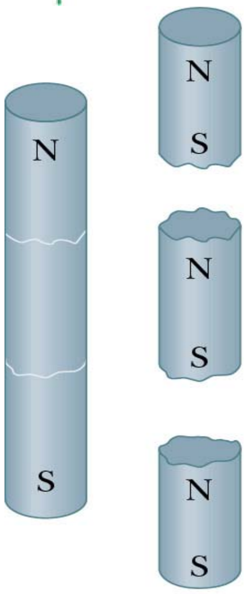
Magnetism

- What makes some materials magnetic?

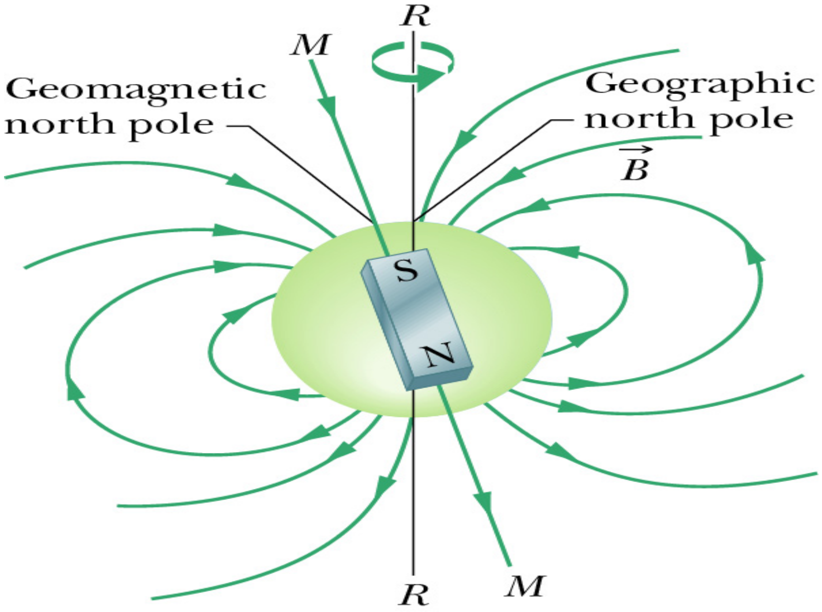
- Magnets are magnetic dipoles - have north and south pole
- If we break a magnet we still have magnetic dipoles
- Magnetic monopoles do not exist

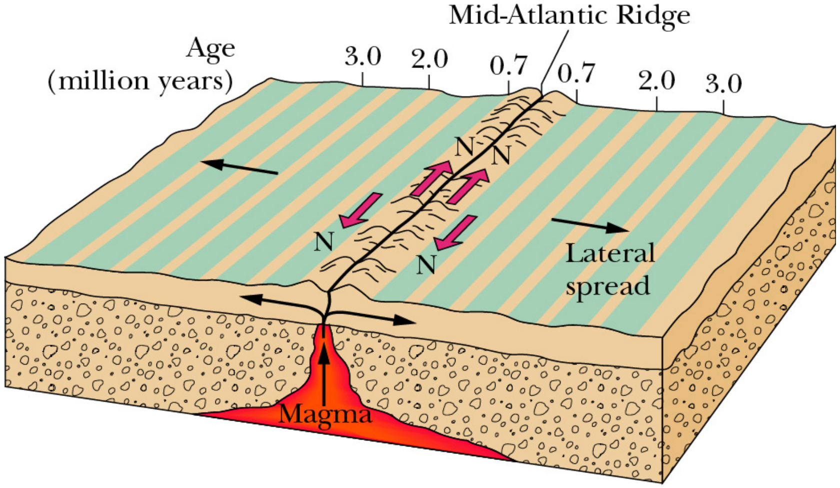




Magnetism of Earth

- Earth acts as huge bar magnet
- Geomagnetic pole at angle of 11.5 degrees from rotational axis
- North pole is actually south pole of Earth's magnetic dipole
- Polarity has reversed about every million years



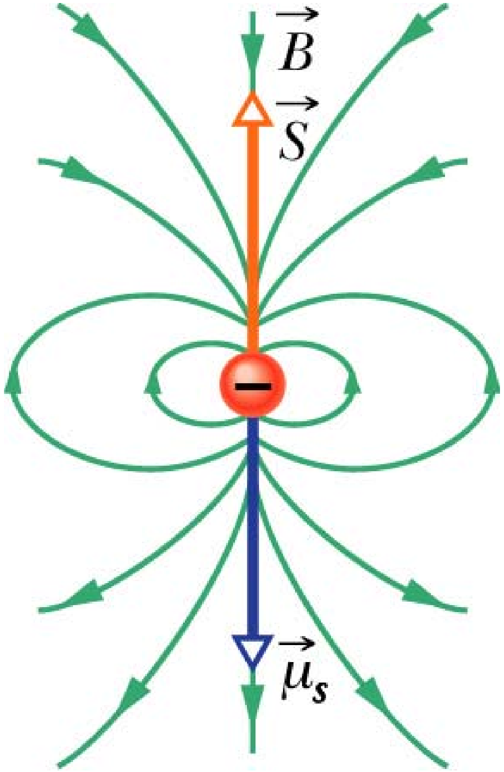


Magnetism

- Electrons moving (a current) set up *B* fields
- Electrons also responsible for *B* fields of magnetic materials

- Electrons have 2 types of magnetic dipoles:
  - Spin magnetic dipole (intrinsic to electron)
  - Orbital magnetic dipole (due to motion of electron around the nucleus)
- Full explanation needs quantum physics

