Gauss' Law (Review)

- Gauss' law – form of Coulomb's law
  - q_{enc} is the total charge enclosed by a Gaussian surface

$$\varepsilon_0 \Phi = q_{enc}$$

- Flux is proportional to # of *E* field lines passing through a Gaussian surface

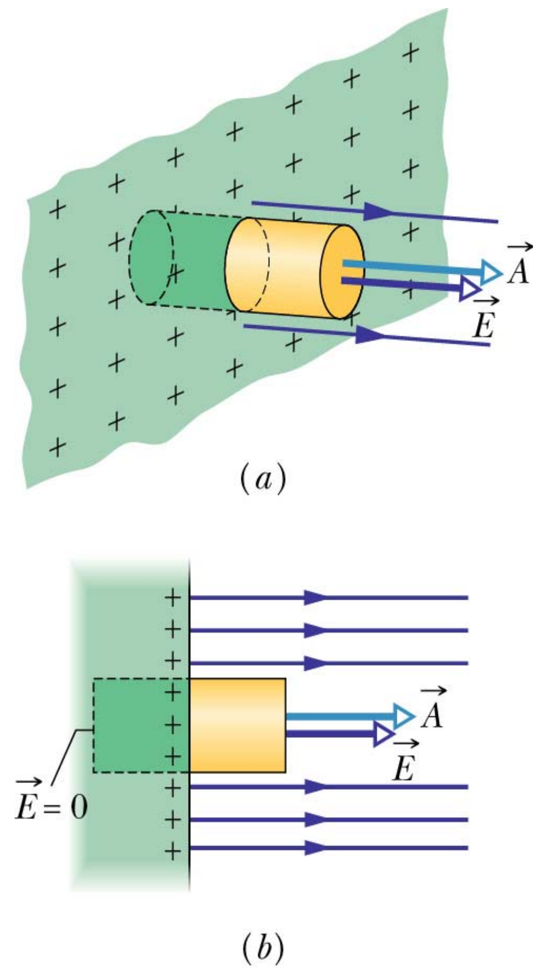
$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

Conductors (Example)

A ball of charge -50e lies at the center of a hollow spherical metal shell that has a net charge of -100e. What is the charge on a) the shell's inner surface and b) its outer surface?

Conductors

- Pick a cylindrical Gaussian surface embedded in the conductor
- Sum the flux through surface
- Inside conductor $E = 0$ so $\Phi = 0$
- Along walls of the cylinder outside the conductor E is $\perp$ to A so $\Phi = 0$
- Outer endcap $\Phi = EA$



Conductors

- Using Gauss' law and $\Phi = EA$

$$\varepsilon_0 \Phi = \varepsilon_0 E A = q_{enc}$$

- If σ is charge per unit area, then

$$q_{enc} = \sigma A$$

- So E for a conducting surface is

$$E = \frac{\sigma}{\varepsilon_0}$$

Conductors

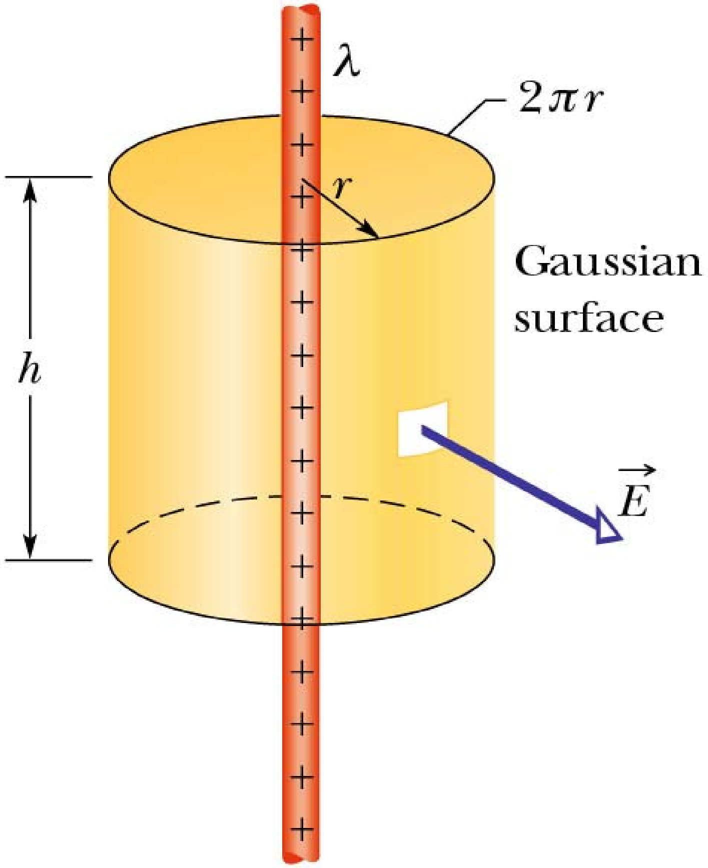
- *E* just outside a conductor is proportional to surface charge density at that location

$$E = \frac{\sigma}{\varepsilon_0}$$

  - If – charge on conductor, *E* toward conductor
  - If + charge on conductor, *E* directed away

