12 - Big Bang Nucleosynthesis
12.1 - The Early Universe

According to the accepted cosmological theories:

• The Universe has cooled during its expansion as

\[ T(t) \sim \frac{1}{R(t)} \]  \hspace{1cm} (12.1)

• In terms of the time evolved from the Big Bang (for a radiation dominated universe \( R \sim t^{-1/2} \))

\[ T(t) \sim \frac{1.3 \times 10^{10} \text{ K}}{t^{1/2} \text{ [s]}} \]  \hspace{1cm} (12.2)

• The particles are in thermal equilibrium. This is guaranteed by reactions which are faster than the expansion rate \((dR/dt)/R\). In this case, particles and antiparticles are in equilibrium through annihilation, i.e. particles + antiparticles \( \leftrightarrow \) photons

• But when \( k_B T \ll mc^2 \), particles and antiparticles (with mass \( m \)) annihilate and photons cannot create them back as their energy is now below the threshold.

• At \( T = 10^{12} \text{ K} \), there was a slight overabundance of matter over antimatter which lead to the violation of baryon/lepton number conservation.

• Also at \( T = 10^{12} \text{ K} \), antinucleons have annihilated with nucleons and the remaining nucleons become the breeding material for primordial nucleosynthesis, or **Big Bang Nucleosynthesis (BBN)**.
The Early Universe

At one-hundredth of seconds of the Universe consisted of an approximately equal number of electrons, positrons, neutrinos and photons, and a small amount of protons and neutrons; the ratio of protons to photons is assumed to have been about $10^{-9}$. The energy density of photons can be calculated from

$$\rho_\gamma = \int E_\gamma \, dn_\gamma$$

(12.3)

where the density of states is given by

$$dn_\gamma = \frac{g_\gamma}{2\pi^2} \frac{k_\gamma^2}{\exp\left(\frac{E_\gamma}{kT}\right) - 1} d\kappa_\gamma$$

(12.4)

and $g_\gamma = 2$ is the number of spin polarizations 1 for the photon while $E_\gamma = \hbar k_\gamma c$ is the photon energy (momentum).

Performing the integration gives

$$\rho_\gamma = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T^4$$

(12.5)

which is the familiar blackbody result.
At large temperatures, when \( k_B T \gg m_i \), the mass \( m_i \) of the particles (electrons, neutrinos, nucleons) are irrelevant and the energy density associated with these particles can also be described by the black-body formula. A straightforward calculation for the density, and number density accounting for all particle degrees of freedom \( g_i \) yields

\[
\rho_i = \frac{\pi^2 k_B^4}{15 \hbar^3 c^3} T^4 \times \begin{cases} 
7/8 & \text{for fermions} \\
1 & \text{for bosons}
\end{cases}
\]  \hspace{2cm} (12.6)

\[
n_i = \frac{\zeta(3) k_B^3}{\pi^2 \hbar^3 c^3} T^3 \times \begin{cases} 
3/4 & \text{for fermions} \\
1 & \text{for bosons}
\end{cases}
\]  \hspace{2cm} (12.7)

as well as the known relation between density and pressure for matter

\[
p_i = \frac{\rho_i}{3} \hspace{2cm} (12.8)
\]

In Eq. (12.7) \( \zeta(3) = 1.202 \) is the Riemann zeta function. The difference for fermions and bosons is because of their different statistical distribution functions.
The Early Universe

As we have seen in Eqs. (3.52) and (3.53), the density of scales with time as \( \rho \sim 1/t^2 \), which together with Eq. (12.6) yields

\[
t \sim \frac{\text{const.}}{T^2} \quad (12.9)
\]

with the precise expression being

\[
t = \left( \frac{90 \hbar^3 c^3}{32 \pi^3 G g_*} \right)^{1/2} \frac{1}{k_B T^2} \quad (12.10)
\]

where \( g_* \) is total the number of degrees of freedom of the particles. Thus, the relation between time and temperature in the early universe depends strongly on the kind of particles present in the plasma. Calculations of time and temperature in the early universe are shown below.