Magnetism of Spin



- Electron has intrinsic spin, *S*, angular momentum
- *S* is quantized – has only a few discrete values
- Its component along any direction is given by

$$\boxed{S_Z = m_S \frac{h}{2\pi}} \qquad \boxed{m_S = \pm\frac{1}{2}}$$

- m_S is spin magnetic quantum number

- $+m_S$ called spin up
- $-m_S$ called spin down

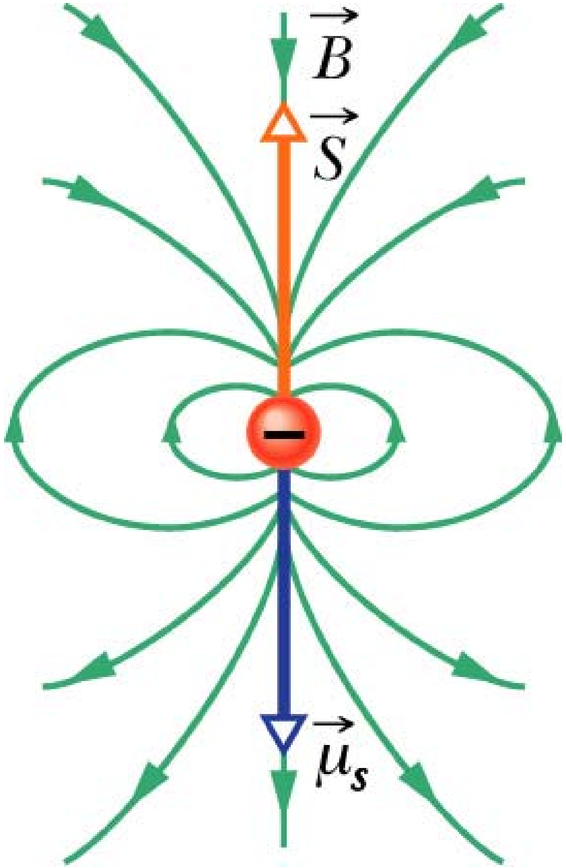
Magnetism of Spin

- spin magnetic dipole moment, μ_S is associated with spin by

$$\boxed{\mu_{S,z} = -\frac{e}{m} S_z}$$

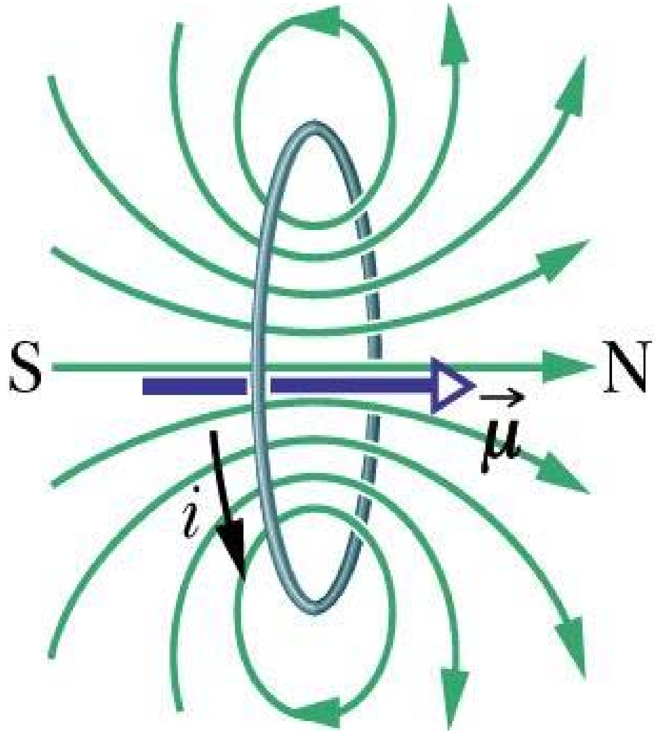
- Minus sign means opposite direction of spin
- Potential energy of an electron in external B field is associated with spin magnetic dipole moment

$$\boxed{U = -\vec{\mu}_S \bullet \vec{B}_{ext}}$$



Magnetism of Orbital Motion

- The orbital motion of electrons around the nucleus generates magnetic dipole fields.
- In some materials these all cancel and there is no net magnetic field.
- In a permanent magnet these are all oriented in the same direction to give the resulting field.



Magnetism of Orbital Motion

- Inside an atom, an electron has orbital angular momentum, L_{orb}
- L_{orb} is quantized
- Its component along any direction is given by

$$L_{orb,Z} = m_l \frac{h}{2\pi}$$

$$m_l = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit})$$

- m_l is orbital magnetic quantum number

Magnetism of Orbital Motion

- orbital magnetic dipole moment, μ_{orb} is associated with orbital angular momentum

$$\mu_{orb,Z} = -\frac{e}{2m} L_{orb,Z}$$

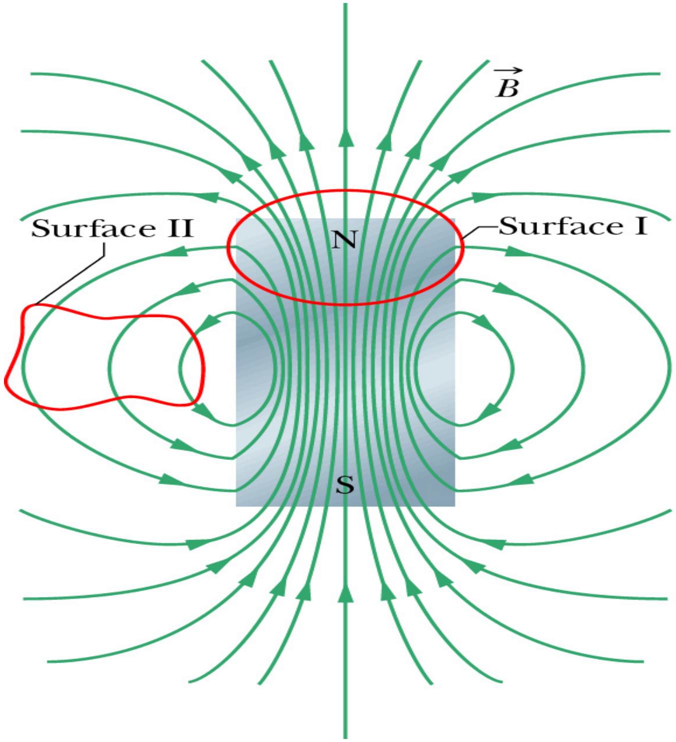
- Minus sign means opposite direction of Lorb
- Potential energy of an atom in external *B* field is associated with orientation of the orbital magnetic dipole moment of each electron in the atom

$$U = -\vec{\mu}_{orb} \bullet \vec{B}_{ext}$$

Gauss's Law of Magnetism

- Magnetic monopoles do not exist
- Express mathematically as

$$\Phi_B = \oint \vec{B} \bullet d\vec{A} = 0$$



- Integral is taken over closed surface
- Net magnetic flux through closed surface is zero
  - As many *B* field lines enter as leave the surface

