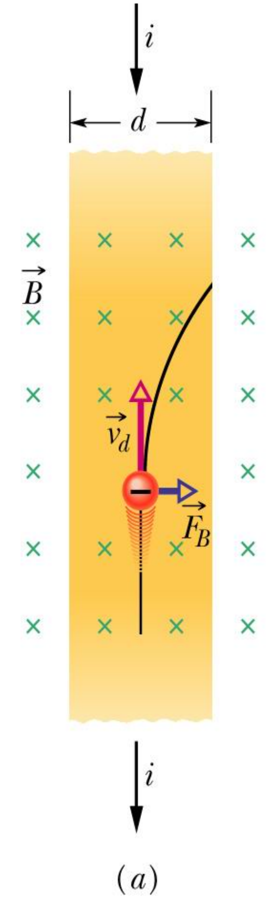
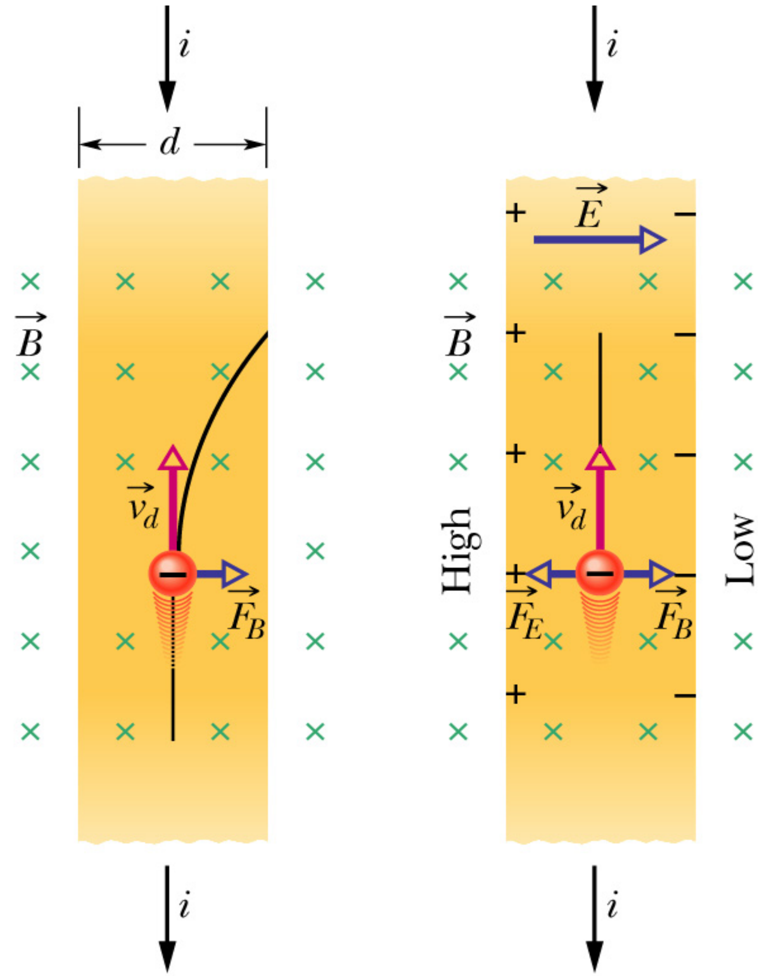
Hall Effect

- Electrons moving in a wire. In this case the wire is a rectangular slab with width, *d*, and thickness, *l*.
- The total cross sectional area of the wire is *A=l d*.
- *B* field points into the screen.