Gauss' Law

- Infinitely long insulating rod with linear charge density λ
- Pick Gaussian surface of cylinder coaxial with rod
- What does E look like?
- $\Phi = 0$ for the endcaps
- $\Phi = EA$ for cylinder



Gauss' Law

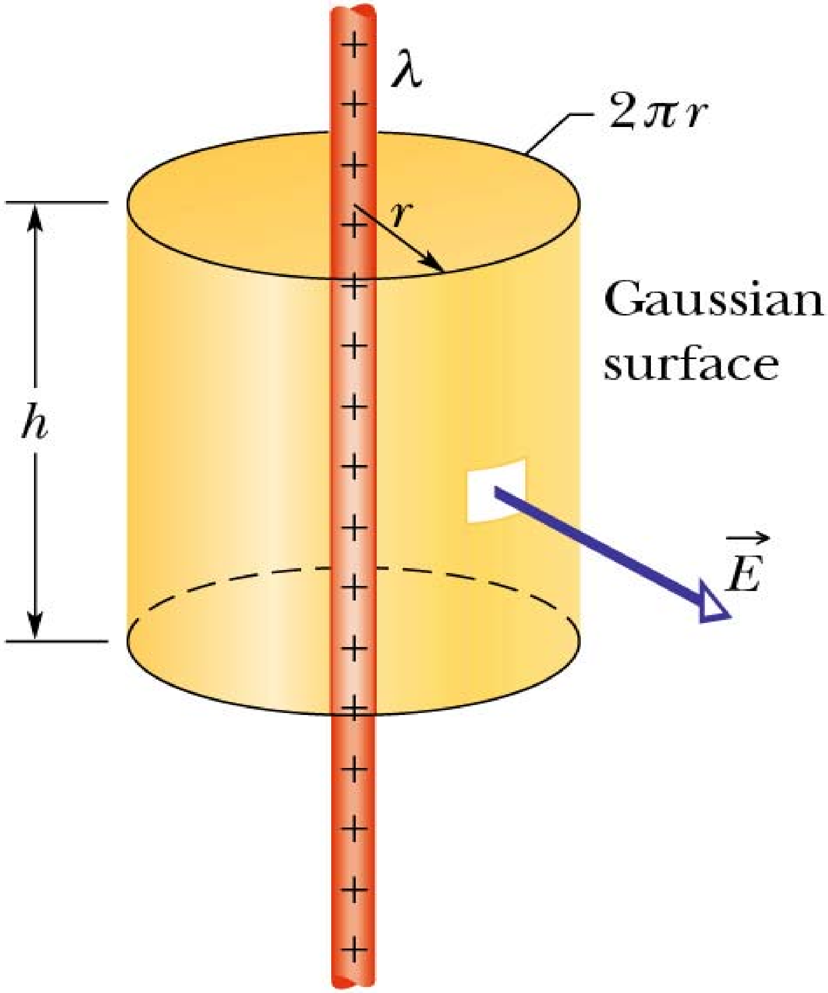
- Substituting in Gauss' law gives

$$\varepsilon_0\Phi = \varepsilon_0 EA = q_{enc}$$

$$A = 2\pi rh \qquad q_{enc} = \lambda h$$

- E for a line of charge is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



Gauss' Law

- Apply Gauss' law to a uniformly charged spherical shell S_2

$$\varepsilon_0 \Phi = q_{enc} = \varepsilon_0 \oint \vec{E} \bullet d\vec{A}$$

