Gauss' Law

- Gauss' Law

$$\varepsilon_0 \Phi = q_{enc}$$

- Also write it as

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

- Net charge q_{enc} is sum of all enclosed charges and may be +, -, or zero

Gauss' Law = Coulomb's Law

- From Gauss' Law

$$\vec{E} = E\,\hat{\vec{r}}, \qquad d\vec{A} = dA\,\hat{\vec{r}}$$

$$\oint \vec{E} \bullet d\vec{A} = E\left(4\pi r^2\right)$$

- Thus,

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = \varepsilon_0 E\left(4\pi r^2\right) = q_{enc}$$

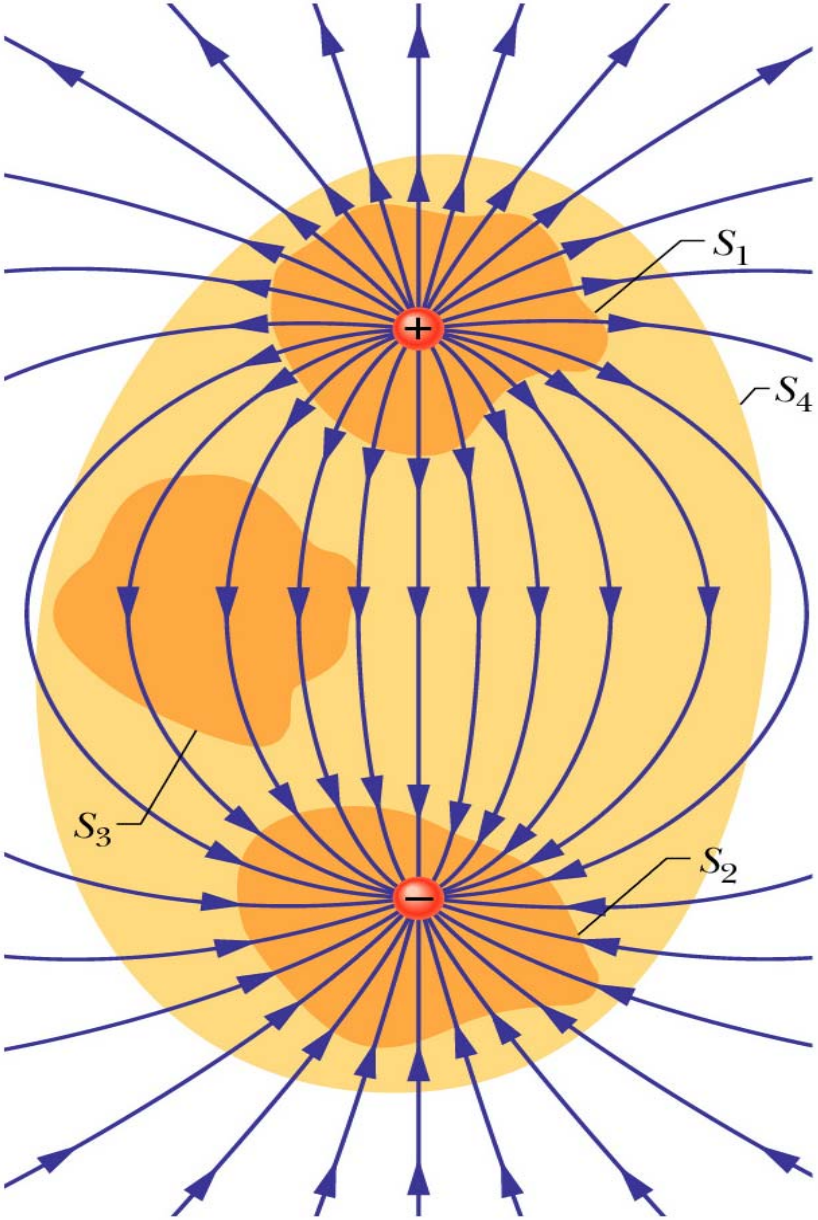
- and we get the Coulomb's law

$$E = \frac{q_{enc}}{4\pi\varepsilon_0 r^2} = k\frac{q_{enc}}{r^2}$$

- What is the flux for each surface?