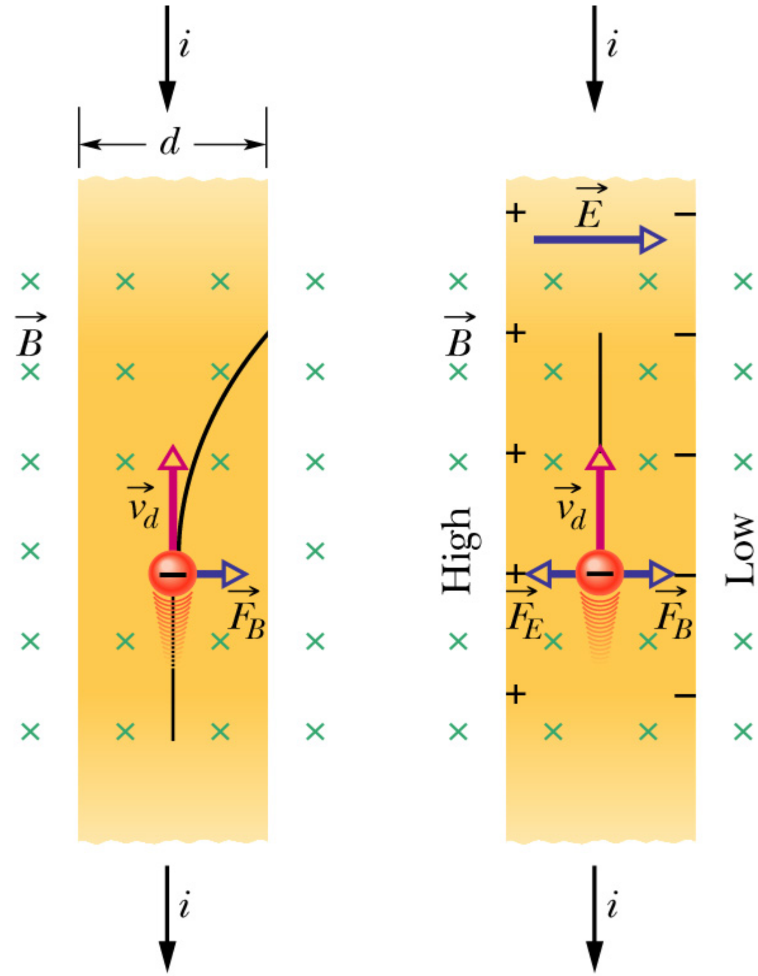
Hall Effect

- Electrons moving in a wire (= current) can be deflected by a *B* field called the Hall effect
- Creates a Hall potential difference, *V*, across the wire
- Can measure the wire's charge density when at equilibrium $F_E = F_B$



Hall Effect

- Electrons have drift velocity, v_d in direction opposite the current, i
- B field into page causes force, F_B to right
- Electrons pile up on right hand side of strip
- Leaves + charges on left and produce an E field inside the strip pointing to right