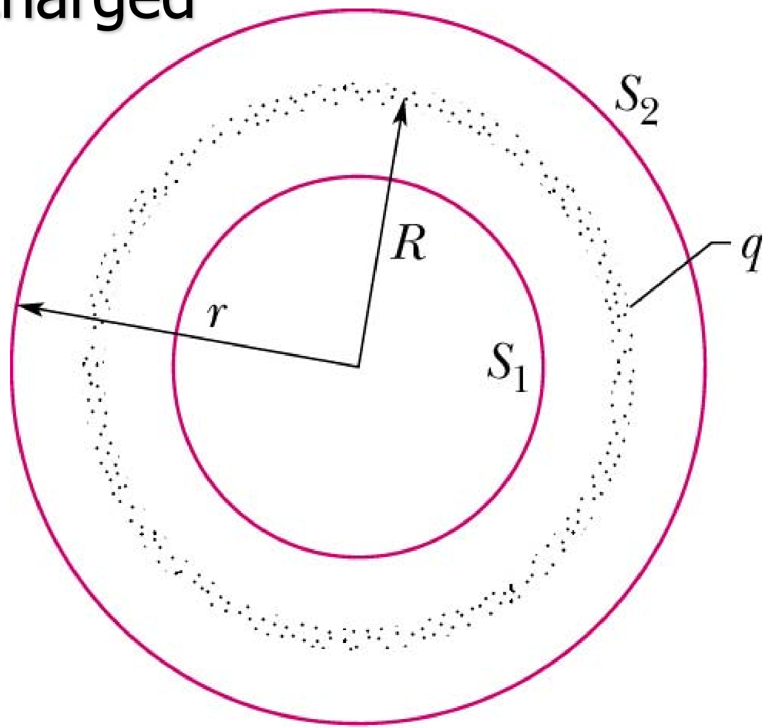
- E radiates out || to A so

$$\oint \vec{E} \bullet d\vec{A} = \oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$A = 4\pi r^2$$

- Substitute to find E

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}, r \geq R$$

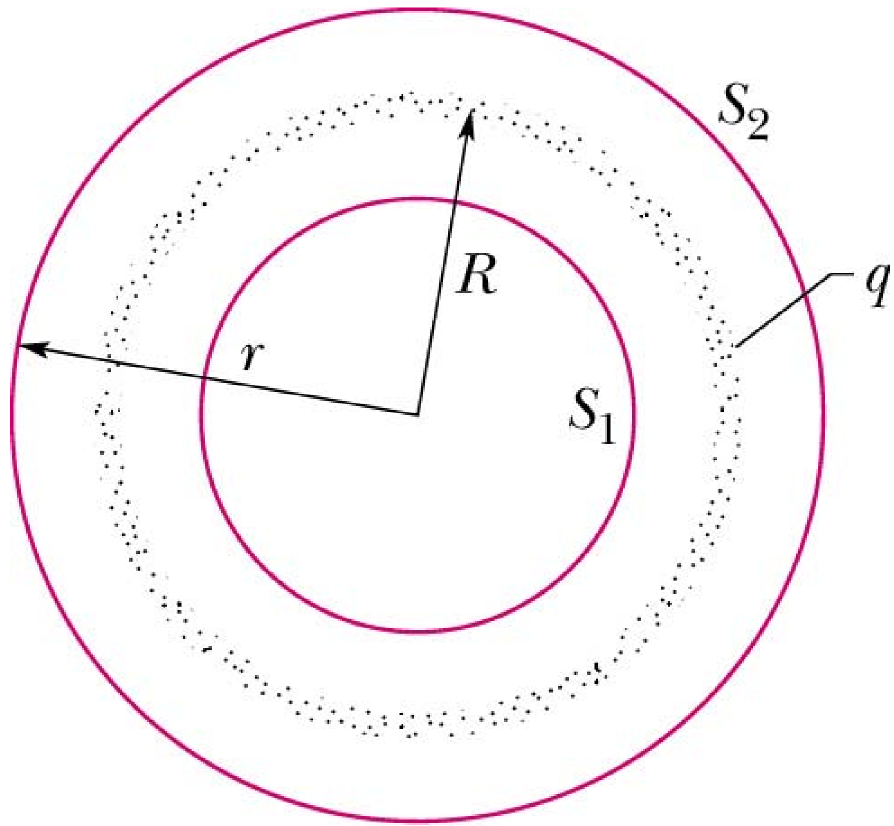


Gauss' Law

$$E = k\frac{q}{r^2}$$

- E outside of a charged spherical shell is same as E of point charge at center of shell.

- Charge inside S_1 is zero, so by Gauss' law E=0 inside shell, $r < R$.
- If a charge is placed inside there will be no force on it.