$$\varepsilon_0 \Phi = q_{enc}$$

- net S_1 - q_{enc} is +
  Φ is outward and +
- S_2 - q_{enc} is -
  Φ is inward and –
- S_3 – q_{enc} is 0
  Φ is 0
- S_4 – total q_{enc} is 0
  Φ is 0



Gauss' Law

- What happens to the flux if I had a charge, Q, outside a Gaussian surface?

$$\varepsilon_0 \Phi = q_{enc}$$

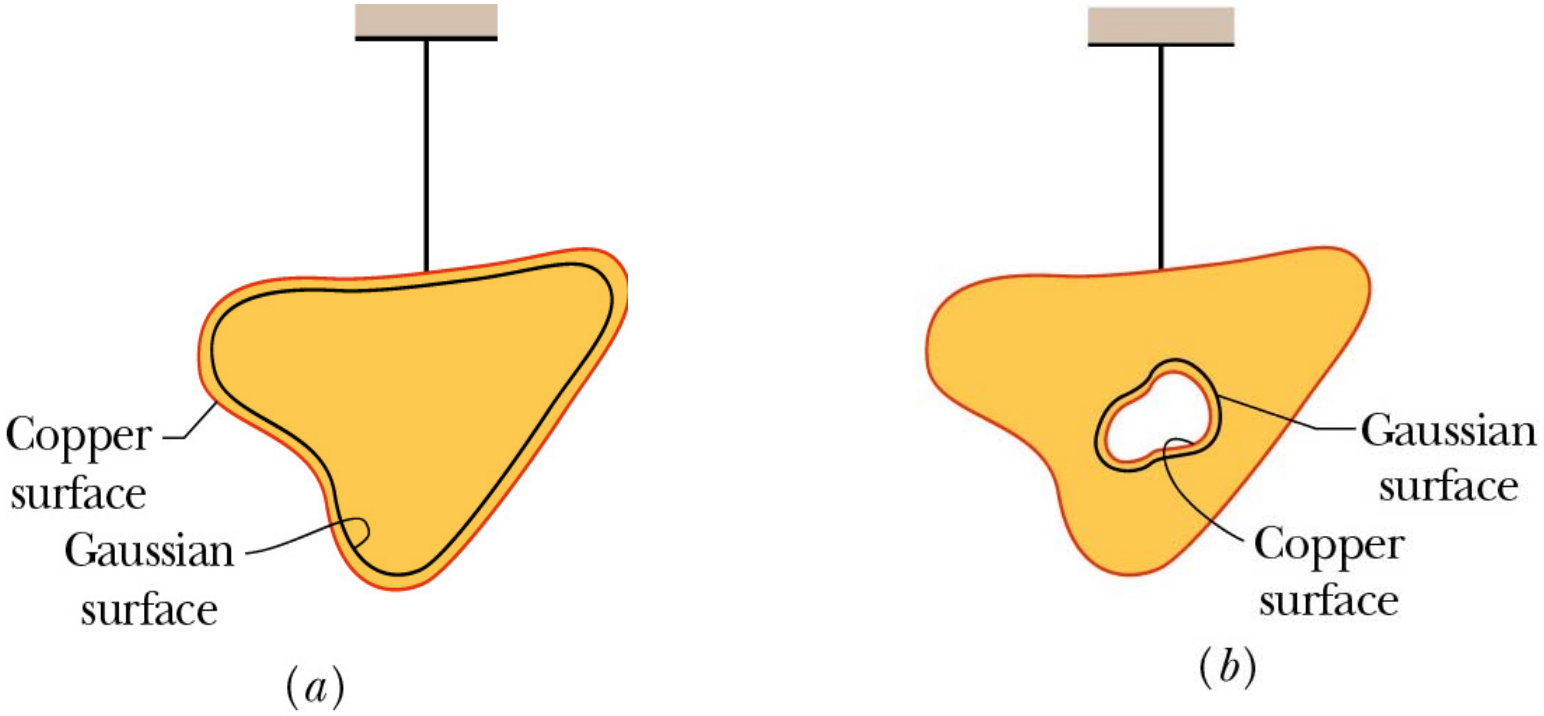
- *Nothing* - q_{enc} does not change

- E field does change but charge outside the surface contributes zero net Φ through surface

Conductors

- Theorem for charged isolated conductor with a net charge Q
  - Charge is always on the surface
  - No charge inside the conductor