Gauss's Law of Magnetism

- Gauss's law for *E* fields

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

- Gauss's law for *B* fields

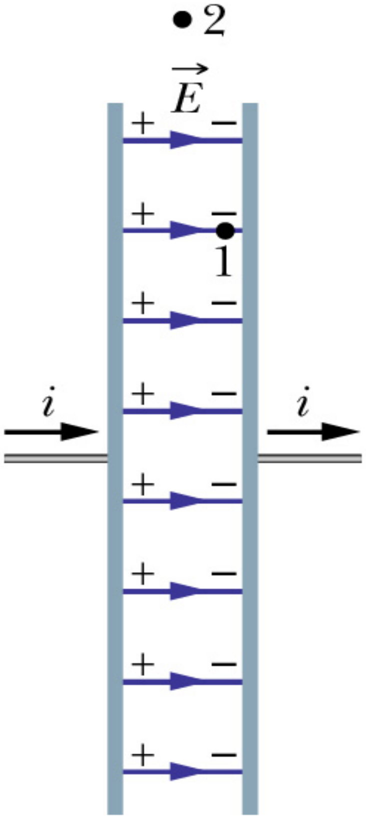
$$\Phi_B = \oint \vec{B} \bullet d\vec{A} = 0$$

- Both cases integrate over closed Gaussian surface

Maxwell’s law of Induction

- Maxwell’s law of induction
  B field is induced along a closed loop by a changing electric flux in region encircled by loop

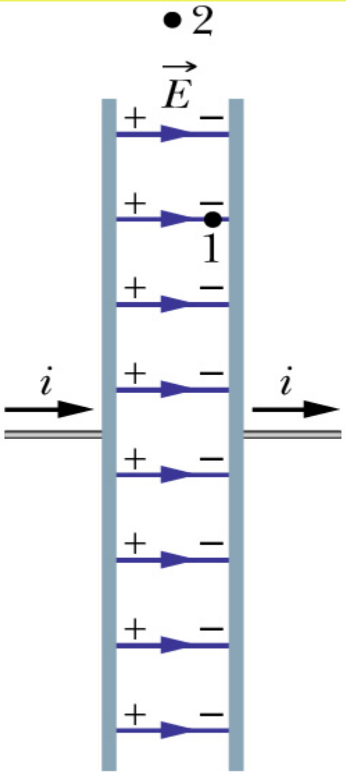
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$



Maxwell’s law of Induction

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Consider circular parallel-plate capacitor with *E* field increasing at a steady rate
- While *E* field changing, *B* fields are induced between plates, both inside and outside (point 1 and 2).
- If *E* field stops changing, *B* field disappears



Maxwell's law of Induction

- Ampere's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Combine Ampere's and Maxwell's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- B field can be produced by a current and/or a changing E field
  - Wire carrying constant current, $d\Phi_E/dt = 0$
  - Charging a capacitor, no current so $i_{enc} = 0$

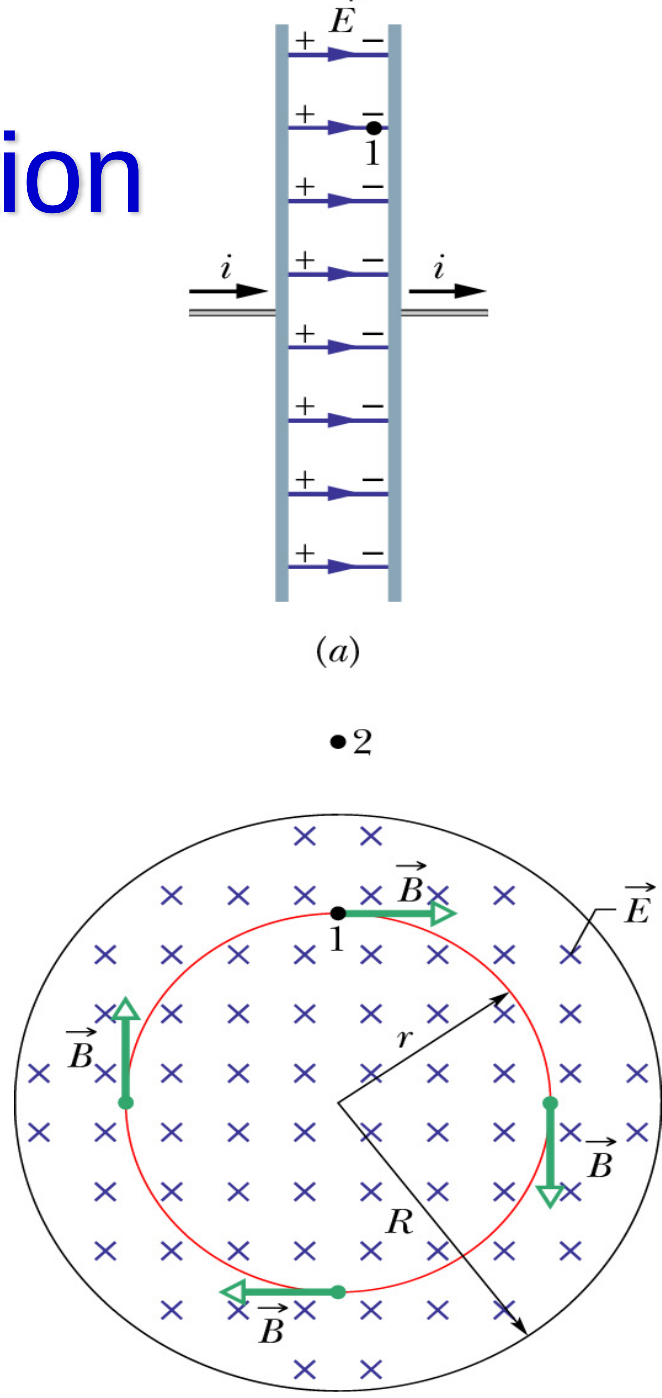
Maxwell's law of Induction

- What is the induced *B* field inside a circular capacitor which is being charged?