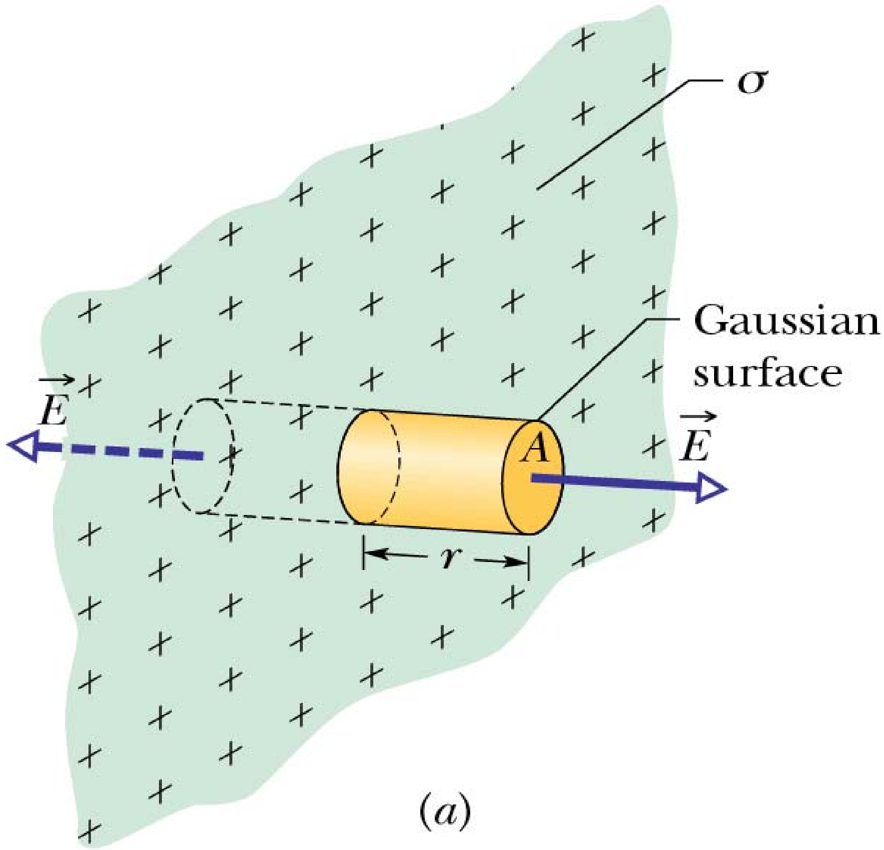
Gauss' Law

- Non-conducting sheet of charge σ

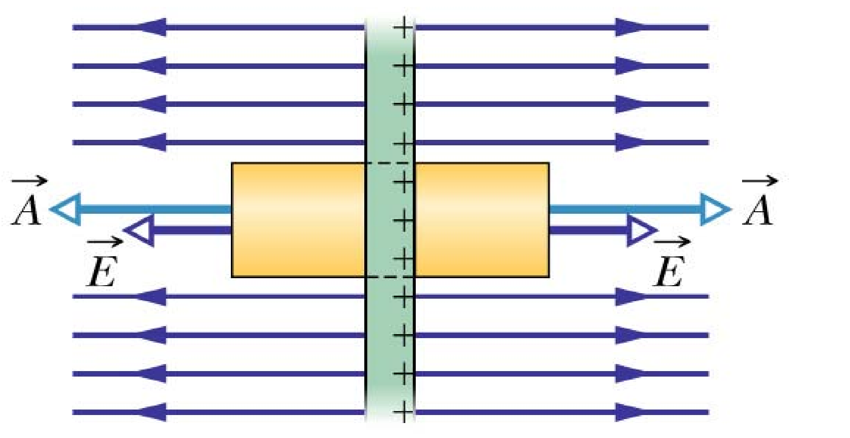
$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

$$\varepsilon_0 (EA + EA) = \sigma A$$

$$E = \frac{\sigma}{2\varepsilon_0}$$



(a)

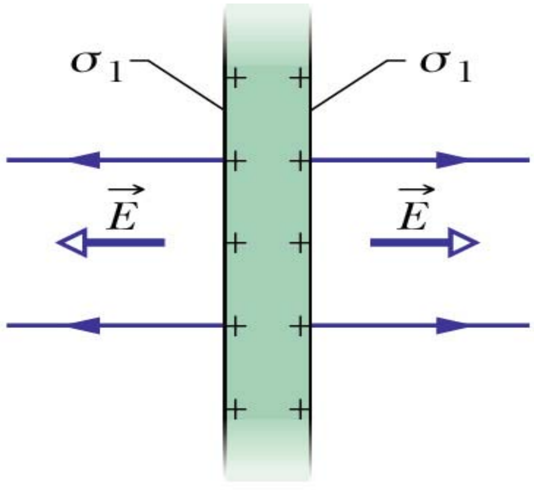


(b)

