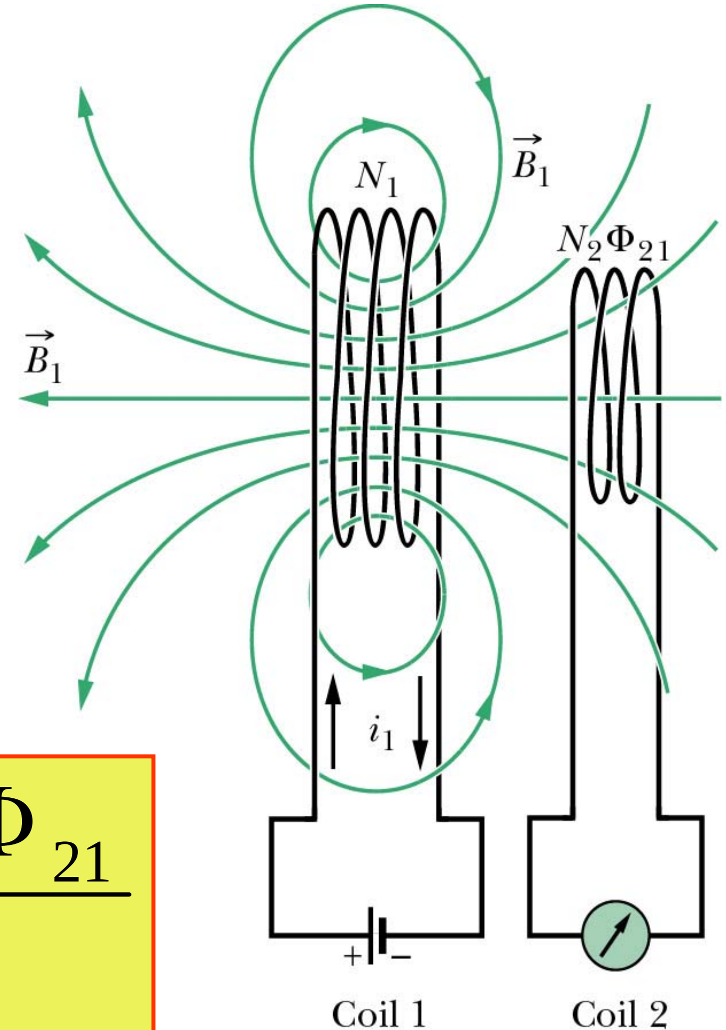
Mutual inductance

- Mutual induction – current in one coil induces emf in other coil
- Distinguish from self-induction
- Mutual inductance, M_{21} of coil 2 with respect to coil 1 is