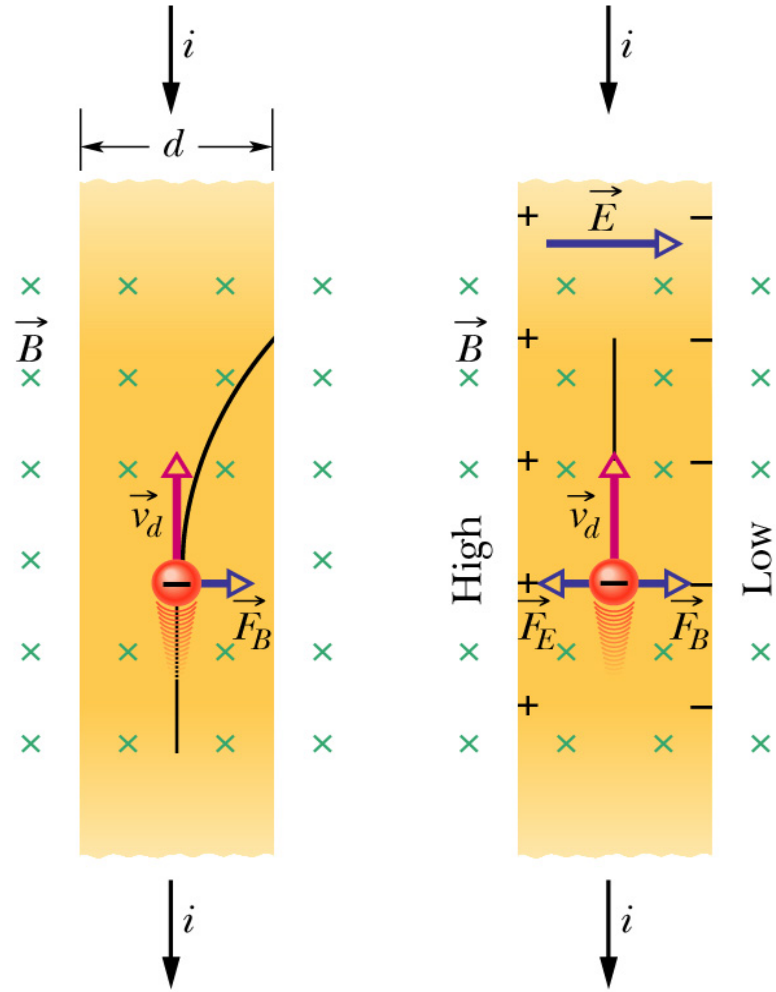


Hall Effect

- E field on electron produces a F_E to the left
- Quickly have equilibrium where $F_E = F_B$
- E field gives a V across the strip

$$V = Ed$$

- Left side is at a higher potential



Hall Effect

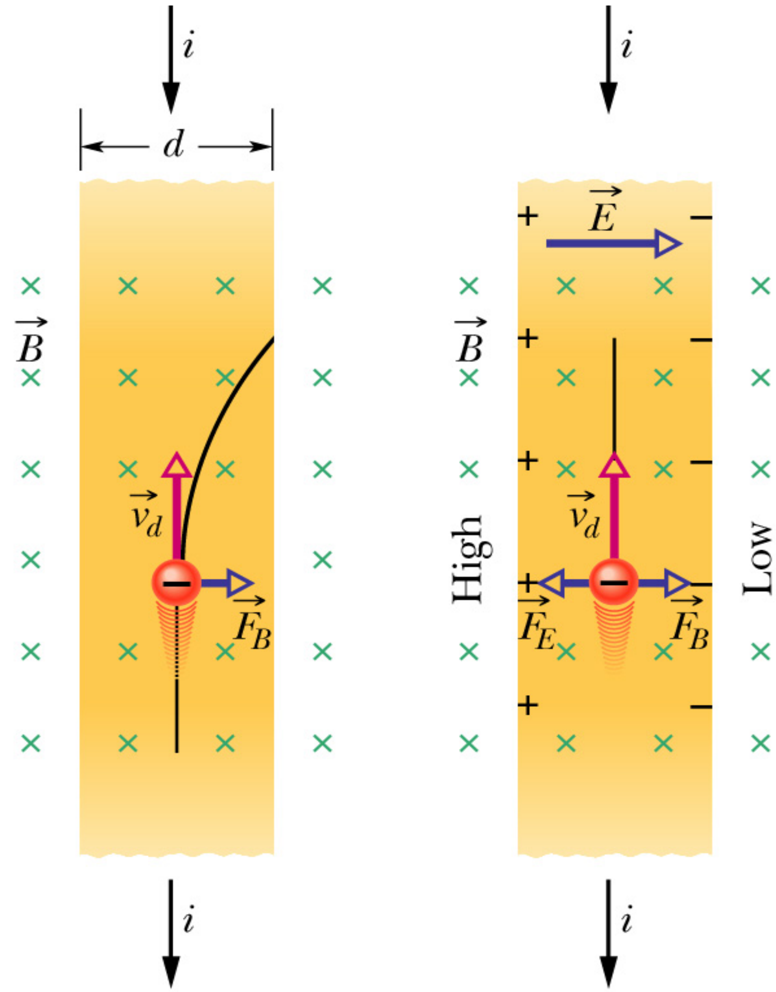
- Can measure the number of charge carriers per unit volume, *n*, at equilibrium