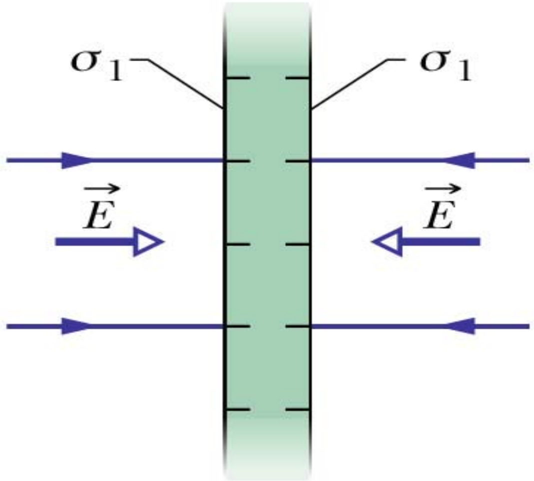
Gauss' Law

- Conducting sheet of charge
  - Total charge spreads over two surfaces
  - σ_1 is charge on one surface,
  - $\sigma_1 = \sigma/2$

$$E = \frac{\sigma_1}{\varepsilon_0}$$



(a)

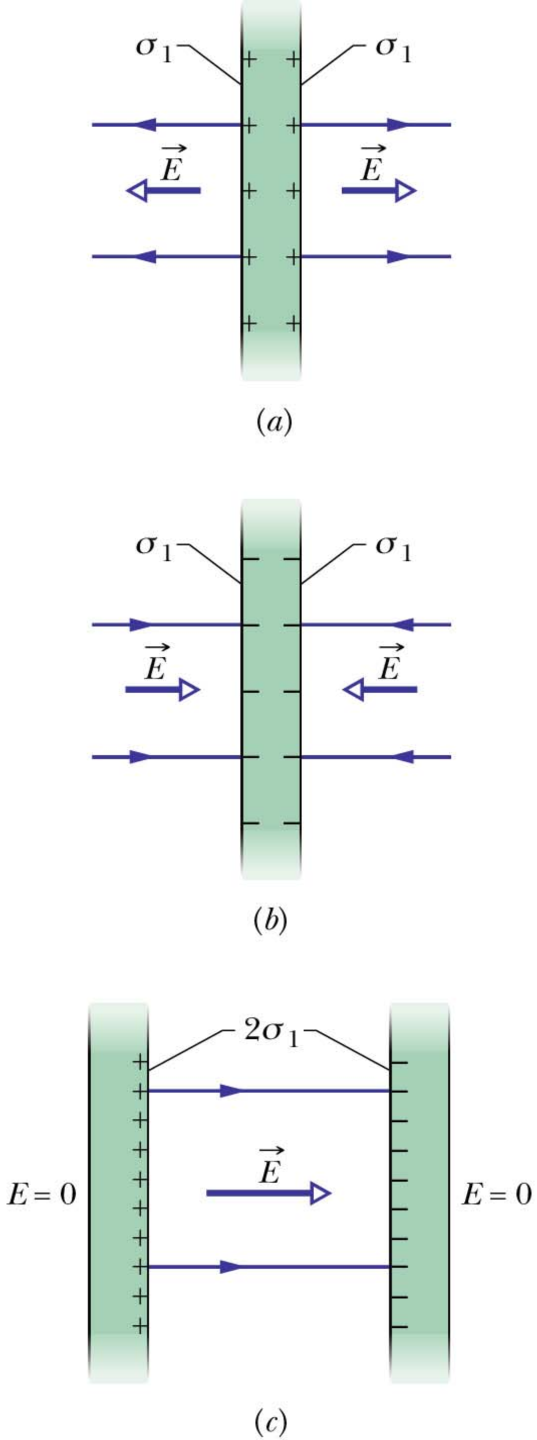


(b)

