Total electric potential energy, *U*, of a system of charges is obtained from the work done by an external *F*, (*W**) to assemble the system, bringing each charge in from ∞. In terms of work done by the field, *W**= -*W*.



• Bring q_1 from ∞ , $W^* = 0$ since no electric F yet

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Potential due to q₁ is

$$V = k \frac{q_1}{r} \qquad \stackrel{q_1}{\xleftarrow} r \xrightarrow{q_2}{} r$$

Bring q₂ in from infinity. From definition of potential energy

$$U = -W = W^* = q_2 V = k \frac{q_1 q_2}{r}$$
 or $U = k \frac{q_1 q_2}{r}$

- Charges of like sign, W^* and U are +
- Charges of opposite sign, W* and U are -

- What is the potential energy when add an additional charge to system?
- Move q₁ from ∞, W* = U = 0
 Move q₂ from ∞

$$W_{12}^* = U_{12} = k \frac{q_1 q_2}{d}$$



• Now bring in q₃

$$W_{13}^* = U_{13} = k \frac{q_1 q_3}{d}$$

Must also remember q₂

$$W^*_{23} = U_{23} = k \frac{q_2 q_3}{d}$$



 Total potential energy is the scalar sum

$$U = U_{12} + U_{13} + U_{23}$$

$$q_1 = +q, \quad q_2 = -4q, \quad q_3 = +2q$$



$$U = k \left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d} \right) = -k \frac{10q^2}{d}$$

Electric Potential for Conductors

Using what we know about conductors

• E = 0 inside

All excess charge is on surface

- All points of a conductor whether inside or on the surface – are at the same potential
 - A conductor is an equipotential

Electric Potential for Conductors





Electric Potential (Exercise)

- An electron moves along 5 different paths between parallel equipotential surfaces
- a) What is the direction of the *E* associated with the surfaces?



 Positive potentials which decrease going to the right.

Electric Potential (Exercise)

 b) Rank the paths by amount of work we do (greatest first).

$$W^* = -W = q\Delta V$$

$$W^* = q \left(V_f - V_i \right)$$

$$90 V \quad 80 V \quad 70 V \quad 60 V \quad 50 V \quad 40 V$$

Electron gives

$$W^*_{Path-1} = -q(70-80) = +10q$$

3, then 1 & 2 & 5, last 4

Uniformly Charged Sphere

- Electric field outside R $E = k \frac{Q}{r^2}$
- Integrate from to r'=r to $r'=\infty$

$$V_{\infty} - V_r = -\int_r^{\infty} E_{r'} dr'$$
$$= -kQ \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$V_r = \frac{kQ}{r}, \quad for \ r > R$$



Uniformly Charged Sphere

• E field inside sphere

$$E = \frac{kQr}{R^3}, r \le R$$

Integrate from r'=r to r=R

 $V_{r} - V_{R} = -\int_{r}^{R} E_{r'} dr'$ $= -\frac{kQ}{R^{3}} \int_{r}^{R} r' dr' = \frac{kQ}{2R^{3}} [r^{2} - R^{2}]$

$$V_{R} = \frac{3kQ}{2R}, \implies$$

$$V_{r} = \frac{kQ}{2R} \left[3 - \frac{r^{2}}{R^{2}} \right] \quad for \quad r < R$$



Read also book examples 25.5, 25.6 and 25.7