$$F_E = F_B$$

$$F_E = qE \qquad F_B = |q\vec{v} \times \vec{B}|$$

$$eE = ev_d B \sin(90)$$

$$E = v_d B$$



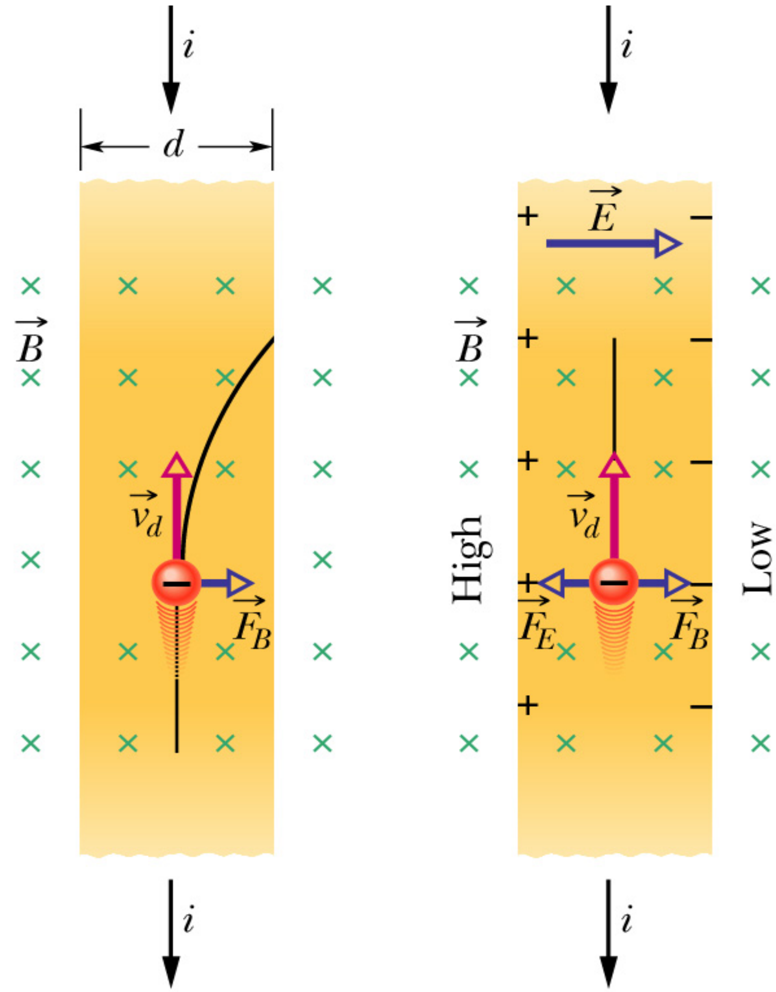
Hall Effect

- Remember from Chpt. 27 that drift speed is

$$v_d = \frac{J}{ne} = \frac{i}{neA}$$

$$E = v_d B = \frac{iB}{neA}$$

$$n = \frac{iB}{EeA}$$

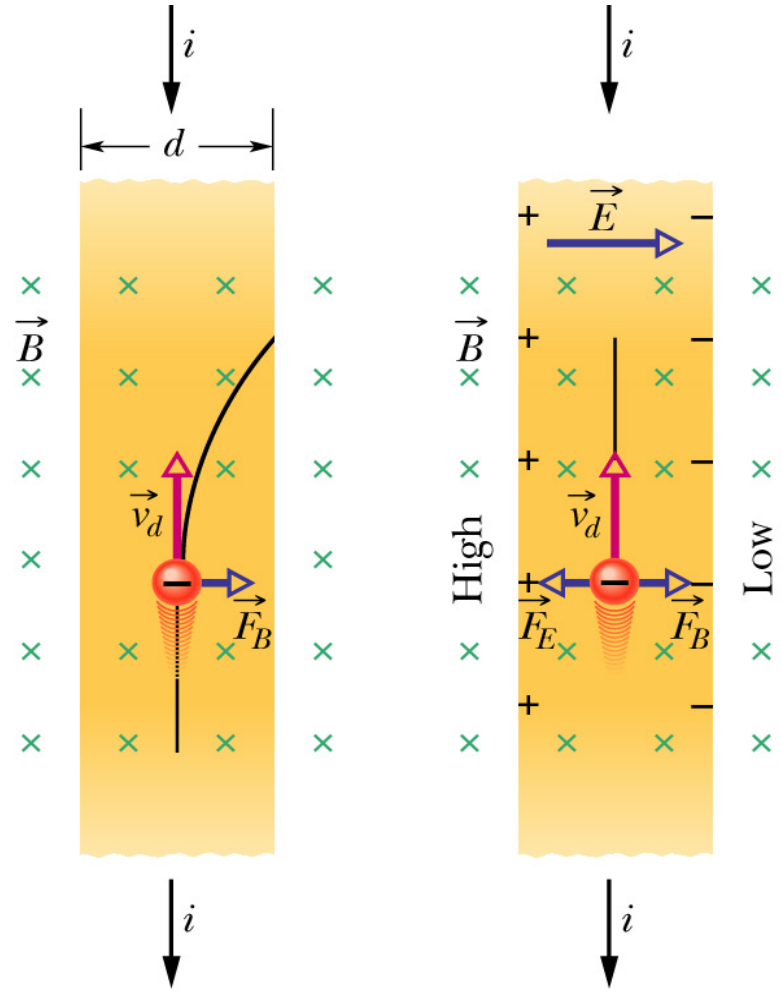


Hall Effect

- Replacing *E* by

$$V = Ed$$

$$n = \frac{iB}{EeA} = \frac{iBd}{VeA}$$



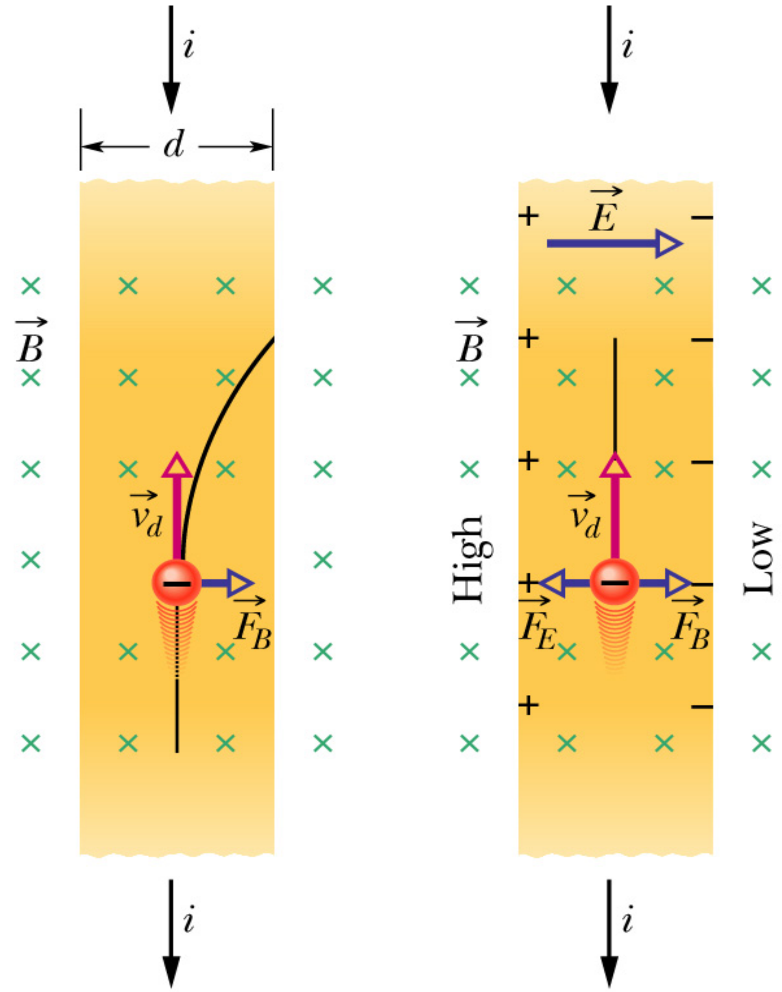
Hall Effect

- If l is the thickness of the strip

$$l = \frac{A}{d}$$

- Finally get

$$n = \frac{iB}{Vle}$$