Gauss' Law

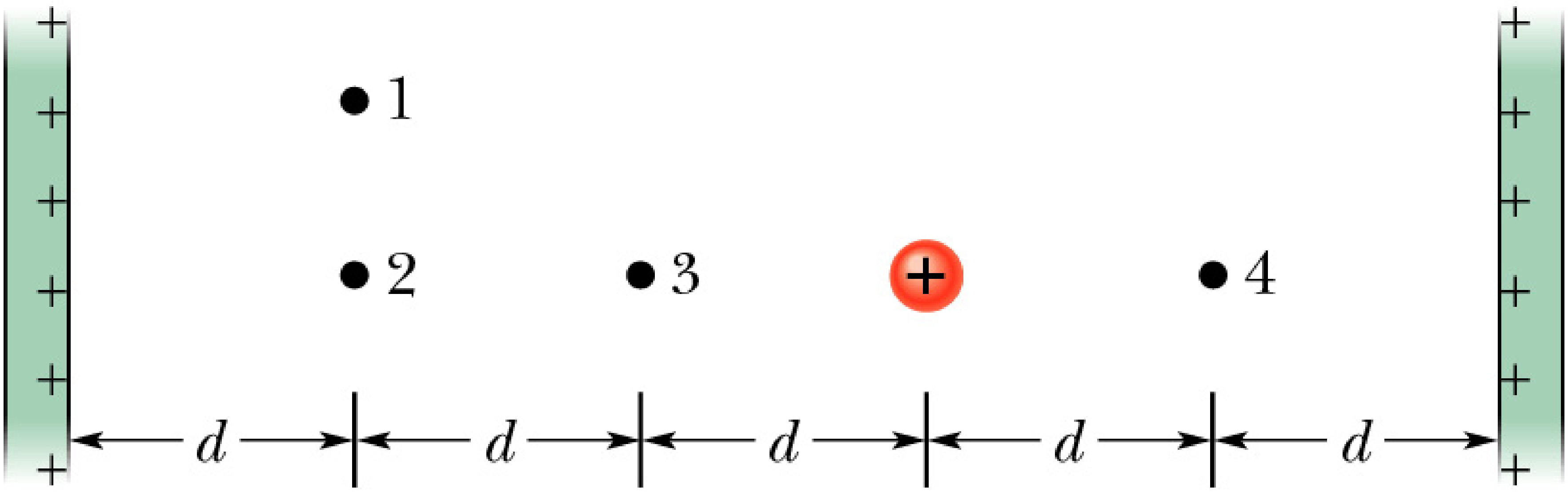
- Positive and negative charged conducting plates put together
  - Excess charges moves to inner faces
  - New total surface density, σ, is equal to $2\sigma_1$

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$



Gauss' Law (Checkpoint)

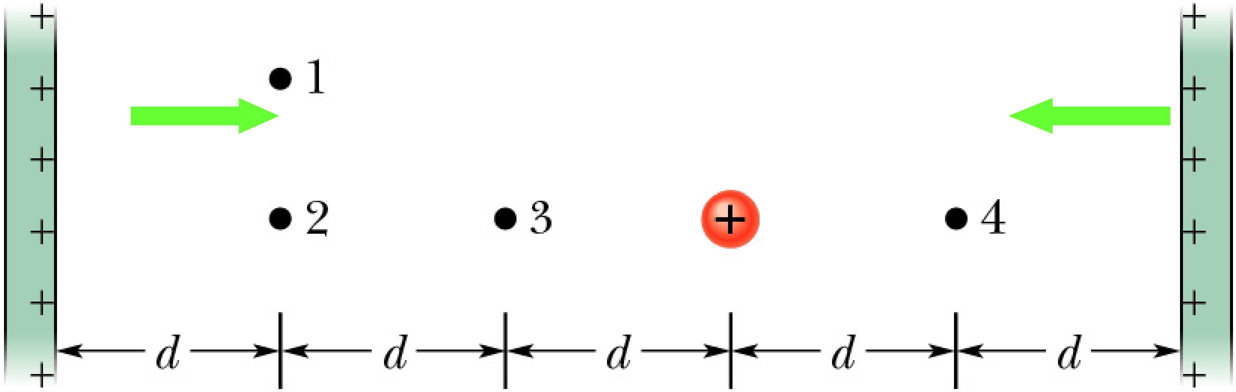
- Two large, parallel, non-conducting sheets with identical + charge and a sphere of uniform + charge. Rank magnitude of net E field for 4 points (greatest first).