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- No current between capacitor plates so $i_{enc} = 0$ and equation becomes

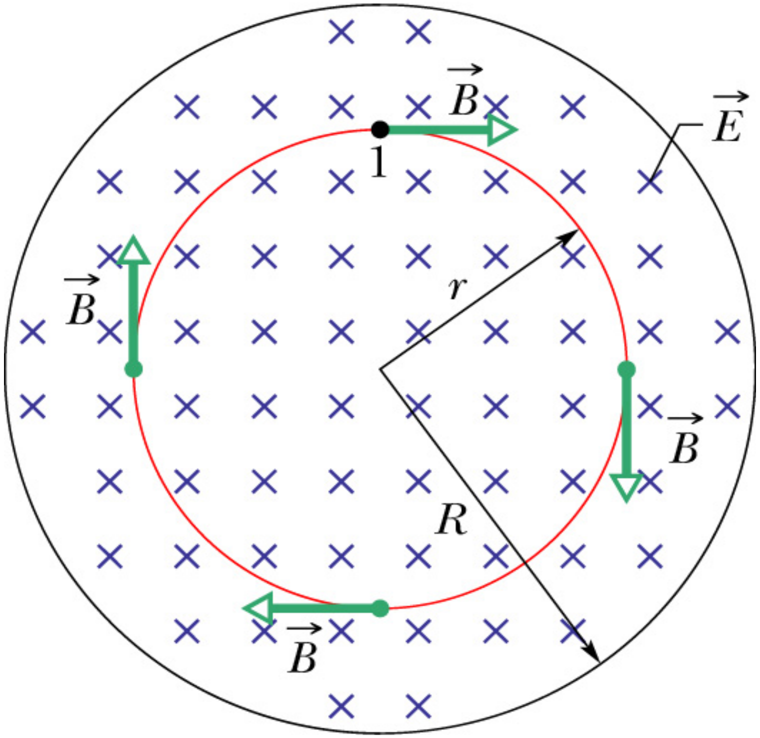
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$



Maxwell's law of Induction

- For left-hand side of equation chose Amperian loop inside capacitor

$$\oint \vec{B} \bullet d\vec{s} = \oint B ds \cos\theta$$



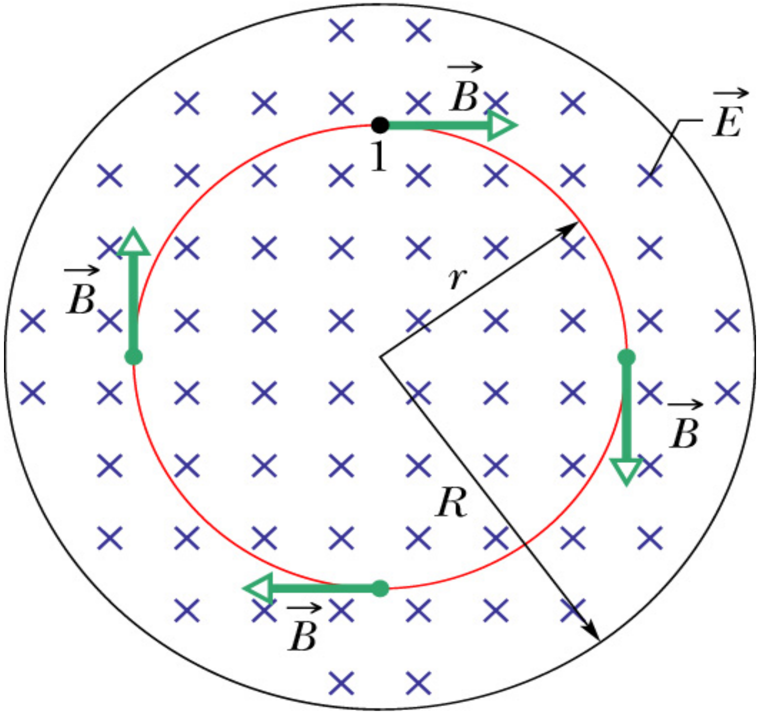
- *B* and *ds* are parallel and *B* is constant so

$$\oint \vec{B} \bullet d\vec{s} = \oint B ds \cos 0 = B \oint ds = B(2\pi r)$$

Maxwell's law of Induction

- For right-hand side of equation find *E* flux through Amperian loop
- *E* uniform between plates and ⊥ to area *A* of loop

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = EA$$



- Right-hand side of equation becomes

$$\mu_0\varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 \frac{d}{dt}(EA) = \mu_0\varepsilon_0 A \frac{dE}{dt}$$