<table>
<thead>
<tr>
<th>T(K)</th>
<th>( a/a_0 )</th>
<th>t(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sim 10 \text{ MeV} )</td>
<td>( 10^{11} )</td>
<td>( 1.9 \times 10^{-11} )</td>
</tr>
<tr>
<td>( \sim 1 \text{ MeV} )</td>
<td>( 10^{10} )</td>
<td>( 1.9 \times 10^{-10} )</td>
</tr>
<tr>
<td>( \sim 100 \text{ keV} )</td>
<td>( 10^9 )</td>
<td>( 2.6 \times 10^{-9} )</td>
</tr>
<tr>
<td>( \sim 10 \text{ keV} )</td>
<td>( 10^8 )</td>
<td>( 2.7 \times 10^{-8} )</td>
</tr>
</tbody>
</table>

For a Universe dominated by relativistic, or radiation-like, particles \( \rho \propto 1/a^4 \) and the above expressions yield \( a \sim T^{-1} \sim t^{1/2} \).
12.1.1 - Time ~ 0.01 sec

At $t \sim 0.01$ s, the temperature is $T \sim 10^{11}$ K, and $k_B T \sim 10$ MeV, which is much larger than the electron mass. Neutrinos, electrons and positrons are easily produced and destroyed by means of weak interactions (i.e., interactions involving neutrinos)

\[ i) \quad n + \nu_e \leftrightarrow p + e^- \]
\[ ii) \quad n + e^+ \leftrightarrow p + \bar{\nu}_e \]
\[ iii) \quad n \leftrightarrow p + e^- + \bar{\nu}_e \]

(12.11)

As long as the weak reactions are fast enough, the neutron-to-proton ratio is given by

\[ [n/p] = \frac{\text{number of neutrons}}{\text{number of protons}} = \frac{N_n(T)}{N_p(T)} = \exp \left[ -\frac{\Delta mc^2}{k_B T} \right] \]

(12.12)

where $m(n) = 939.5$ MeV, $m(p) = 938.3$ MeV, and $\Delta m = 1.294$ MeV.

At $T = 10^{11}$ K, $k_B T = 8.62$ MeV, yielding $n/p = 0.86$.

This temperature is far above the temperature of nucleosynthesis, but the $n/p$ ratio already begins to drop.
12.1.2 - Time ~ 0.1 sec

At $t \sim 0.1$ s, the temperature is $T \sim 3 \times 10^{10}$ K, and $k_B T \sim 2.6$ MeV. Neutrinos, electrons and positrons are still in equilibrium according to Eq. (12.11). The lifetime for destruction of a neutron by means of these reactions can be calculated from

$$r(T) = n_v \left\langle \sigma v \right\rangle_v$$  \hspace{1cm} (12.13)

This is not an easy calculation, as it requires the knowledge of $\sigma$ for neutrino induced interactions. A detailed calculation yields

$$r(T) = \frac{0.76}{\text{sec}} \left( \frac{k_B T}{\text{MeV}} \right)^5$$  \hspace{1cm} (12.14)

At $T \sim 3 \times 10^{10}$ K, this yields a neutron destruction lifetime of 0.01 sec.

Thus, the weak rates drop very fast, as $T^5$. At some $T$ the weak rates are so slow that they cannot keep up with Universe expansion rate. The Hubble rate at this epoch is found to be

$$H(T) = \frac{0.67}{\text{sec}} \left( \frac{k_B T}{\text{MeV}} \right)^2$$  \hspace{1cm} (12.15)

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12.1.3 - Decoupling

When the Hubble expansion rate is equal to the neutron destruction rate, i.e., when Eqs. (12.14) and (12.15) are equal, we find $k_B T \sim 1$ MeV.

As temperature and density decreases beyond this point, the neutrinos start behaving like free particles. Below $10^{10}$ K they cease to play any major role in the reactions. That is, they decouple and matter becomes transparent to the neutrinos.

At $k_B T \sim 1$ MeV (twice the electron mass), the photons also stop produce positron/electron pairs

$$\gamma \leftrightarrow e^+ + e^- \quad (12.16)$$

The $e^-e^+$-pairs begin to annihilate each other, leaving a small excess of electrons. However, the thermal energies are still high enough to destroy any formed nuclei. At this point

$$[n/p] = \exp \left[ -\frac{\Delta mc^2}{kT} \right] \sim 0.25 \quad (12.17)$$

As the temperature drops, neutrino-induced reactions continue creating more protons. In the next 10 secs the n/p ratio will drop to about 0.17 $\sim 1/6$. And after the neutron percentage continues to decrease because of neutron beta-decay. When nucleosynthesis starts the n/p ratio is 1/7.
12.1.4 - Baryon to photon ratio

Neutrons and protons were created at an earlier stage of the universe, when quarks and gluons combined to form them. As with the other particles, very energetic photons produced baryons (nucleons) and antibaryon pairs and were also produced by the inverse reactions. As the temperature decreased, baryons annihilated each other and more photons were created. One does not know why a small number of baryons remained. The resulting baryon/photon ratio at this time was

$$\frac{\rho_b}{\rho_\gamma} = \eta \sim 10^{-9} \quad (12.18)$$

Usually $\eta$ is considered the only parameter of BBN. However, BBN is also sensitive to two other parameters: the neutron lifetime and the number of light neutrino families.