  - $E = 0$ inside the conductor

- At the surface of a charged conductor the E field is $\perp$ to the surface

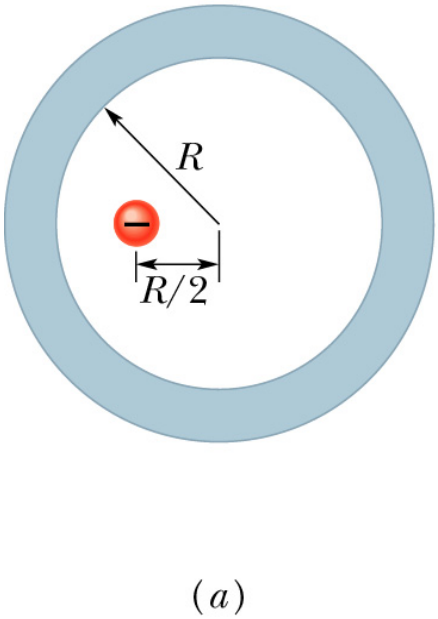

Copper
surface
Gaussian
surface
(a)
Gaussian
surface
Copper
surface
(b)

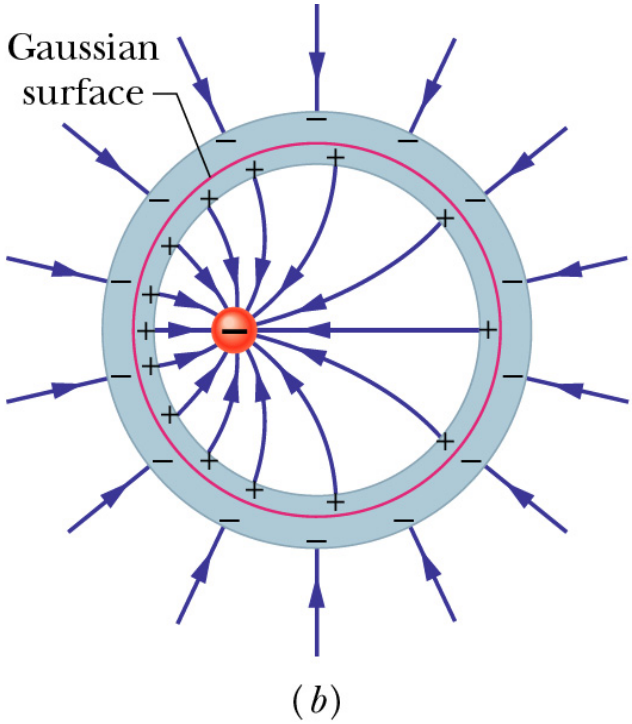
Conductors

- Usually charge on conductor is not uniform (except for a sphere)
- Charge will accumulate more at sharp points on an irregularly shaped conductor

Example 1a

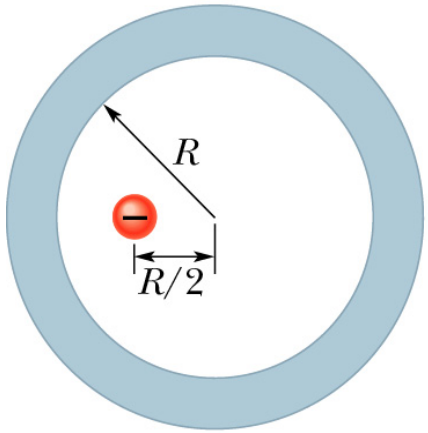
- Have point charge of -5.0μC not centered inside an electrically neutral spherical metal shell
- What are the induced charges on the inner and outer surfaces of the shell?



