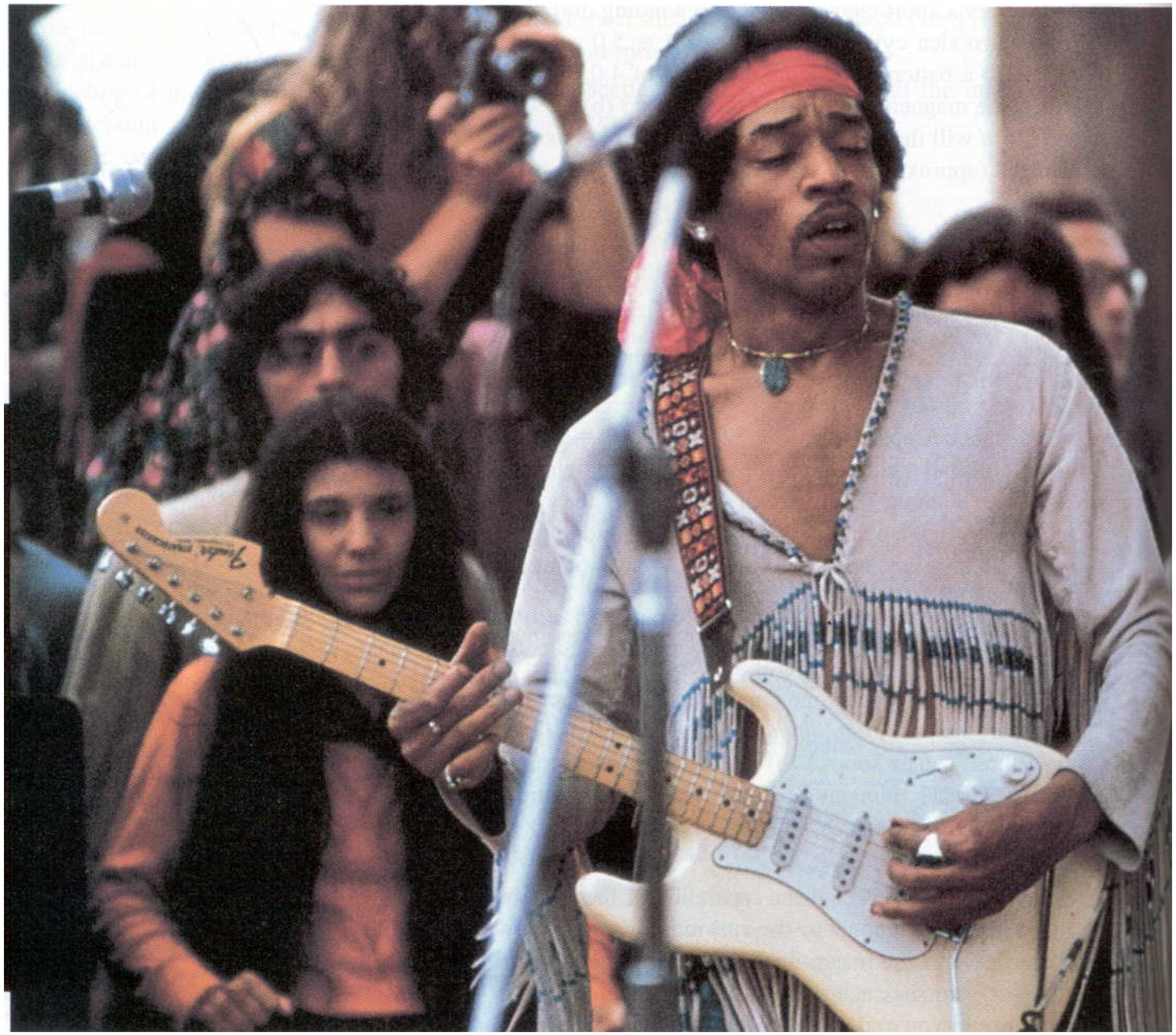
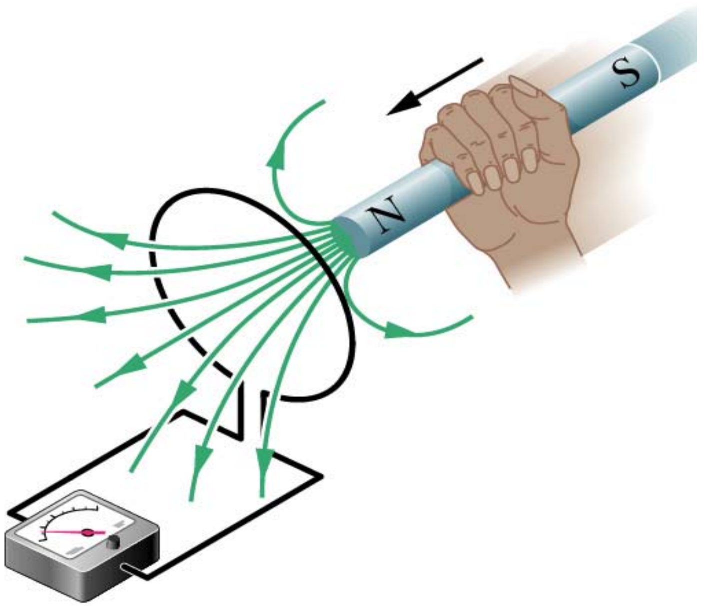