(a) neutron lifetime $\tau_{1/2}(n)$ - An increase in $\tau_{1/2}(n)$ theoretically implies a decrease of all weak rates which convert protons and neutrons. The freeze-out would happen at a higher temperature and would lead to a larger $n/p$ ratio. As all neutrons essentially end up in $^4\text{He}$, the $^4\text{He}$ abundance grows if $\tau_{1/2}(n)$ is increased.

(b) number of neutrino families - The energy density of the early Universe depends on the number of neutrino families ($n_\nu$): the larger $n_\nu$, the larger and the faster the expansion rate of the Universe $H = (dR/dt)/R$. An increase in $H$ leads to an earlier freeze-out and hence more $^4\text{He}$ abundance.
Neutron-to-proton (n/p) ratio as a function of time and temperature. The dashed curve is given by \(\exp(-\Delta m/k_B T)\). The dotted curve is the free-neutron decay curve, \(\exp(-t/\tau_n)\). The solid curve indicates the resulting n/p ratio as a combination of the two processes. BBN starts at \(t \sim 4\) min.
12.2 - Big Bang Nucleosynthesis (BBN)

A summary of the BBN when the temperature of the universe allowed deuteron to be formed without being immediately destroyed by photons is:

1. The light elements (deuterium, helium, and lithium) were produced in the first few minutes after the Big Bang.

2. Elements heavier than $^4$He were produced in the stars and through supernovae explosions.

3. Helium and deuterium produced in stars do not match observation because stars destroy deuterium in their cores.

4. Therefore, all the observed deuterium was produced around three minutes after the big bang, when $T \sim 10^9$ K.

5. A simple calculation based on the n/p ratio shows that BBN predicts that 25% of the matter in the Universe should be helium.

6. More detailed BBN calculations predict that about 0.001% should be deuterium.
The deuteron bottleneck

As the temperature of the Universe decreased, neutrons and protons started to interact and fuse to a deuteron

\[ n + p \rightarrow d + \gamma \tag{12.19} \]

The binding energy of deuterons is small \((E_B = 2.23 \text{ MeV})\). The baryon-to-photon ratio, called \(\eta\), at this time is also very small \((< 10^{-9})\). As a consequence, there are many high-energy photons to dissociate the formed deuterons, as soon as they are produced.

The temperature for nucleosynthesis at the start of is about 100 keV, when we would have expected \(\sim 2 \text{ MeV}\), the binding energy of deuterium. The reason is the very small value of \(\eta\). The BBN temperature, \(\sim 100 \text{ keV}\), corresponds according to Eq. (12.10) to timescales less than about 200 sec. The cross-section and reaction rate for the the reaction in Eq. (12.19) is

\[ \sigma v \sim 5 \times 10^{-20} \text{ cm}^3 / \text{sec} \tag{12.20} \]

So, in order to achieve appreciable deuteron production rate we need \(\rho \sim 10^{-17} \text{ cm}^{-3}\). The density of baryons today is known approximately from the density of visible matter to be \(\rho_0 \sim 10^{-7} \text{ cm}^{-3}\) and since we know that the density \(\rho\) scales as \(R^{-3} \sim T^3\), the temperature today must be \(T_0 = (\rho_0/\rho)^{1/3} T_{\text{BBN}} \sim 10K\), which is a good estimate.
The deuteron bottleneck

The bottleneck implies that there would be no significant abundance of deuterons before the Universe cooled to about $10^9$ K.

Other important facts are:

1 - The nucleon composition during BBN was proton-rich.

2 - The most tightly bound light nucleus is $^4$He.

3 - There is no stable nucleus with mass numbers $A = 5$ and $A = 8$.

4 - The early universe was too cold and not dense enough to overcome the Coulomb barriers to produce heavier nuclides.

5 - The BBN network is active until all neutrons are bound in $^4$He. As the BBN mass fraction of neutrons was $X_n = N_n / (N_n + N_p) = 1/8$, it follows that the mass fraction of $^4$He after BBN is about $X_{^4\text{He}} = 2X_n = 25\%$. 
BBN Predictions - The Helium abundance

BBN predicts that when the universe had $T = 10^9$ K (1 minute old), protons outnumbered neutrons by 7:1. When $^2\text{H}$ and He nuclei formed, most of the neutrons formed He nuclei. That is, one expects 1 He nucleus for every 12 H nuclei, or 75% H and 25% He. This is the fraction of He and $^2\text{H}$ we observe today.
12.2.1 - The BBN reaction network

After deuterons are produced at $T \sim 10^9$ K, a successive chain of nuclear reactions occur. The most important are

1: $n \rightarrow p$
2: $n(p, \gamma)d$
3: $d(p, \gamma)^3He$
4: $d(d, n)^3He$
5: $d(d, p)^3H$
6: $^3H(d, n)^4He$
7: $^4He(^3H, \gamma)^7Li$
8: $^3He(n, p)^3H$
9: $^3He(d, p)^4He$
10: $^4He(^3He, \gamma)^7Be$
11: $^7Li(p, ^4He)^4He$
12: $^7Be(n, p)^7Li$

Except for the $^7Be$ electron capture, all reactions are fast. The binding energies of $^3He$, $^3H$, $^4He$ are significantly larger than the one of deuterons. Thus these nuclei are not dissociated again.