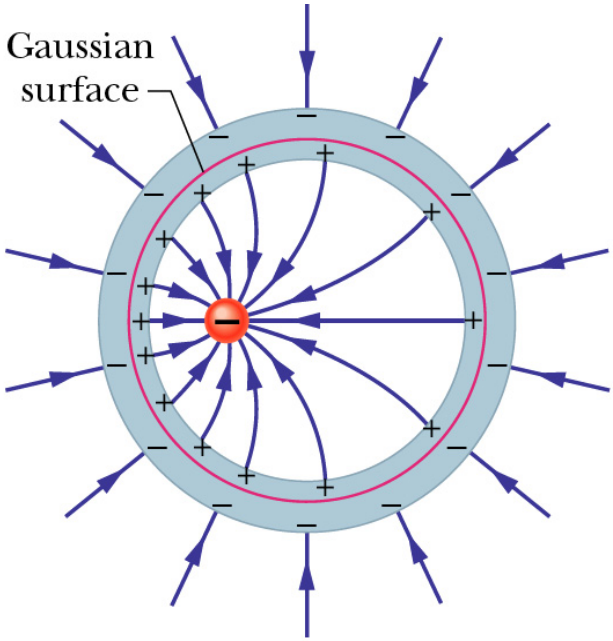


Example 1b

- E=0 inside conductor
- Thus Φ=0 for Gaussian surface
- So net charge enclosed must be 0
- Induced charge of +5.0μC lies on inner wall of sphere
- Shell is neutral so charge of -5.0μC on outer wall



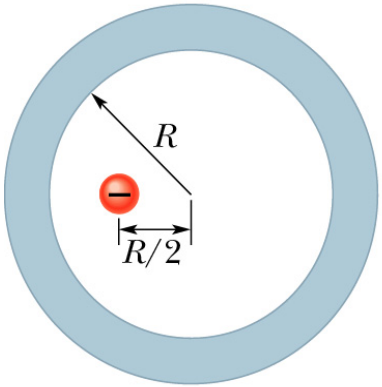
(a)



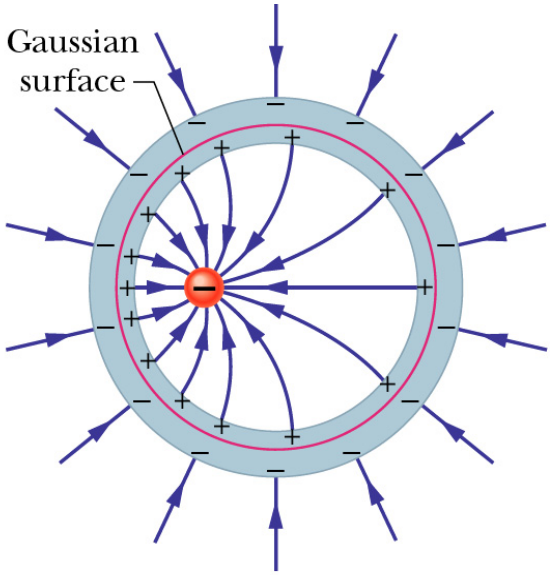
(b)

Example 1c

- Are the charges on the sphere surfaces uniform?
- Charge is off-center so more + charge collects on inner wall nearest point charge
- Outer wall the charge is uniform
  - No E inside shell to affect distribution
  - Spherical shape



(a)



(b)