Maxwell's law of Induction

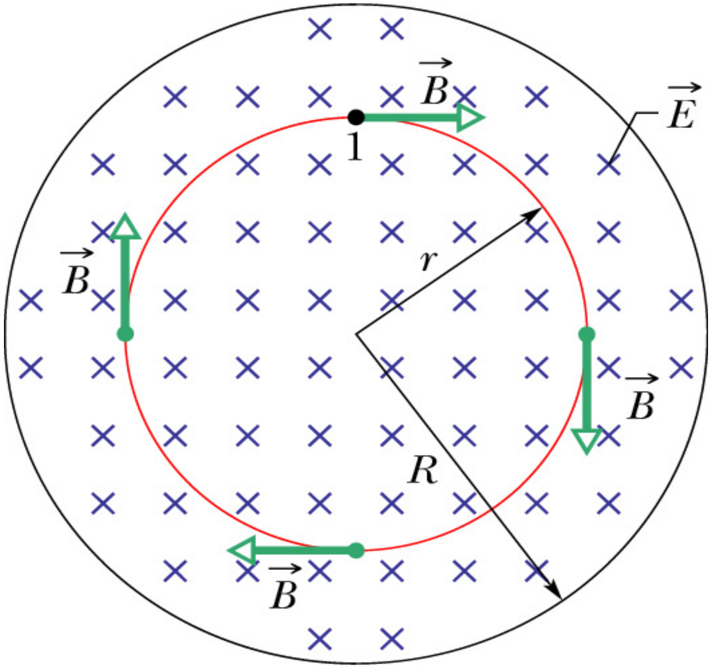
- Equating two sides gives

$$B(2\pi r) = \mu_0 \varepsilon_0 A \frac{dE}{dt}$$

- A is area of loop $A = \pi r^2$

- Solving for *B* field inside capacitor gives

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt}$$

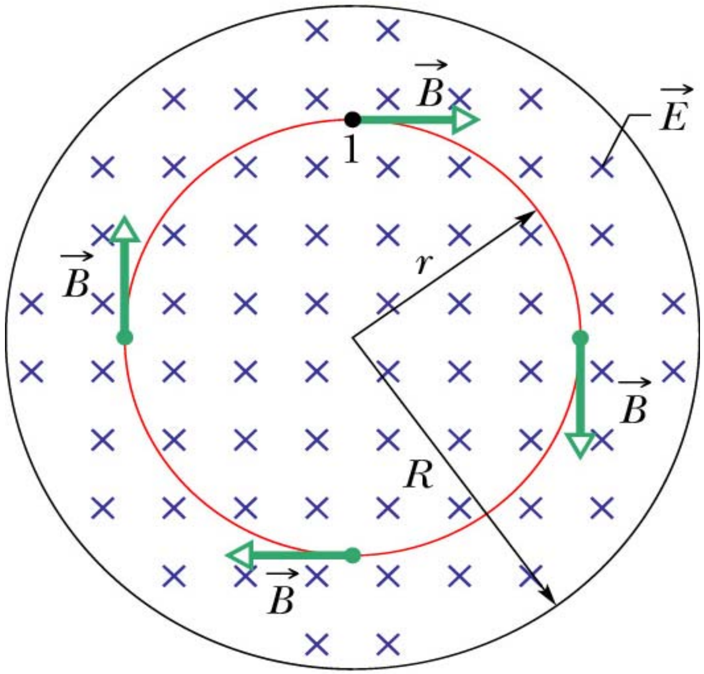


- *B* increases linearly with radius
- *B = 0* at center and max at plate edges

Maxwell’s law of Induction

- What is the induced *B* field outside a circular capacitor which is being charged?

$$\oint \vec{B}\bullet d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$



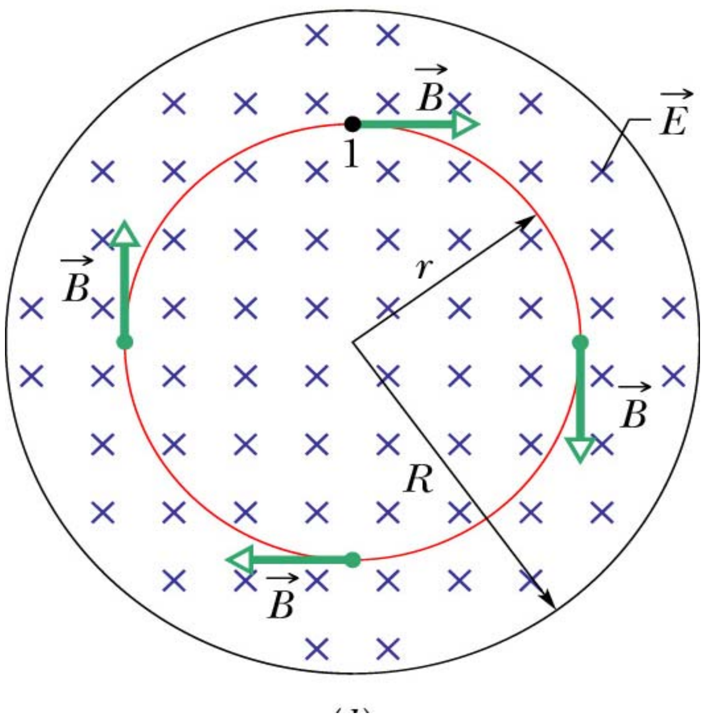
- Realize $i_{enc} = 0$ and find same relations

$$\oint \vec{B}\bullet d\vec{s} = B(2\pi r)$$

$$\mu_0\varepsilon_0\frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 A\frac{dE}{dt}$$

Maxwell’s law of Induction

$$B(2\pi r)=\mu_0\varepsilon_0 A\frac{dE}{dt}$$



- *E* field only exists between plates so area of E field is not full area of loop, only area of plates

$$A=\pi R^2$$

- *B* field becomes

$$B=\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}$$

- Outside capacitor, *B* decreases with radial distance from a max value at *r = R*

Maxwell's law of Induction

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- Can represent change in electric flux with a fictitious current called the displacement current, i_d

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Ampere-Maxwell's law becomes

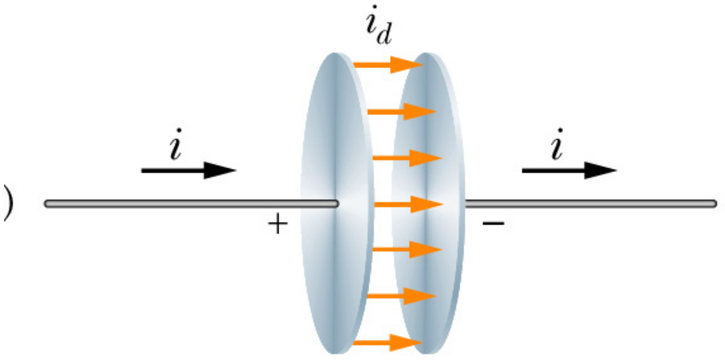
$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}$$