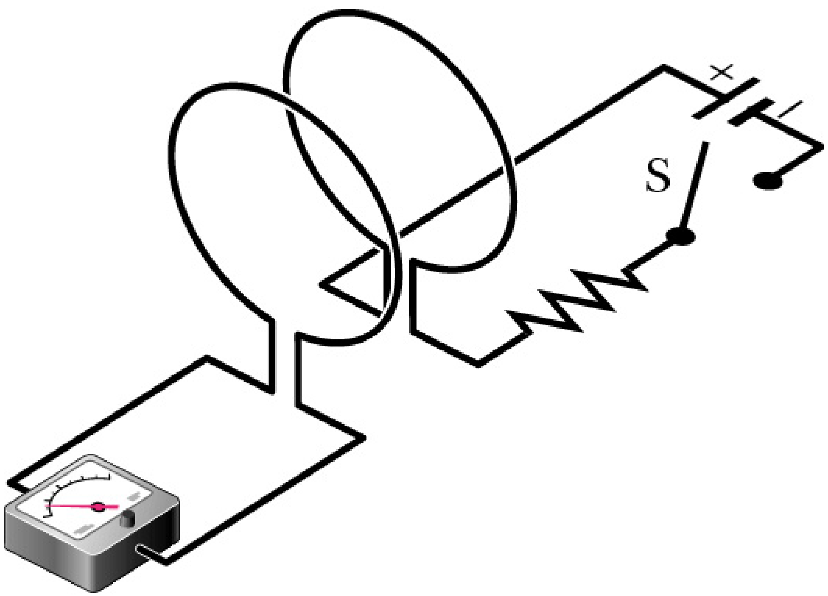
Inducing currents

- A current can produce a *B* field
- Can a *B* field generate a current?
- Move a bar magnet in and out of loop of wire
  - Moving magnet towards loop causes current in loop
  - Current disappears when magnet stops
  - Move magnet away from loop current again appears but in opposite direction
  - Faster motion produces a greater current