Gauss' Law (Checkpoint)

- *E* due to sheets

$$E = 0$$



- *E* due to point charge

$$E = k\frac{q}{r^2}$$

- Magnitude depends on distance r from point charge

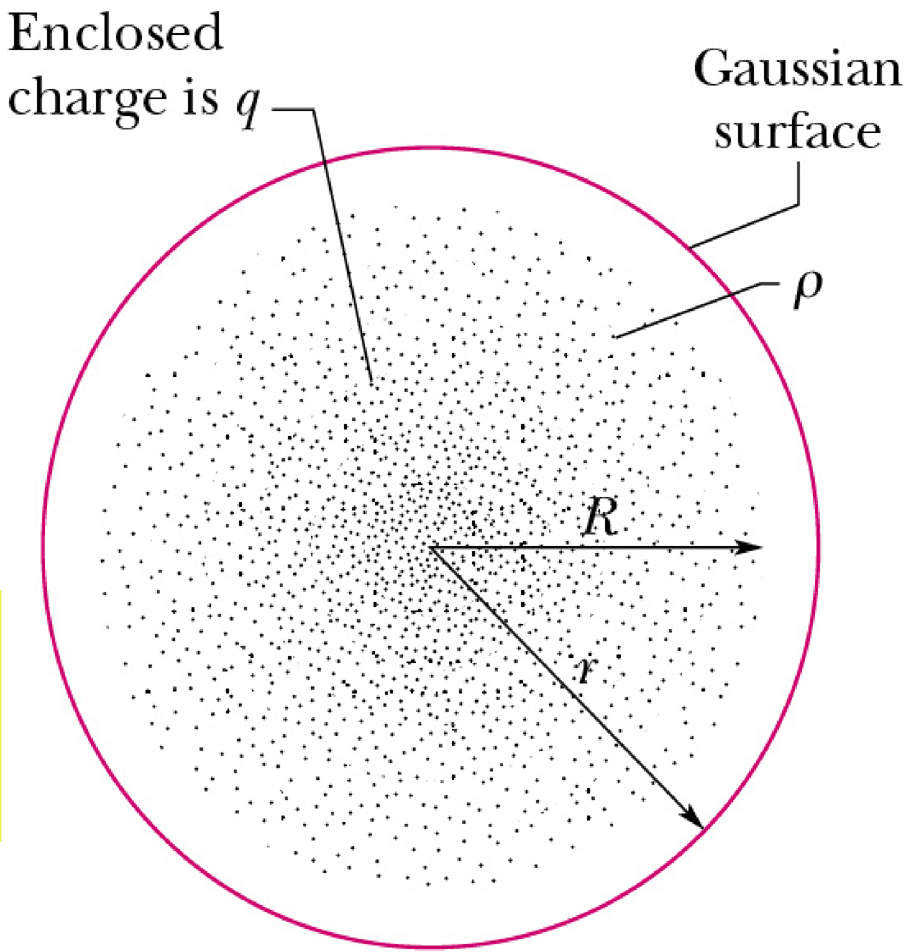
3 and 4 tie, then 2, then 1

Gauss' Law

- Non-conducting solid sphere of radius R and total (uniform) charge q
- Gaussian sphere outside sphere

$$E = k\frac{q}{r^2}, \quad r \geq R$$

- Same as shell



(a)