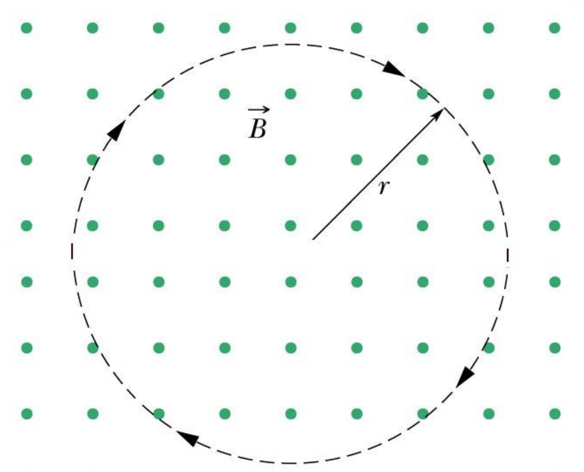


Magnetic Fields: Circular Motion

- F_B continually deflects path of charged particles

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- If v and B are ⊥, F_B causes charged particles to move in a circular path



- If B points towards you
  - + particles move clockwise.
  - – particles move counter clockwise.

Magnetic Fields: Circular Motion

- Derive radius of circular path for particle of charge, *q*, and mass, *m*, moving with velocity, *v*, which is ⊥ to *B* field

$$F_B = |q\vec{v} \times \vec{B}| = qvB\sin\phi = qvB$$