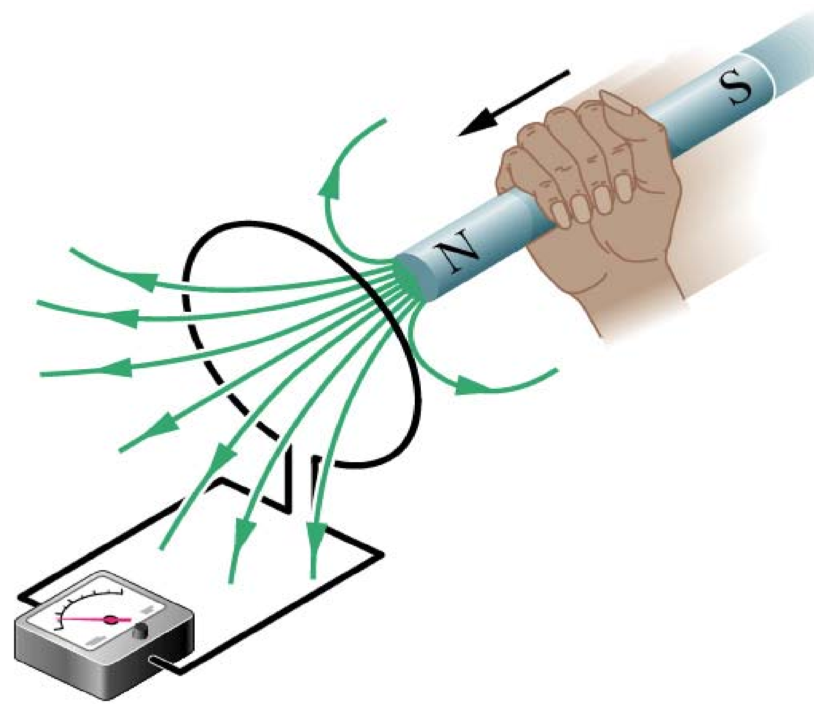
Inducing currents

- Have 2 conducting loops near each other
  - Close switch so current flows in one loop, briefly register a current in other loop
  - Open switch, again briefly register current in other loop but in opposite direction



Inducing currents

- Current produced in the loop is called induced current
- The work done per unit charge to produce the current is called an induced emf
- Process of producing the current and emf is called induction

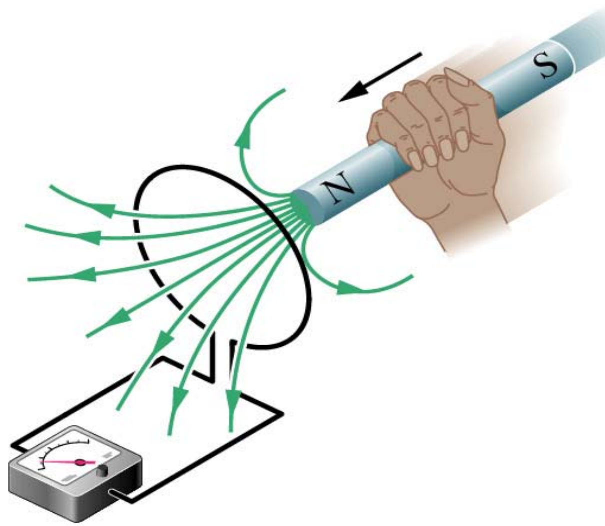


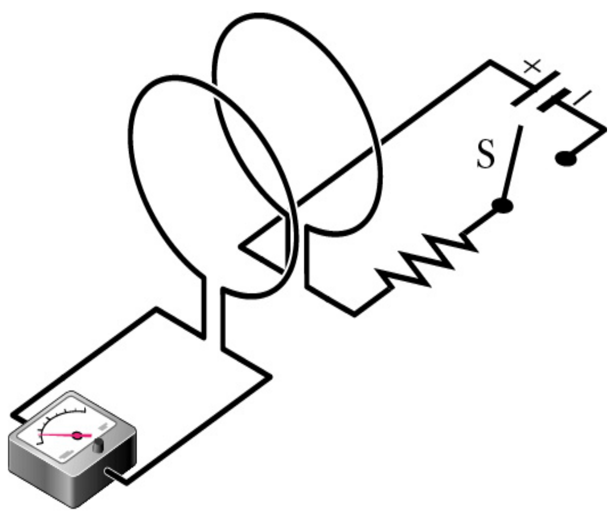
Faraday's law

- Faraday observed that an induced current (and an induced emf) can be generated in a loop of wire by:
  - Moving a permanent magnet in or out of the loop
  - Holding it close to a coil (solenoid) and changing the current in the coil
  - Keep the current in the coil constant but move the coil relative to the loop
  - Rotate the loop in a steady B field
  - Change the shape of the loop in a B field

Faraday's law

- Faraday concluded that an emf and a current can be induced in a loop by changing the amount of magnetic field passing through the loop
- Need to calculate the amount of magnetic field through the loop so define magnetic flux analogous to electric flux





Faraday's law

- Magnetic flux through area *A*

$$\boxed{\Phi_B = \int \vec{B} \bullet d\vec{A}}$$

- *dA* is vector of magnitude *dA* that is ⊥ to the differential area, *dA*

- If *B* is uniform and ⊥ to *A* then

$$\Phi_B = BA$$

- SI unit is the weber, Wb

$$\boxed{1Wb = 1T \cdot m^2}$$

Faraday's law

- Faraday's law of induction – induced emf in loop is equal to the rate at which the magnetic flux changes with time
- Minus signs means induced emf tends to oppose the flux change
- If magnetic flux is through a closely packed coil of N turns

$$\mathsf{E} = -\frac{d\Phi_B}{dt}$$

$$\boxed{\mathsf{E} = -N\frac{d\Phi_B}{dt}}$$

Faraday's law

- Can change the magnetic flux through a loop (or coil) by
  - If B is constant within coil