$$L = \frac{N\Phi_B}{i}$$

$$M_{21} = \frac{N_2\Phi_{21}}{i_1}$$

Mutual inductance

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1}$$

- Rearrange equation

$$M_{21} i_1 = N_2 \Phi_{21}$$

- Vary i_1 with time

$$M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt}$$

- Faraday's law

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{21}}{dt}$$

- Induced emf in coil 2 due to i in coil 1 is

$$\boxed{\mathcal{E}_2 = -M_{21} \frac{di_1}{dt}}$$

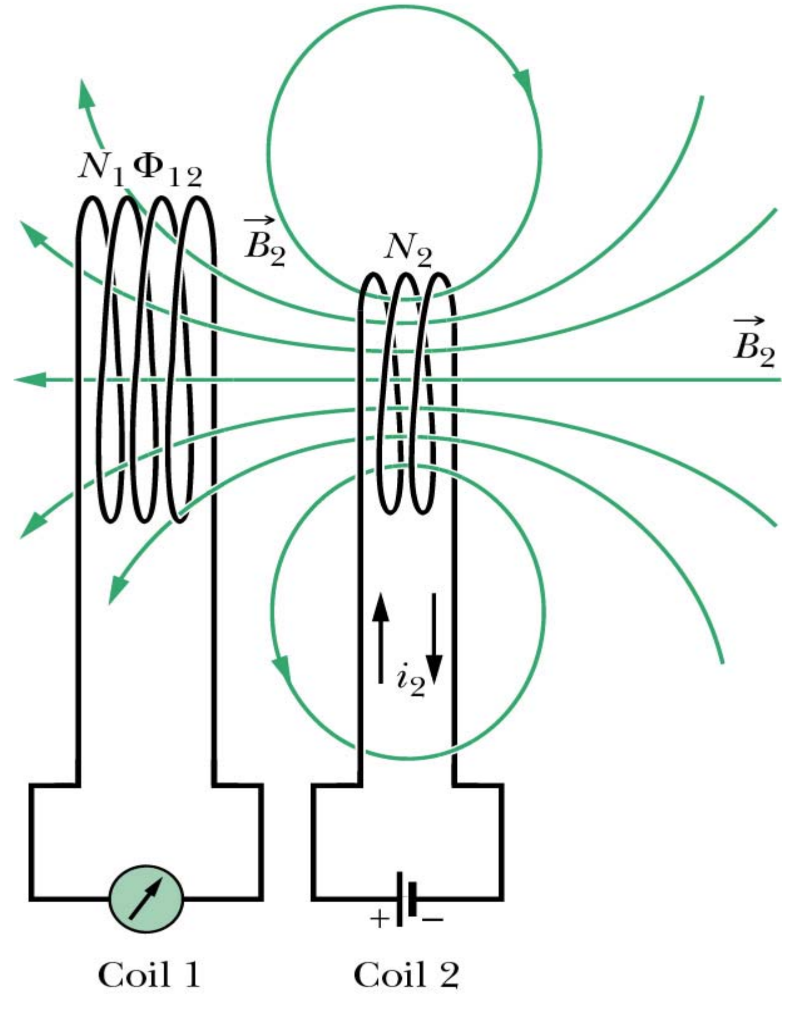
- Obeys Lenz's law (minus sign)

Mutual inductance

- Reverse roles of coils
- What is induced emf in coil 1 from a changing current in coil 2?
- Same game as before

$$M_{12} = \frac{N_1 \Phi_{12}}{i_2}$$

$$\mathcal{E}_1 = -M_{12} \frac{di_2}{dt}$$



Mutual inductance

- The mutual inductance terms are equal

$$M_{12} = \frac{N_1 \Phi_{12}}{i_2} \qquad M_{21} = \frac{N_2 \Phi_{21}}{i_1} \qquad M_{21} = M_{12} = M$$

- Rewrite emfs as

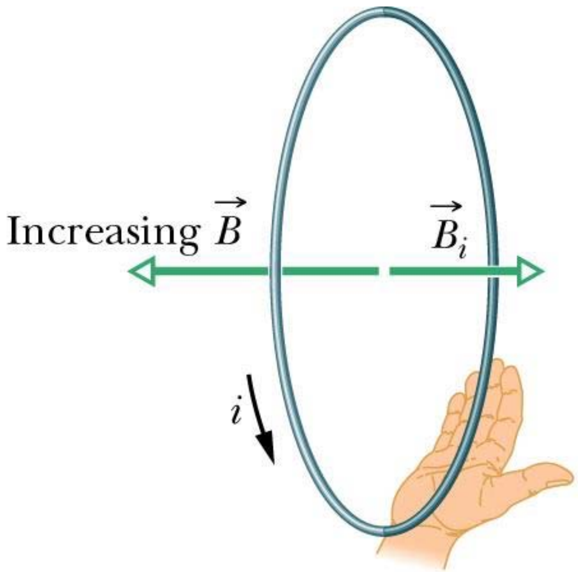
$$\mathcal{E}_2 = -M \frac{di_1}{dt} \qquad \mathcal{E}_1 = -M \frac{di_2}{dt}$$