- Newton's second law for circular motion is

$$F = ma = m\frac{v^2}{r}$$

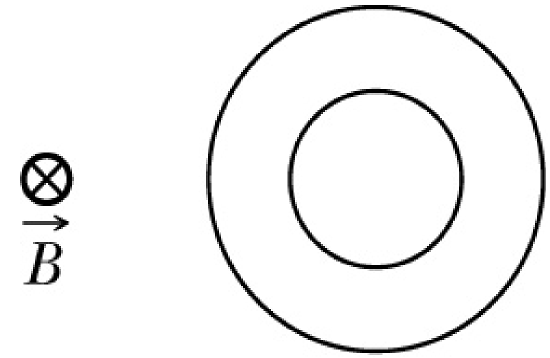
- Setting the forces equal and solving for *r*
  - Faster particles move in larger circles

$$qvB = m\frac{v^2}{r}$$

$$r = \frac{mv}{qB}$$

Exercise

- A proton and an electron travel at same *v* (in the plane of the page).



- There is a *B* field into the page.

- A) Which particle follows the smaller circle?

  $r \propto m/q$, $|q\_e|=|q\_p|=e$, and $m_p > m_e$,
  so the electron has the smaller circle

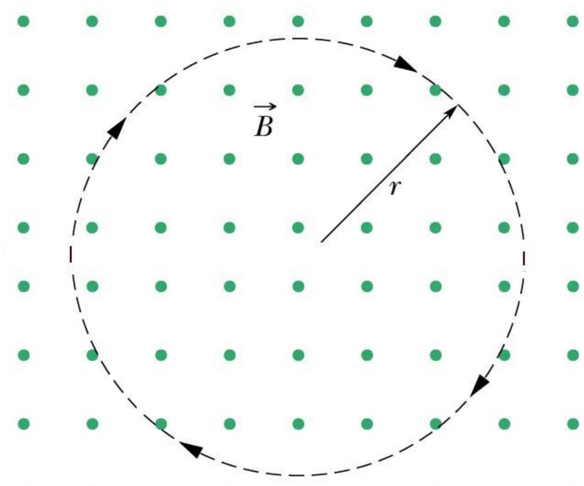
- B) What direction does the electron move in?