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = BA\cos\theta$$

  - Change magnitude of B field within coil
  - Change area of coil, or portion of area within B field
  - Change angle between B field and area of coil (e.g. rotating coil)

$$\mathsf{E} = -N\frac{d\Phi_B}{dt}$$

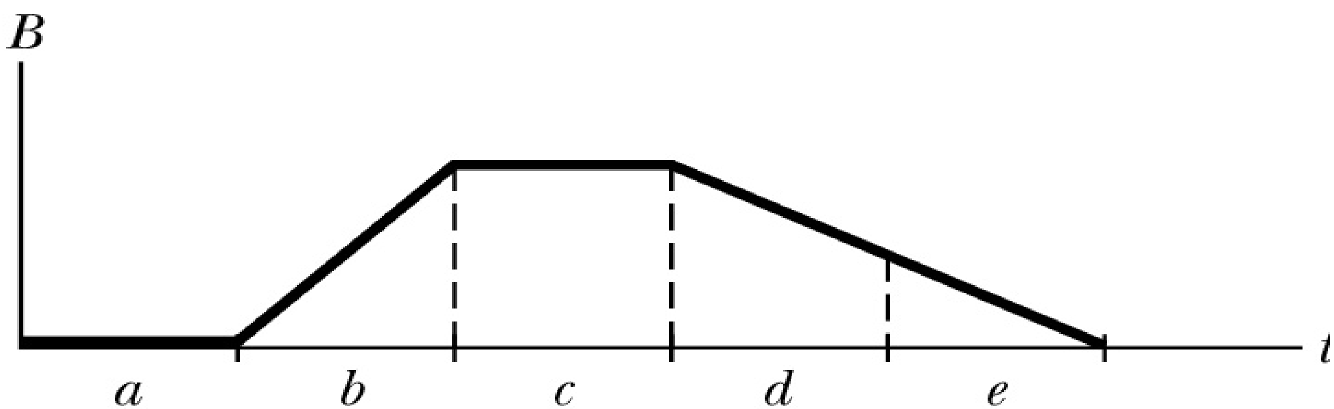
$$\mathsf{E} = -NA\cos\theta\frac{dB}{dt}$$

$$\mathsf{E} = -NB\cos\theta\frac{dA}{dt}$$

$$\mathsf{E} = -NBA\frac{d(\cos\theta)}{dt}$$

Exercise

- Graph shows magnitude *B(t)* of uniform *B* field passing through loop, ⊥ to plane of the loop. Rank the five regions according to magnitude of emf induced in loop, greatest first.