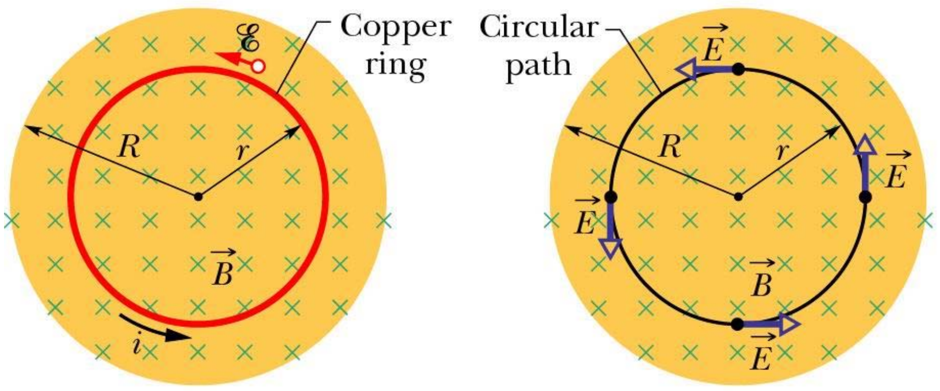
- Notice same form as self-induced emf

$$\mathcal{E}_L = -L \frac{di}{dt} \qquad L = \frac{N \Phi_B}{i}$$

Induced Electric Fields

- Put a copper ring in a uniform *B* field which is increasing in time so the magnetic flux through the copper ring is changing
- By Faraday's law an induced emf and current are produced
- If there is a current there must be an *E* field present to move the conduction electrons around ring

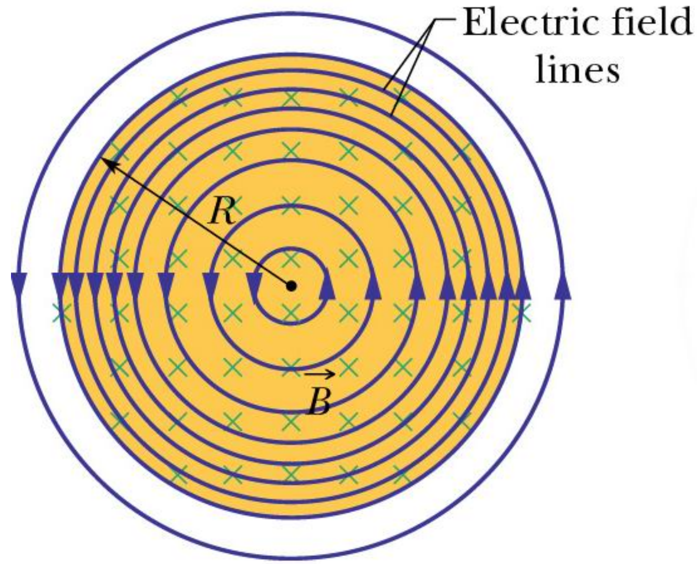




(*a*) (*b*)

Induced Electric Fields

- Induced *E* field acts the same way as an *E* field produced by static charges, it will exert a force, *F=qE*, on a charged particle
- True even if there is no copper ring (the picture shows a region of magnetic field increasing into the board which produces circular electric field lines).
- Restate Faraday's law – A changing *B* field produces an *E* field given by