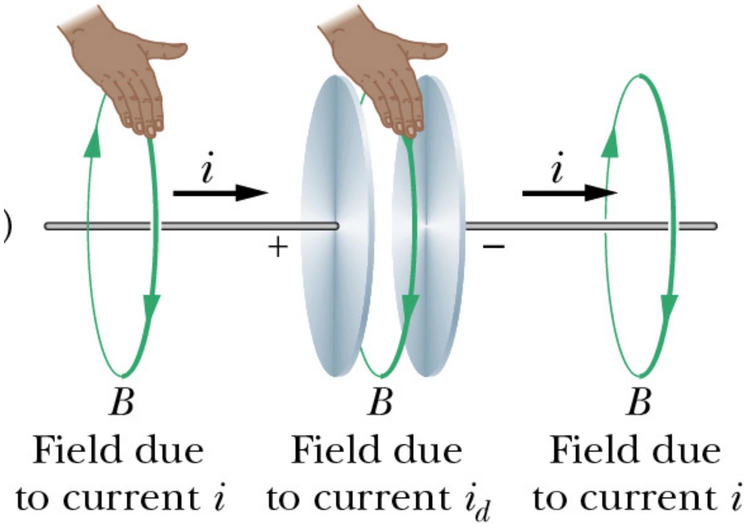
Maxwell’s law of Induction

$$\oint \vec{B}\bullet d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}$$

- Think of displacement current as fictional current between plates

- Use right-hand rule to find direction of *B* field for both currents





Maxwell’s law of Induction

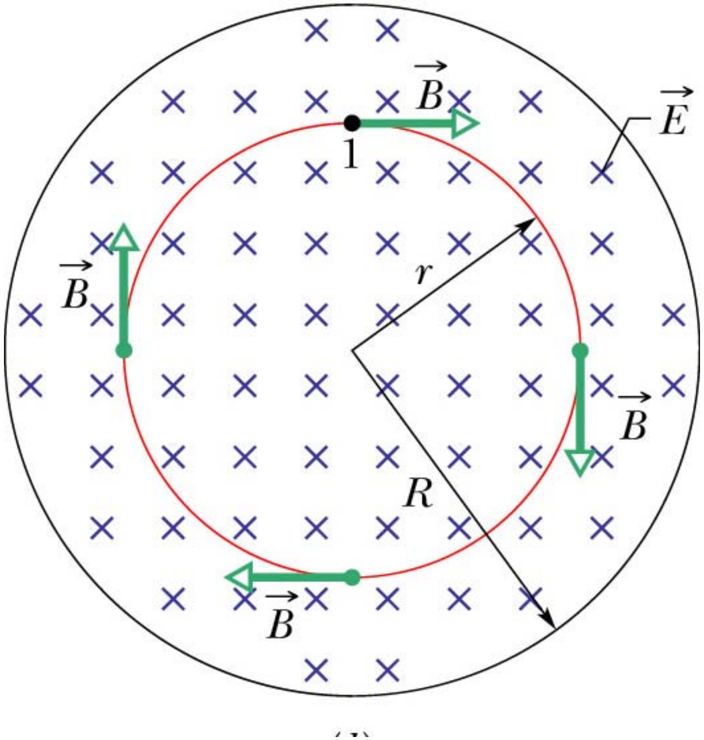
- Used Ampere’s law to calculate *B* field inside a long straight wire with current *i*

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2} \right) r$$

- Find *B* field inside a circular capacitor just replace *i* with displacement current, ***i_d***

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2} \right) r$$



Maxwell’s law of Induction

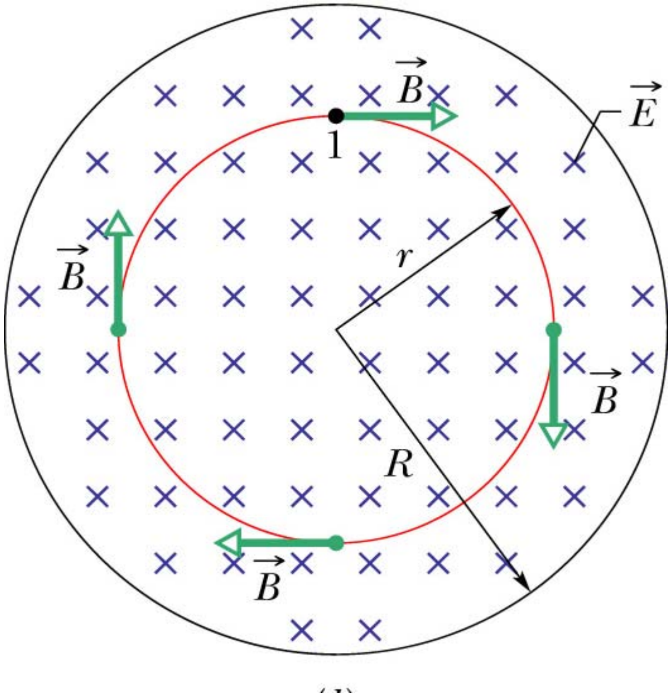
- Used Ampere’s law to calculate *B* field outside a long straight wire with current *i*

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

$$B = \frac{\mu_0 i}{2\pi r}$$

- Find *B* field outside a circular capacitor just replace *i* with displacement current, ***i*_d_**

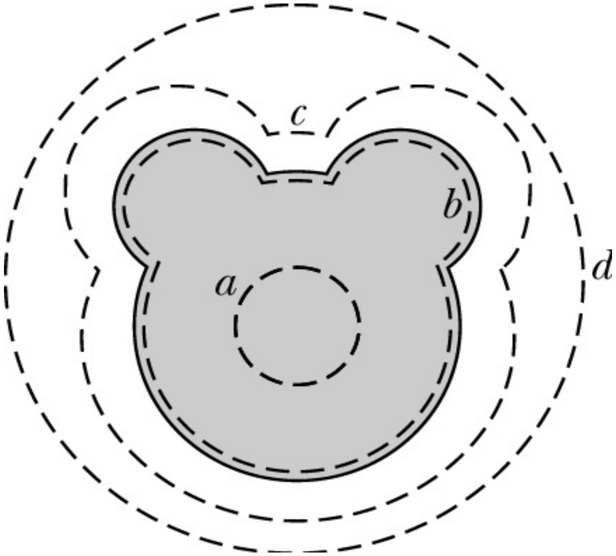
$$B = \frac{\mu_0 i_d}{2\pi r}$$



Exercise

- Parallel-plate capacitor of shape shown. Dashed lines are paths of integration. Rank the paths according to the magnitude of integral *Bds* when capacitor is discharging, greatest first.