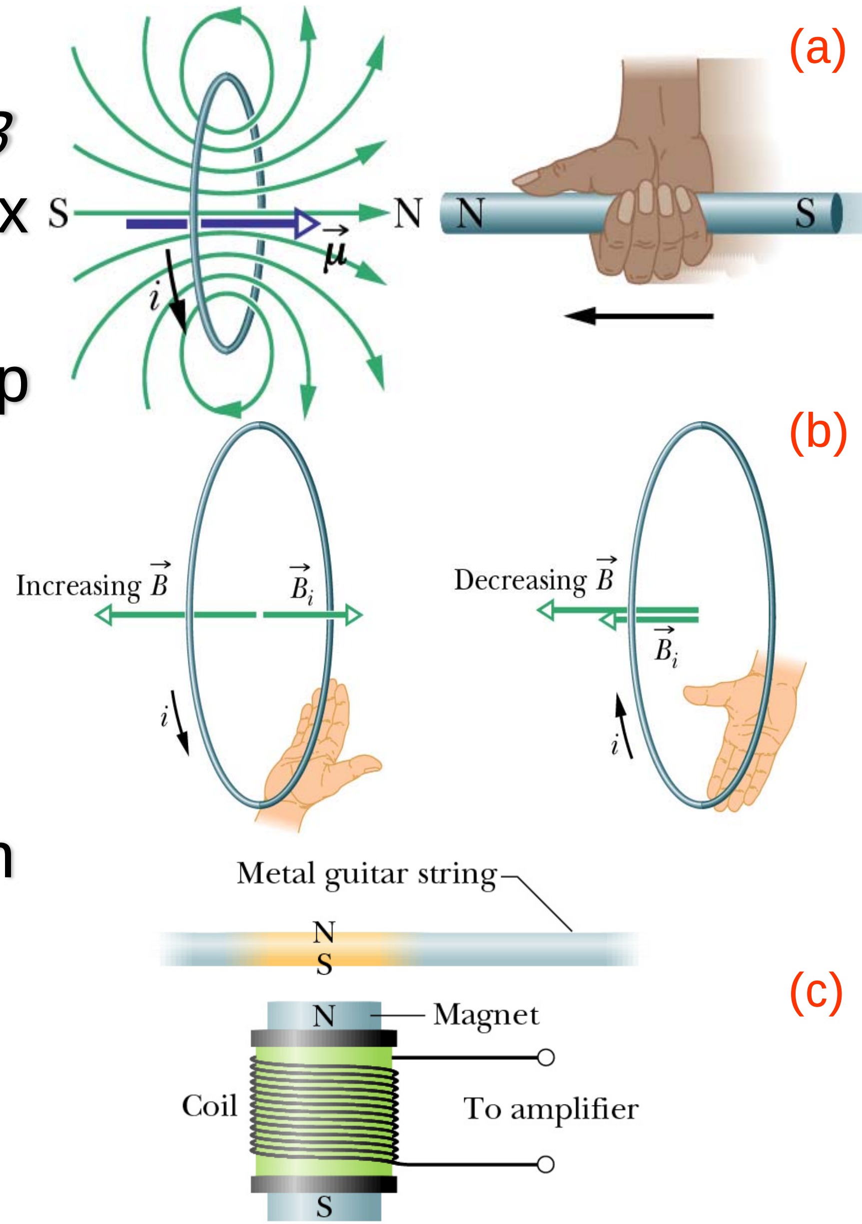


☞ $$= -NA\cos\theta\frac{dB}{dt} = -NA\frac{dB}{dt}$$

b, then d & e tie,
then a & c (zero)

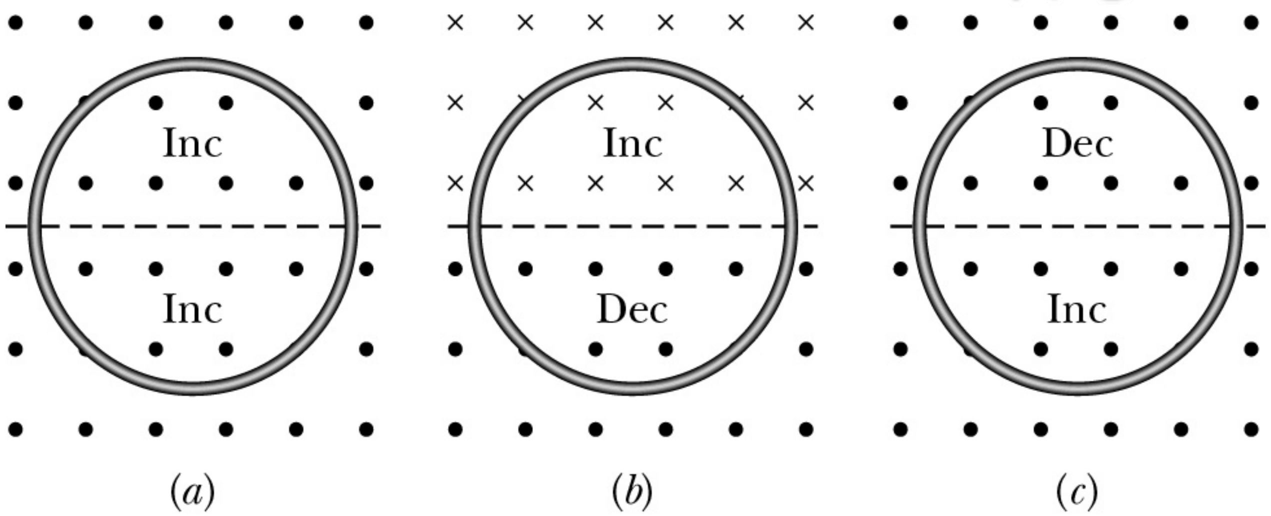
Lenz's law

- Lenz's law – An induced emf gives rise to a current whose *B* field opposes the change in flux that produced it
- (a) Magnet moves towards loop the flux in loop increases so induced current sets up *B* field opposite direction
- (b) Magnet moves away from loop the flux decreases so induced current have *B* field in same direction to oppose the decrease
- (c) Example: electric guitar



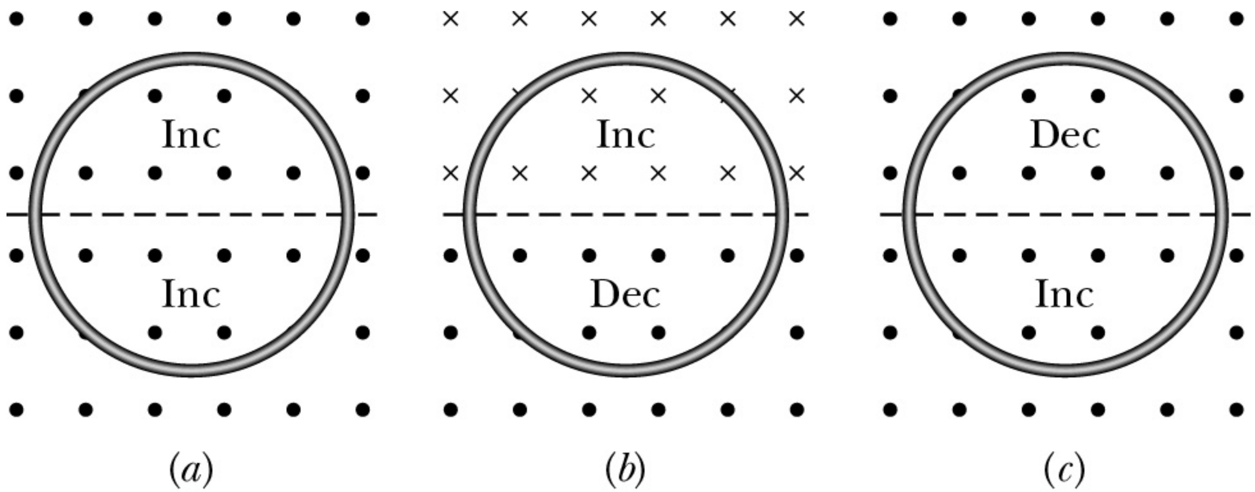
Exercise

- Three identical circular conductors in uniform B fields that are either increasing or decreasing in magnitude at identical rates. Rank according to magnitude of current induced in loop, greatest first.



  - Use Lenz's law to find direction of B_i
  - Use right-hand rule to find direction of current

Exercise

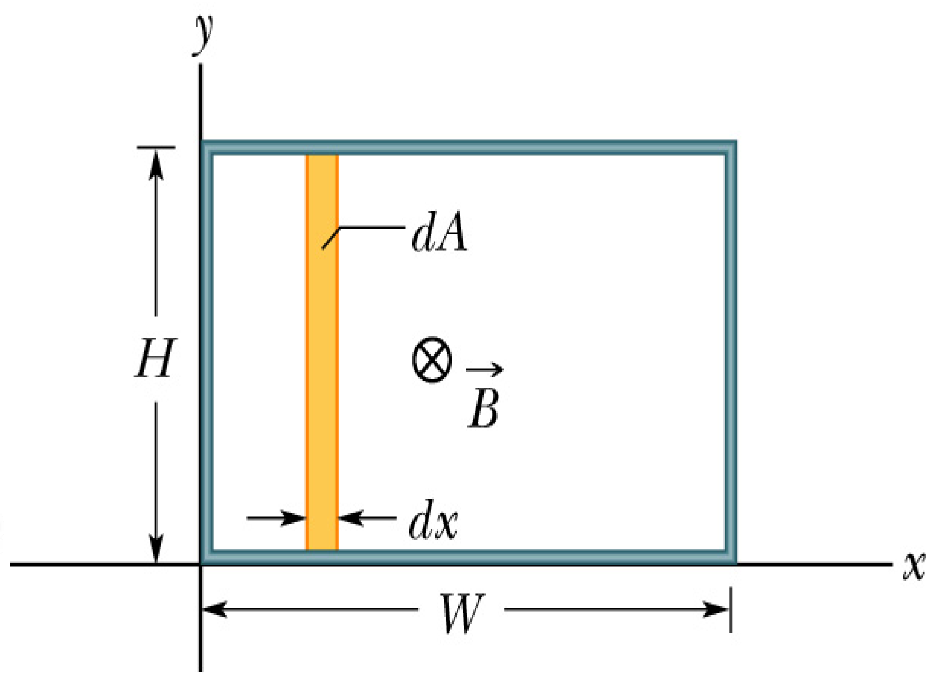


- Situation a –
  - From Lenz's law, B_i from induced current opposes increasing B so B_i is into page
  - From right-hand rule, induced current is clockwise in both sections of circle
- Do same for situation b and c

a & b tie, then c (zero)

Exercise

- What is magnitude and direction of induced emf around loop at t=0.10s?
- Loop has width W=3.0m and height H=2.0m
- Loop in non-uniform and varying *B* field ⊥ to loop and directed into the page



$$B = 4t^2x^2$$

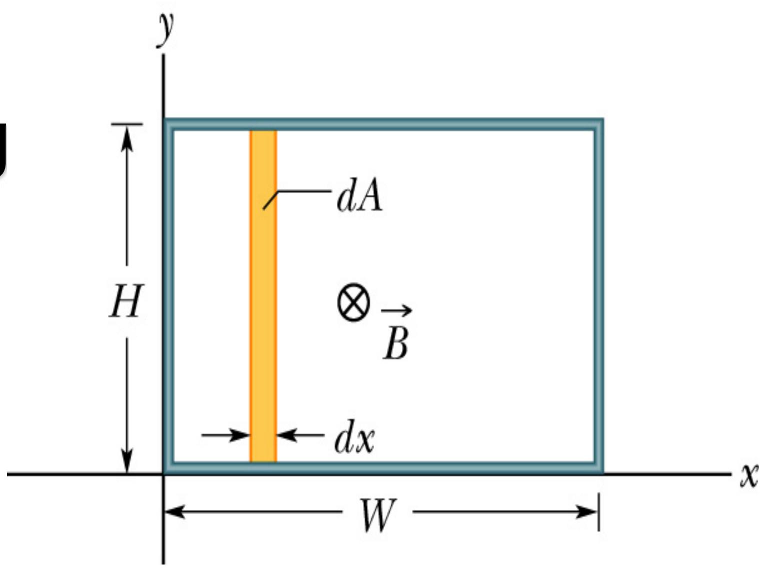
- Since magnitude *B* is changing in time, flux through the loop is changing so use Faraday's law to calculate induced emf

$$☜ = -\frac{d\Phi_B}{dt}$$

Problem

- B is not uniform so need to calculate magnetic flux using

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$



- B ⊥ to plane of loop and only changes in x direction

$$\vec{B} \bullet d\vec{A} = BdA = BHdx$$

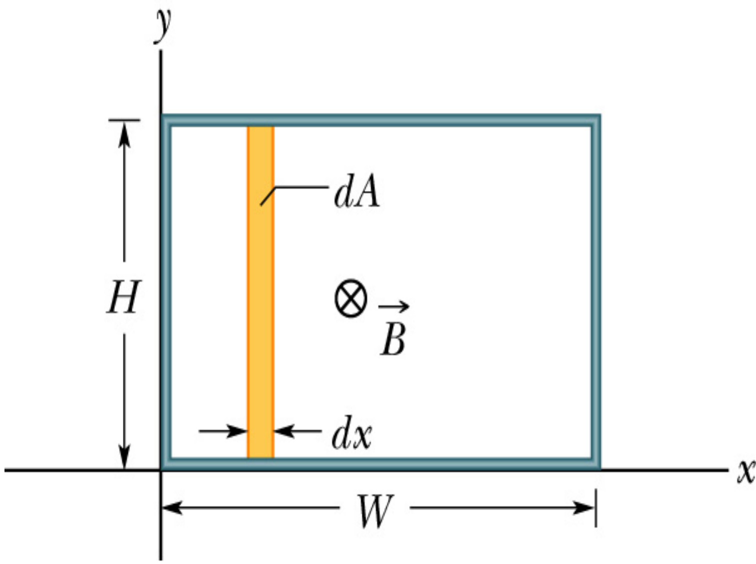
- Treat time as constant so

$$\Phi_B = \int BHdx = 4t^2 H \int_0^3 x^2 dx = 4t^2 H \left[\frac{x^3}{3} \right]_0^3 = 72\, t^2$$

Problem

- Now use Faraday's law to find the magnitude of the induced emf

$$= \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t$$



- At t=0.10s, emf = 14 V
- Find direction of emf by Lenz's law
  - *B* is increasing so B_i is in opposite direction - out of the page
  - Right-hand rule – current (and emf) are counterclockwise