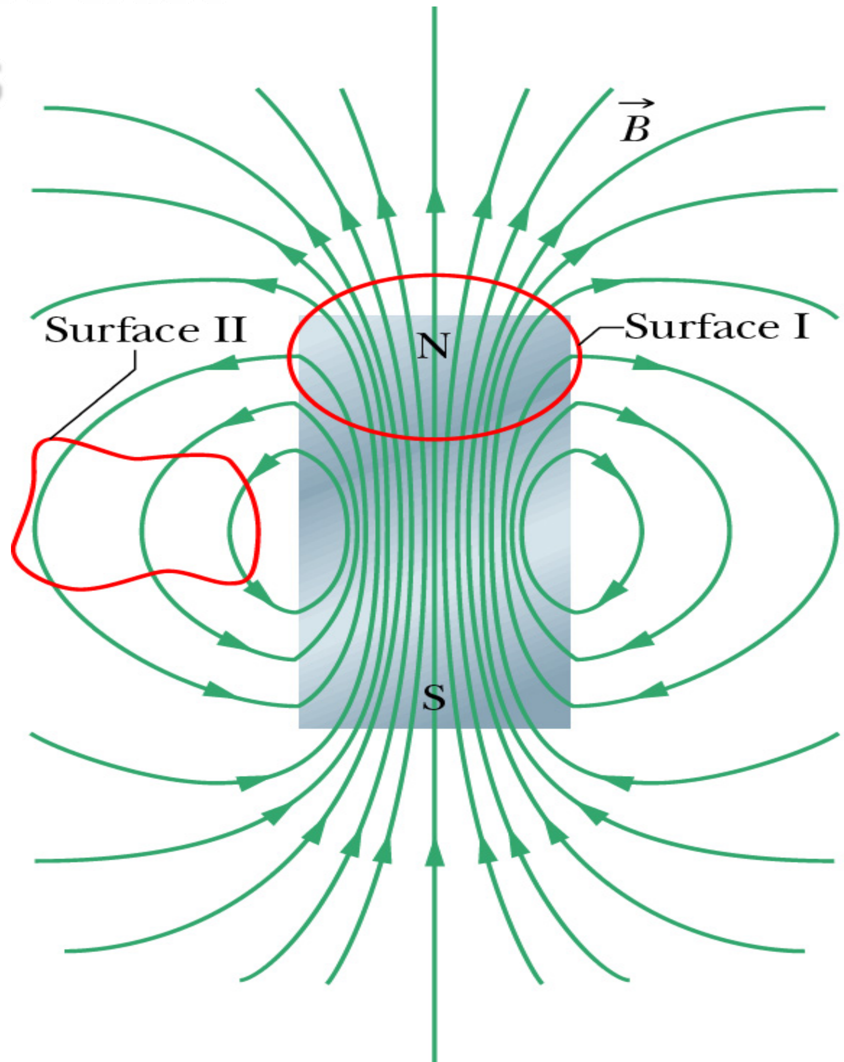
$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

No magnetic monopoles

- Magnetic monopoles do not exist
- Express mathematically as

$$\Phi_B = \oint \vec{B} \bullet d\vec{A} = 0$$

- Integral is taken over closed surface
- Net magnetic flux through closed surface is zero
  - As many *B* field lines enter as leave the surface



Electric charge x no magnetic charge

- Gauss's law for *E* fields

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

- Gauss's law for *B* fields

$$\Phi_B = \oint \vec{B} \bullet d\vec{A} = 0$$

- Both cases integrate over closed Gaussian surface

Faraday's and Maxwell's laws

- Faraday's law of induction *E* field is induced along a closed loop by a changing magnetic flux encircled by that loop

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

- Is the reverse true?
- Maxwell's law of induction *B* field is induced along a closed loop by a changing electric flux in region encircled by loop

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere's + Maxwell's law

- Ampere's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Combine Ampere's and Maxwell's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- *B* field can be produced by a current and/or a changing *E* field
  - Wire carrying constant current, $d\Phi_E/dt = 0$
  - Charging a capacitor, no current so $i_{enc} = 0$

Maxwell's laws

- Basis of all electrical and magnetic phenomena can be described by 4 equations called Maxwell's equations
- As fundamental to electromagnetism as Newton's law are to mechanics
- Einstein showed that Maxwell's equations work with special relativity
- Maxwell's equations basis for most equations studied since beginning of semester and will be basis for most of what we do the rest of the semester

Maxwell's laws

Maxwell's 4 equations are

- Gauss' Law $$\oint \vec{E} \bullet d\vec{A} = \frac{q}{\varepsilon_0}$$
- Gauss' Law for magnetism $$\oint \vec{B} \bullet d\vec{A} = 0$$
- Faraday's Law $$\oint \vec{E} \bullet d\vec{s} = -\frac{d}{dt}\int \vec{B} \bullet d\vec{A}$$
- Ampere-Maxwell Law $$\oint \vec{B} \bullet d\vec{s} = \varepsilon_0 \mu_0 \frac{d}{dt}\int \vec{E} \bullet d\vec{A} + \mu_0 i$$

Maxwell's laws in differential form

- Gauss' Law $$\vec{\nabla} \bullet \vec{E} = \frac{\rho}{\varepsilon_0}; \qquad \rho = \text{charge density}$$
- Gauss' Law for magnetism $$\vec{\nabla} \bullet \vec{B} = 0$$
- Faraday's Law $$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$
- Ampere-Maxwell Law $$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{d\vec{E}}{dt} + \mu_0 \vec{J}$$

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}; \qquad \vec{J} = \text{current density}$$