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}$$



- Only displaced current in capacitor

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{d,enc}$$

- What is i_d for each path?

b, c, d all tie, then a

Magnetism

- Three types of magnetism:
- Ferromagnetism
  - Property of iron, nickel, neodymium
  - Strongest type of magnetism
- Paramagnetism
  - Exhibited by materials containing transition, rare earth or actinide elements
- Diamagnetism
  - Exhibited by all common materials but masked if other two types of magnetism are present

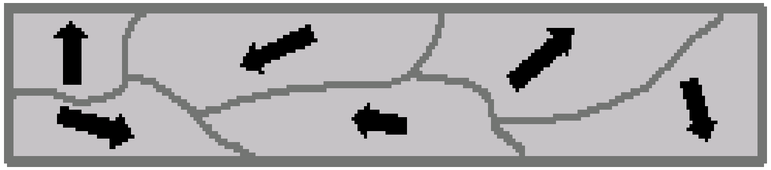
Ferromagnets

- Electron spins of one atom in the material interact with those of neighboring atoms
- Process of coupling causes alignment of magnetic dipole moments of the atoms despite thermal agitations
- This alignment gives material its permanent magnetism

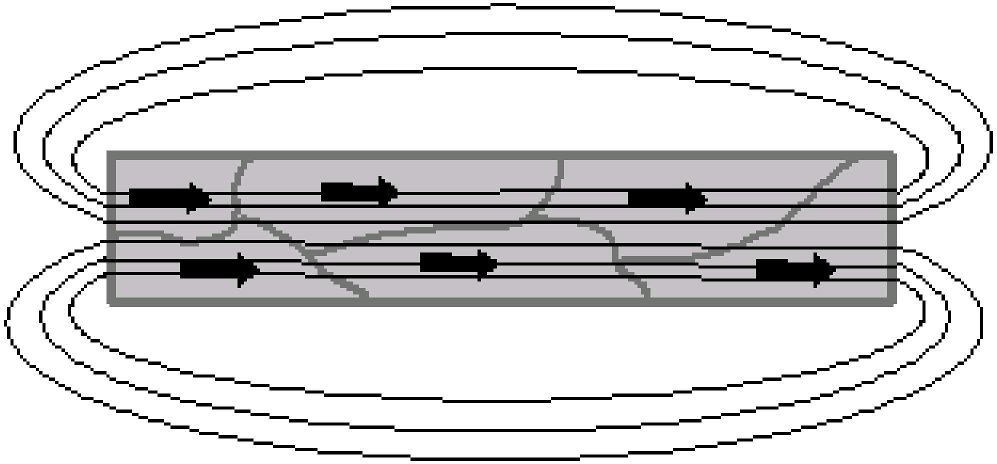
Ferromagnets

- If coupling produces strong alignment of adjacent atomic dipoles, why aren’t all pieces of iron strong magnets?

- As a whole the material's magnetic domains are oriented randomly and effectively cancel each other out



- If B_{ext} applied, domains align giving a strong net B field in same direction as B_{ext}



- Net B field partially exists even when B_{ext} is removed

Ferromagnets

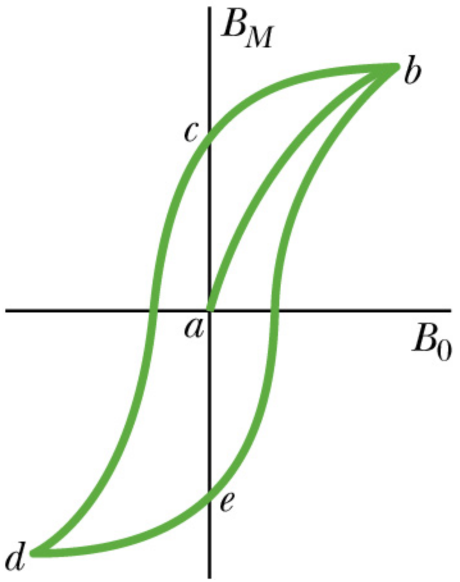
- If we place ferromagnetic material (e.g. iron) inside a solenoid with field B_0, increase the total B field inside coil to

$$B = B_0 + B_M \qquad B_0 = \mu_0 i n$$

- B_M is magnitude of B field contributed by iron core
- B_M result of alignment of the domains
- B_M increases total B by large amount - iron core inside solenoid increases B by typically about 5000 times
- For the electromagnetic core we use “soft” iron where the magnetism is not permanent (goes away when the external field is turned off).

Ferromagnets

- If we increase and then decrease external field, B_0, the magnetization curves for "permanent" magnets are not the same
- Lack of retraceability is called hysteresis
- Change of magnetic domains orientations are not totally reversible, retain some memory of their alignment
- This is the origin of the permanent magnetic
- Used for magnetic storage of information on tapes, cds, etc