At $T \sim 10^8$ K BBN terminates because
- the temperature and density are too low
- the Coulomb barriers too high
12.2.2 - BBN Nuclei in Stars

Deuteron

- In stellar processes deuteron is quickly converted to $^3\text{He}$
- Astronomers look at quasars: bright atomic nuclei of active galaxies, ten billion light years away.

$^3\text{He}$

- Star account for only 0.1% of all He.
- The $^3\text{He}$ abundance in stars is difficult to deduce. Its abundance is increasing in stellar fusion.
- Scientists look to our own galaxy.

$^7\text{Li}$

- $^7\text{Li}$ can form when "cosmic rays" collide with interstellar gas.
- Observations can be made on old, cool stars in our own galaxy.
- $^7\text{Li}$ is destroyed more than it is created inside of stars.
- Very old stars have low oxygen content, and their outermost layers still contain mostly primordial $^7\text{Li}$. 
12.2.3 - Experimental S-factors for BBN reactions
12.2.4 - Time-evolution of BBN – Mass fractions

Mass fractions of light nuclei as a function of time during the BBN.
Primordial abundances

NOTE: Light elements have been made and destroyed since the Big Bang.

• Some are made in:
  • stars ($^3\text{He}$, $^4\text{He}$),
  • spallation (scattering) ($^6$,$^7\text{Li}$, Be, B)
  • supernova explosions ($^7\text{Li}$, $^{11}\text{B}$)

• Some are destroyed:
  • d, Li, Be, B are very fragile; they are destroyed in the center of stars → observed abundances (at surface of stars) do not reflect the destruction inside

$^4\text{He}$ : $Y_p = 0.2421 \pm 0.0021$

$D : D/H = (2.78 + 0.44 - 0.38) \times 10^{-5}$

$^7\text{Li} : \text{Li}/H = (1.23 + 0.68 - 0.32) \times 10^{-10}$

$\Omega_B h^2 = 0.0224 \pm 0.0009$
12.2.5 - BBN predictions: Neutron lifetime

Helium mass fraction calculated with the BBN model as a function of the baryon-to-photon ratio parameter $\eta$.

$\eta_{\text{WMAP}} = 6.2 \times 10^{-10}$

$\eta$

$Y_4$

904 s
886.7 s
869 s

Neutron life time
Experiments on Neutron Lifetime

\[ \tau_n \text{ measurements vs. time} \]

885.7 ± 0.8 sec !!!

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Helium mass fraction calculated with the BBN model as a function of the number of neutrino families.
BBN incredibly successful, except for Lithium problem

SBBN: one parameter

baryon-to-photon ratio $\eta$

$\eta = (6.225^{+0.157}_{-0.154}) \times 10^{-10}$

(WMAP 2010)

<table>
<thead>
<tr>
<th></th>
<th>BBN</th>
<th>Observation</th>
</tr>
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<tbody>
<tr>
<td>$^4\text{He}/\text{H}$</td>
<td>0.242</td>
<td>0.242</td>
</tr>
<tr>
<td>$\text{D}/\text{H}$</td>
<td>$2.62 \times 10^{-5}$</td>
<td>$2.78 \times 10^{-5}$</td>
</tr>
<tr>
<td>$^3\text{He}/\text{H}$</td>
<td>$0.98 \times 10^{-5}$</td>
<td>$(0.9 - 1.3) \times 10^{-5}$</td>
</tr>
<tr>
<td>$^7\text{Li}/\text{H}$</td>
<td>$4.39 \times 10^{-10}$</td>
<td>$1.2 \times 10^{-10}$</td>
</tr>
</tbody>
</table>
BBN incredibly successful, except for Lithium problem

The SBBN model explains very well the abundance of light elements, except for the observed $^6$Li and $^7$Li. While the $^6$Li abundance is difficult to explain because of "astration", i.e., $^6$Li reprocessing in stars, no theory or model can explain why the observed $^7$Li abundance is so much smaller than predicted.

This has led to a large number of speculations, such as if the MB distribution is valid for the BBN scenario, or if another statistics should be adopted. See, e.g.,

Other possibilities even include electron screening, the effect of dark matter, or parallel universes. See, e.g.,

For a recent status of the lithium problem, see: