# Inductance (units)

- Inductor is a device used to produce and store a desired $B$ field (e.g. solenoid)
- A current, $i$, in an inductor with $N$ turns produces a magnetic flux, $\Phi_B$, in its central region
- Inductance, $L$ is defined as

$$L = \frac{N\Phi_B}{i}$$

- SI unit is henry, H

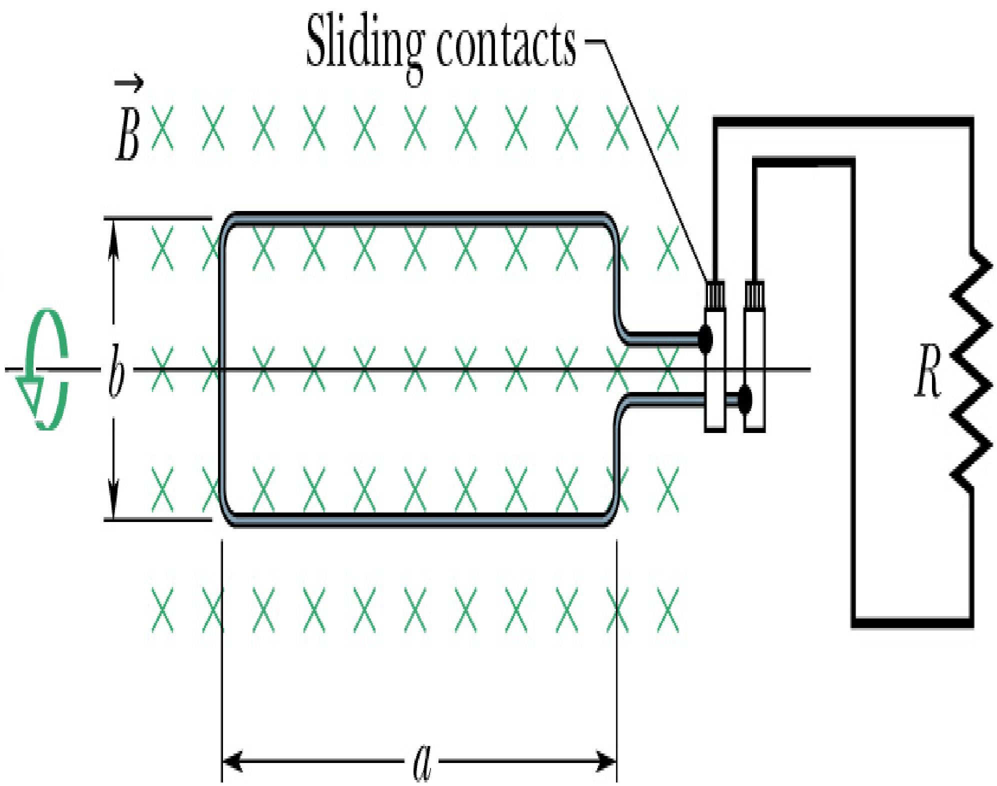
$$H = T \cdot m^2 / A$$

# Inductance of a solenoid

- What is inductance of a solenoid? $$L = \frac{N\Phi_B}{i}$$
- First find flux of single loop in solenoid
$$\Phi_B = \int \vec{B} \bullet d\vec{A} = BA = \mu_0 niA$$
- # of turns (*N*) per unit length (*l*) $$n = N / l$$
- Thus $$L = l\mu_0 n^2 A$$ or $$\frac{L}{l} = \mu_0 n^2 A$$
- Depends only on the physical properties of the solenoid

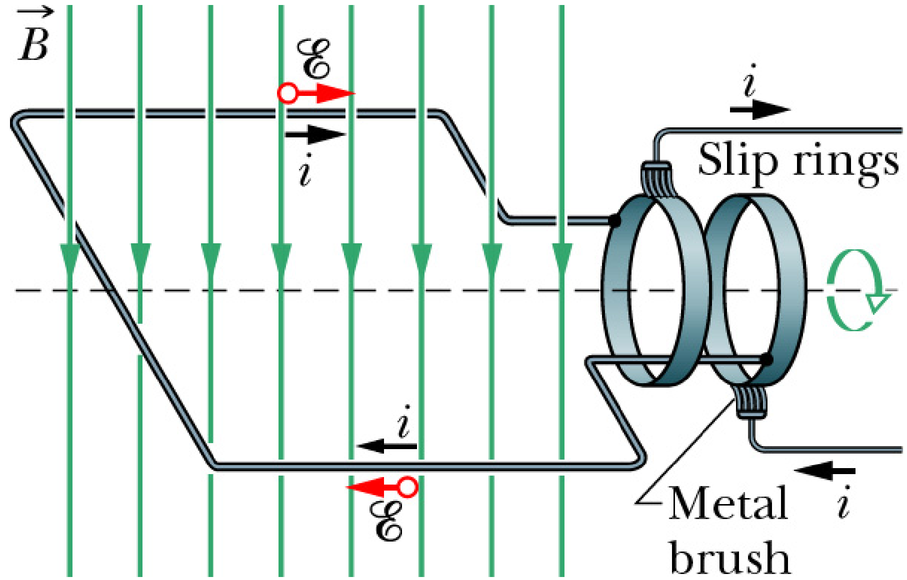
# Generators

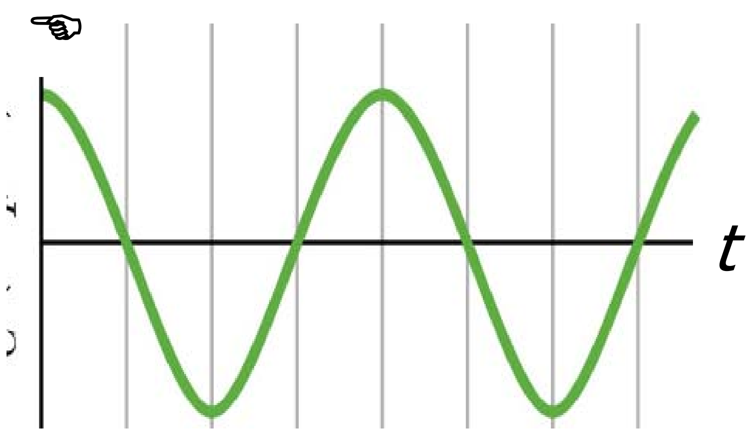
- Generators – convert mechanical energy to electrical energy
- External agent rotates loop of wire in *B* field
  - Hydroelectric plant
  - Coal burning plant
- Changing $\Phi_B$ induces an emf and current in an external circuit



# Generators

- Alternating current (ac) generator
  - Ends of wire loop are attached to slip rings which rotate with loop
  - Stationary metal brushes are in contact with slip rings and connected to external circuit
  - emf and current in circuit alternate in time



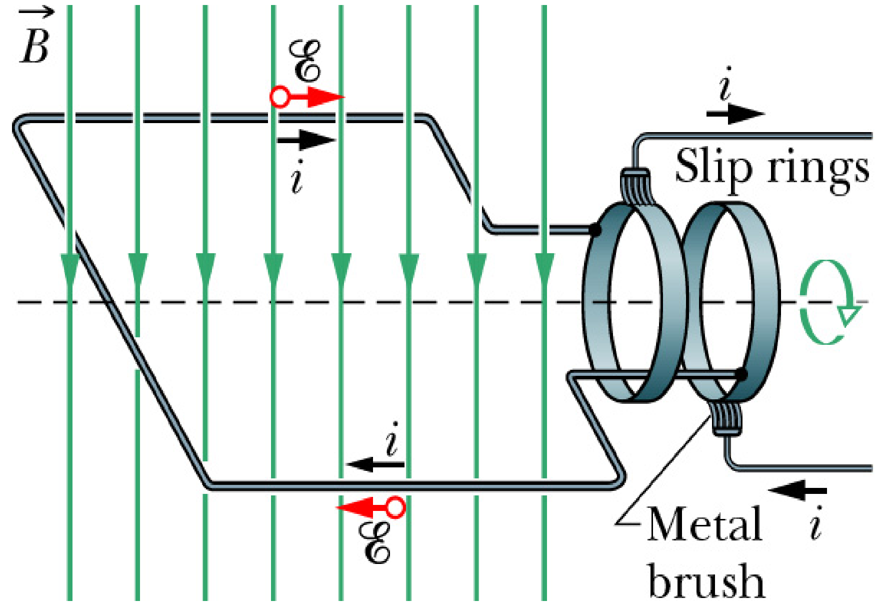
# Generators

- Calculate emf for generator with N turns of area A and rotating with constant angular velocity, ω
- Magnetic flux is

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = BA\cos\theta$$

- Relate angular dis-placement to angular velocity

$$\theta = \omega t$$




- Flux through one loop is

$$\Phi_B = BA\cos\omega t$$

# Generators

- Faraday's law says

$$\mathrm{E} = -N\frac{d\Phi_B}{dt}$$

- Substitute

$$\Phi_B = BA\cos\omega t$$

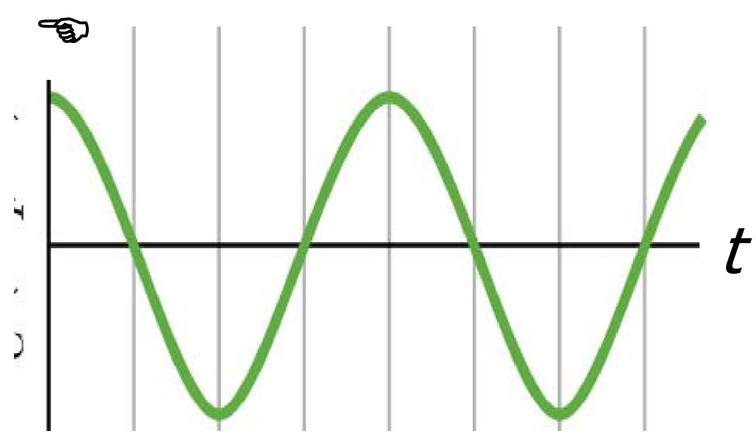
$$\mathcal{E} = -NBA\frac{d}{dt}\left(\cos\omega t\right)$$

$$\mathcal{E} = NBA\omega\sin\omega t$$

- Maximum emf is when $\omega t = 90$ or 270 degrees

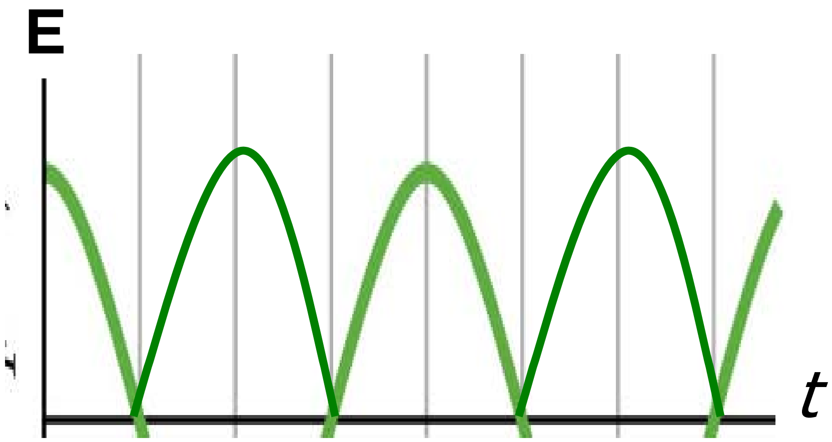
$$\mathcal{E}_{\max} = NBA\omega$$

- Emf is 0 when $\omega t = 0$ or 180 degrees



# Generators

- Direct current (dc) generator
  - Ends of loop are connected to a single split ring
  - Metal brush contacts to split ring reverse their roles every half cycle
  - Polarity of induced emf reverses but polarity of split ring remains the same



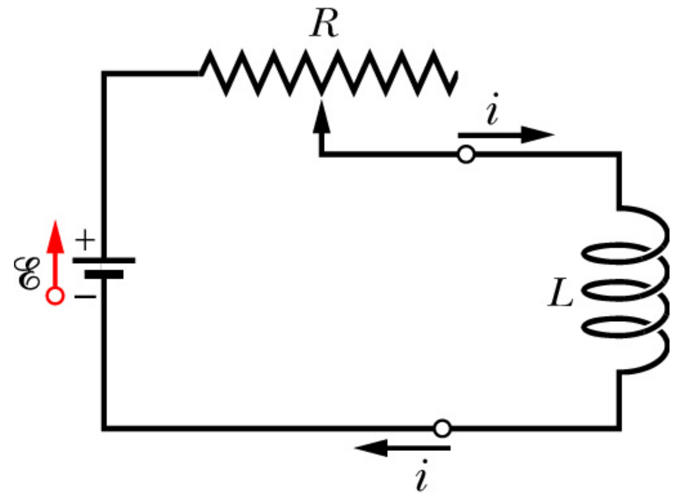

- Not suitable for most applications
  - Can use to charge batteries
- Commercial dc gen. use out of phase coils