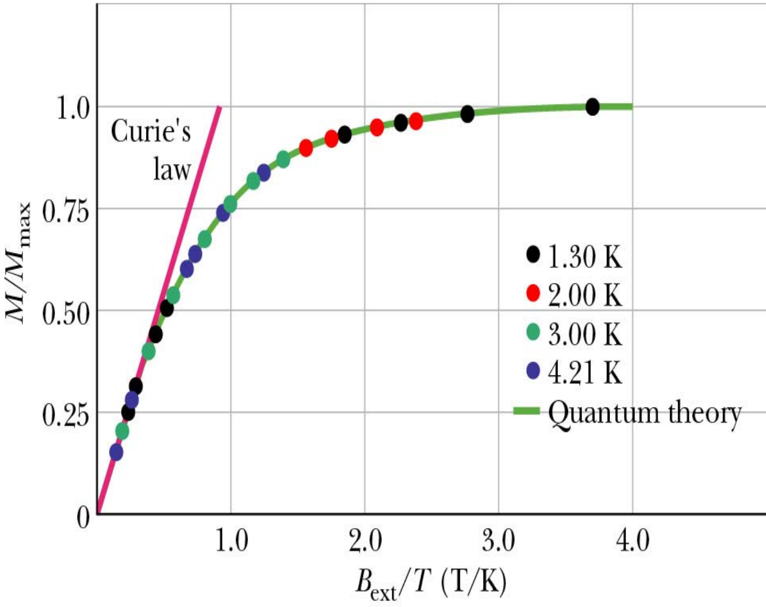


Paramagnetism

- Each atom has a permanent net magnetic dipole moment from spin and orbital dipole moments of its electrons
- Atomic dipole moments are randomly oriented so material has no net magnetic field
- If B_{ext} present, partially align the atomic dipole moments giving the material a net B field in the direction of B_{ext}
- The dipole alignment and their net B field disappear when B_{ext} is removed
- Random collisions of atoms due to thermal agitation prevent total alignment of atomic dipoles thus weakening material's B field

Paramagnetism

- Paramagnetism
  - Stronger than diamagnetism
  - Random collisions of atoms due to thermal agitation prevent total alignment of atomic dipoles thus weakening material's B field
  - Curie's law relates magnetization, M, of sample to B_{ext} and temperature, T
    - Only valid when ratio B_{ext}/T not too large



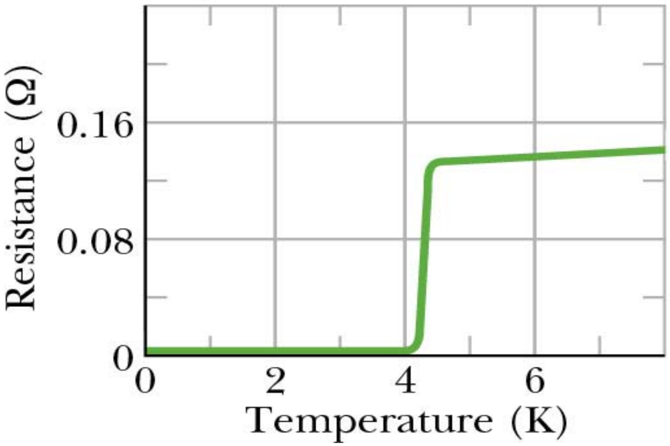
$$M = C\frac{B_{ext}}{T}$$

Diamagnetism

- Atoms in some materials lack net magnetic dipole moment
- If external B field present, induce a weak net B field in material directed opposite to B_{ext}
- Dipole moments and their net B field disappear when B_{ext} is removed
- Organic material (animals, humans) exhibit diamagnetism (picture in the book of a floating frog).

Superconductivity

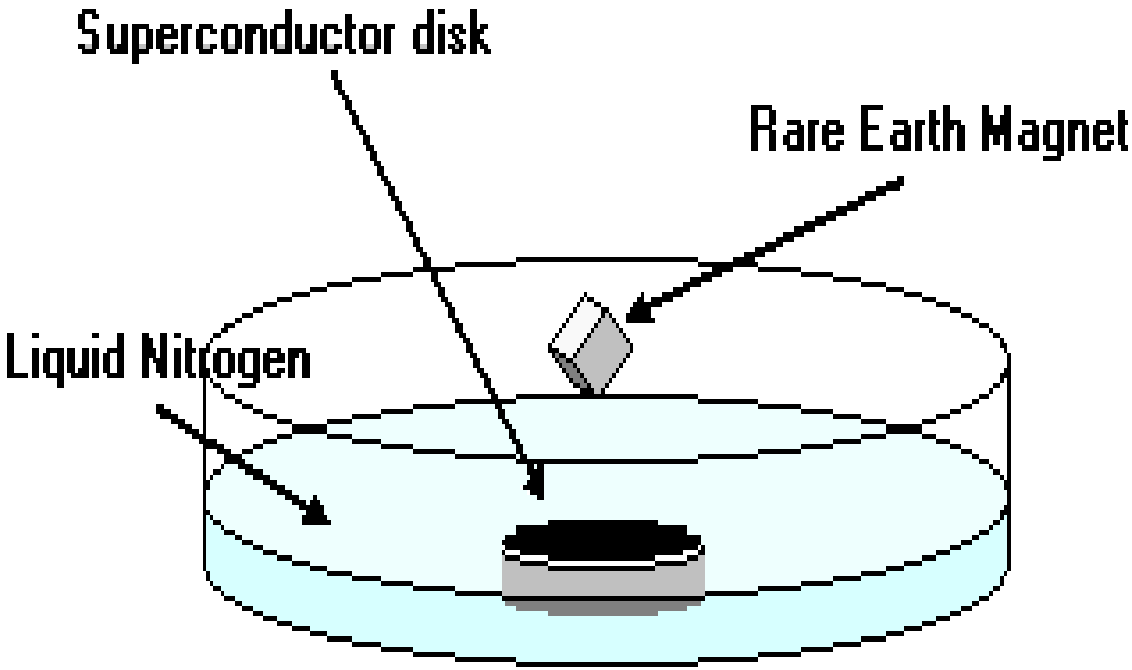
- **Superconductor** – a material whose resistance disappears at very low temperatures
- Collisions of electrons in material are suppressed
- Explain effect using Cooper pairs (pairs of electrons)
- Doesn't explain high-temp superconductors
- Purely quantum effect



- **Meissner effect** – in a small B_{ext} field, a superconductor will exclude all B fields from within its bulk

Meissner effect demo

- A super-conducting material repels all magnetic field (extreme diamagnetism).
- Thus a magnet will float
- We use the most powerful modern permanent magnets made out of a neodymium (rare-earth)-nickel-cobalt compound.
- May be the basis of future transportation - levitated trains, etc



Superconductor disk

Rare Earth Magnet

Liquid Nitrogen

The Meissner Effect