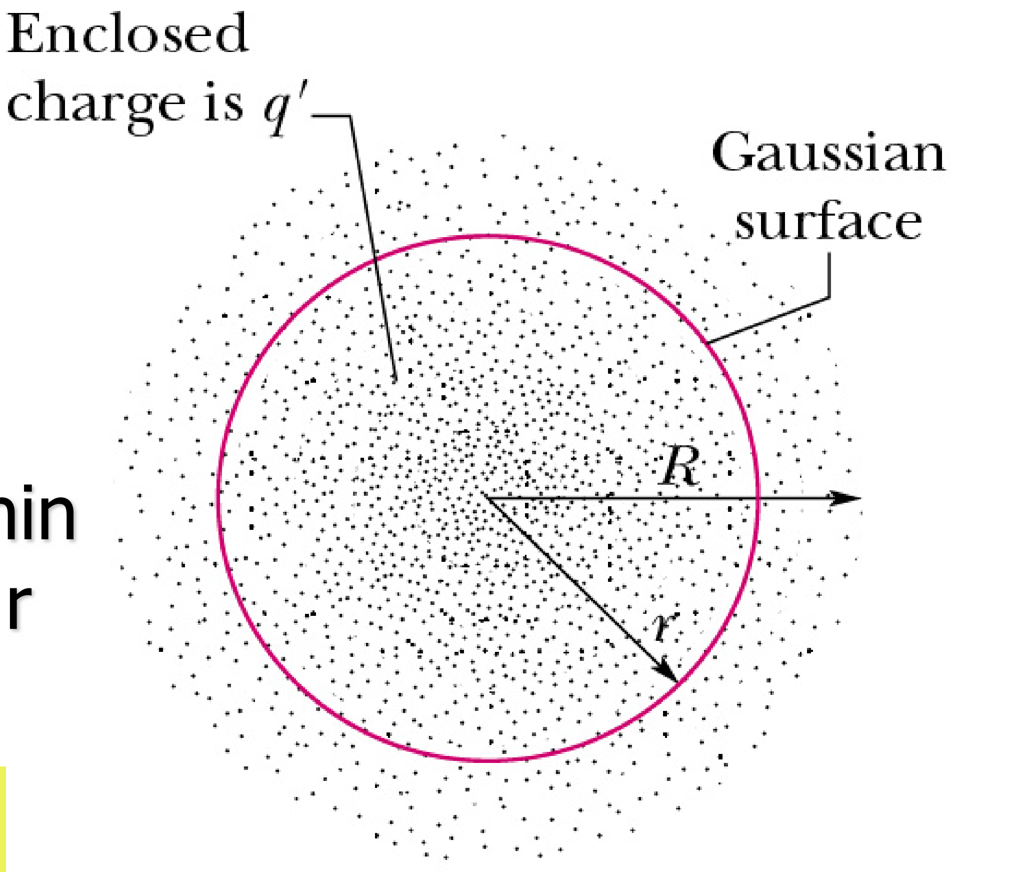
Gauss' Law

- Use series of Gaussian spheres for inside

$$E = k\frac{q'}{r^2}$$

- Full charge enclosed within R is uniform so q' within r is proportional to q

$$\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}$$



(b)

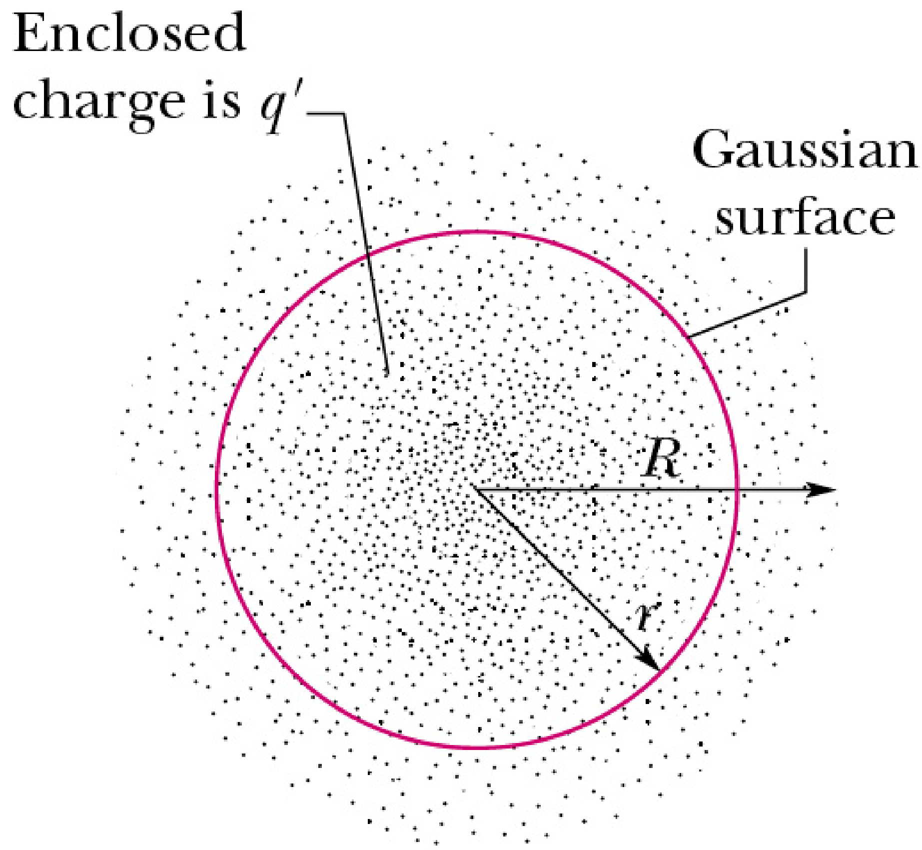
Gauss' Law

- Enclosed charge at r is

$$q' = q\frac{r^3}{R^3}$$

- *E* field inside sphere

$$E = \frac{kqr}{R^3}, r \le R$$



(b)