# Motors

- Motors – converts electrical energy to mechanical energy
  - Generator run in reverse
  - Current is supplied to loop and the torque acting on the current-carrying loop causes it to rotate
  - Do mechanical work by using the rotating armature
  - As loop rotates, changing $B$ field induces an emf
  - Induced emf (back emf) reduces the current in the loop – remember Lenz's law
  - Power requirements are greater for starting a motor and for running it under heavy loads

# Self-induction

- A changing current in a coil generates a self-induced emf, $\varepsilon_L$ in the coil
- Process is called self-induction
- Change current in coil using a variable resistor, $\varepsilon_L$ will appear in coil only while the current is changing




$$L = \frac{N\Phi_B}{i}$$

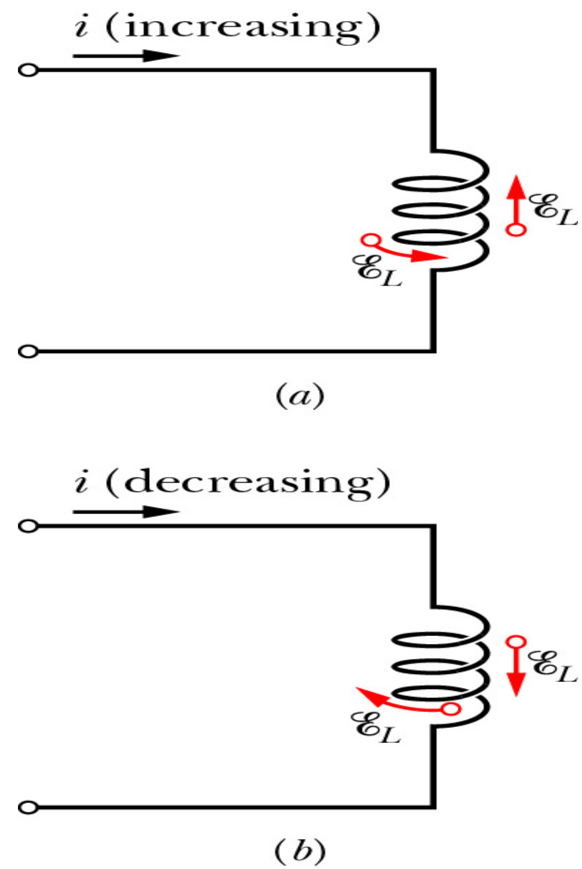
$$\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt} = -L\frac{di}{dt}$$

# Self-induction

- Induced emf only depends on rate of change of current, not its magnitude
- Direction of $\varepsilon_L$ follows Lenz's law and opposes the change in current
- Self-induced $V_L$ across inductor
  - Ideal inductor $V_L = \mathcal{E}_L$
  - Real inductor (like real battery) has some internal resistance

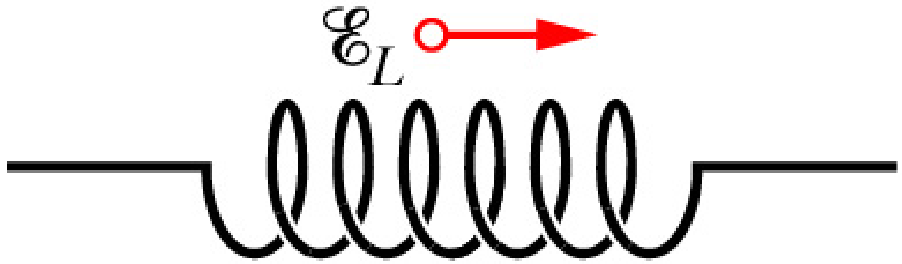
$$V_L = \mathcal{E}_L - iR$$

$$\mathcal{E}_L = -L\frac{di}{dt}$$



# Exercise

- Have an induced emf in a coil. What can we tell about the current through the coil? Is it moving right or left and is it constant, decreasing or increasing?



Only get $\varepsilon_L$ if current changing

Decreasing and rightward

OR

Increasing and leftward