  Clockwise

Magnetic Fields: Circular Motion

- Period, *T*, is the time for one full revolution

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$



- Frequency, *f*, is the number revolutions per unit time

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

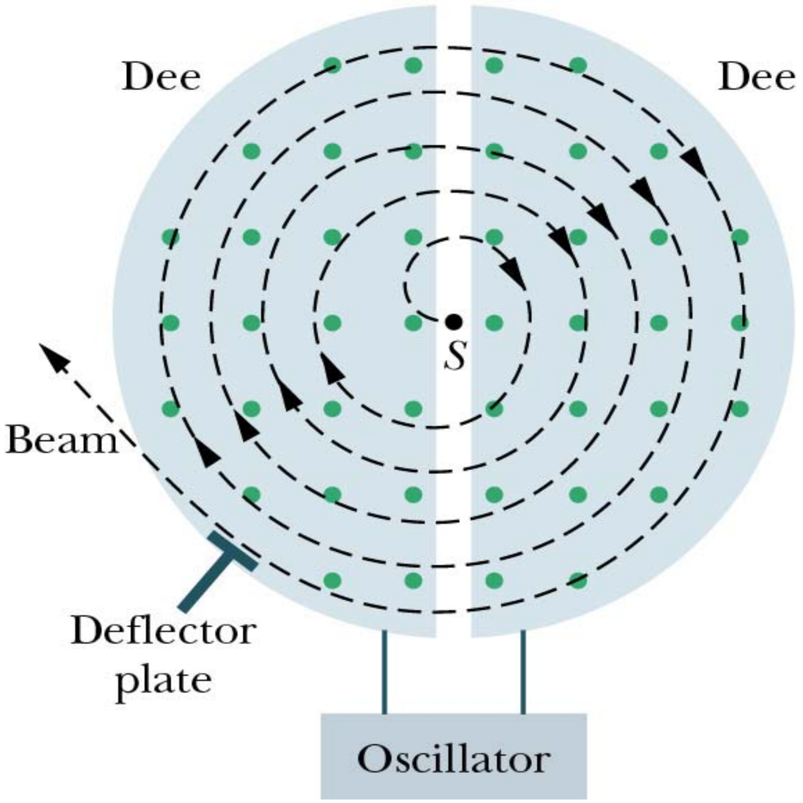
- Angular frequency, ω, is

$$\omega = 2\pi f = \frac{qB}{m}$$

- Only depend on *q* and *m* but not *v*

Cyclotron

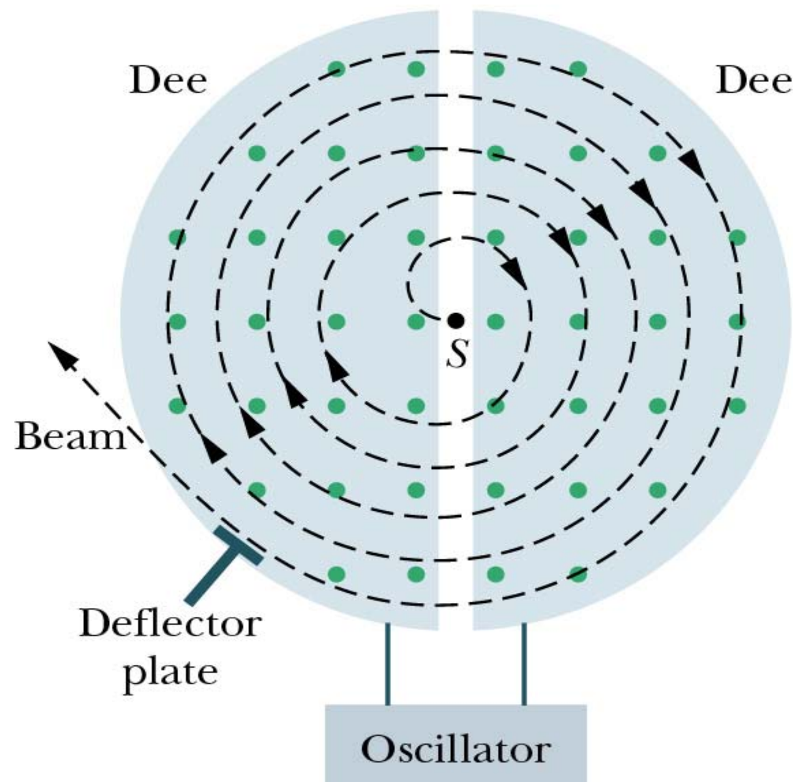
- Cyclotron
  - Particles starts at the center.
  - They circulate inside 2 hollow metal D shaped objects
  - Alternate the electric sign of the Dees so *V* across gap alternates (the oscillator does this).
  - Whole thing immersed in magnetic field *B* (green dots pointing out of page) ⊥ to *v*
  - *B* approximately 1-10 T (tesla).



Cyclotron

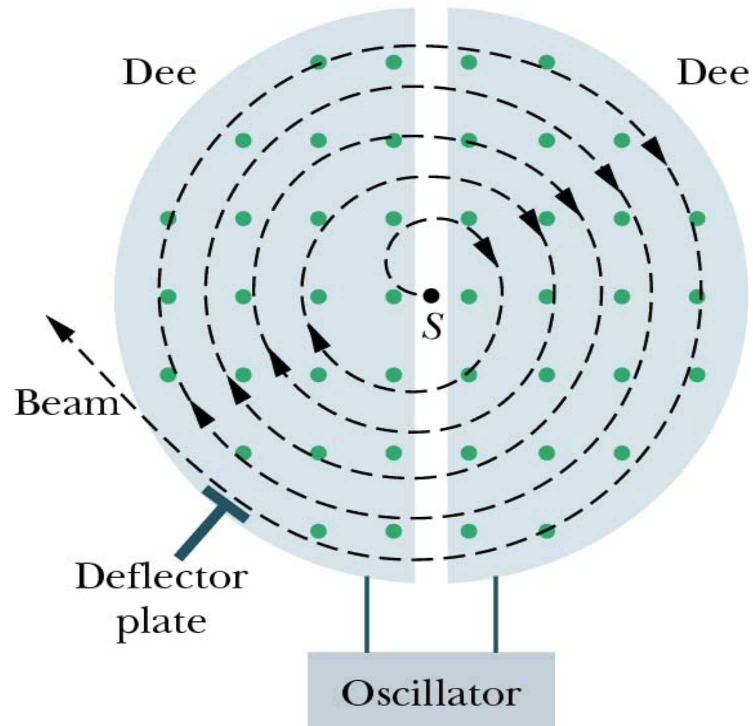
- Cyclotron
  - Proton starting in center will move toward negatively charged Dee
  - Inside Dee *E* field = 0 (inside conductor) but *B* field causes proton to move in circle with radius which depends on *v*

$$r = \frac{mv}{qB}$$



Cyclotron

- Cyclotron
  - When proton enters gap between Dees *E* field is flipped so proton is again attracted to negatively charged Dee
  - Every time proton enters gap the polarity of the Dees is changed and the proton is given another kick (accelerated)



Cyclotron

- Cyclotron
  - Key is that the frequency, *f*, of the proton does not depend on *v* and must equal the f_{osc} of the Dees

$$f = f_{osc}$$

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$qB = 2\pi m f_{osc}$$