14 - Supernovae (short overview)

The core-collapse of a supernova

The core of a pre-supernova is made of nuclei in the iron-mass range $A \sim 45 - 65$ and electrons. The pressure is mainly due to the degenerate electrons. As long as the core mass is smaller than the Chandrasehkhar mass $M_{ch} = 1.44(2Y_e)^2 M_\odot$ (see Stellar Structure II), the core can be stabilized by the degeneracy pressure of the electrons.

But silicon burning continues around the iron core, adding mass to the iron core, and electrons can be captured by nuclei

$$e^- + (Z, A) \rightarrow (Z-1, A) + \nu_e \qquad (14.1)$$

This process reduces the pressure and cools the core. Neutrinos leave, reducing the electron fraction, Y_e and consequently also the Chandrasekhar mass $M_{ch.}$

The core collapses.

The temperature during this phase is high enough so that all reactions are in equilibrium, except for the reactions mediated by the weak interaction (beta-decay, neutrino interactions). The neutrino interactions with matter have to be considered in details.

Neutrino scattering and electron capture

For a relativistic degenerate electron gas ($P \sim \rho^{4/3}$) one can rewrite Eq. (6.15) as

$$\frac{P}{\rho} \sim \frac{1}{4} Y_e E_F \qquad (14.2)$$

Where Y_e is the electron fraction, ρ is the total density and E_F the electron Fermi energy.

In terms of the density given in units of 10^7 g/cm^3, we have

$$E_F \sim 1.1 \left(\rho_7 Y_e\right)^{1/3} \quad \text{MeV} \qquad (14.3)$$

This is about 1 MeV at $\rho_7 = 1$ and hence is in the nuclear energy scale. It is then energetically more favorable to capture high-energy electrons by protons or nuclei.

The cross section for electron capture on protons is

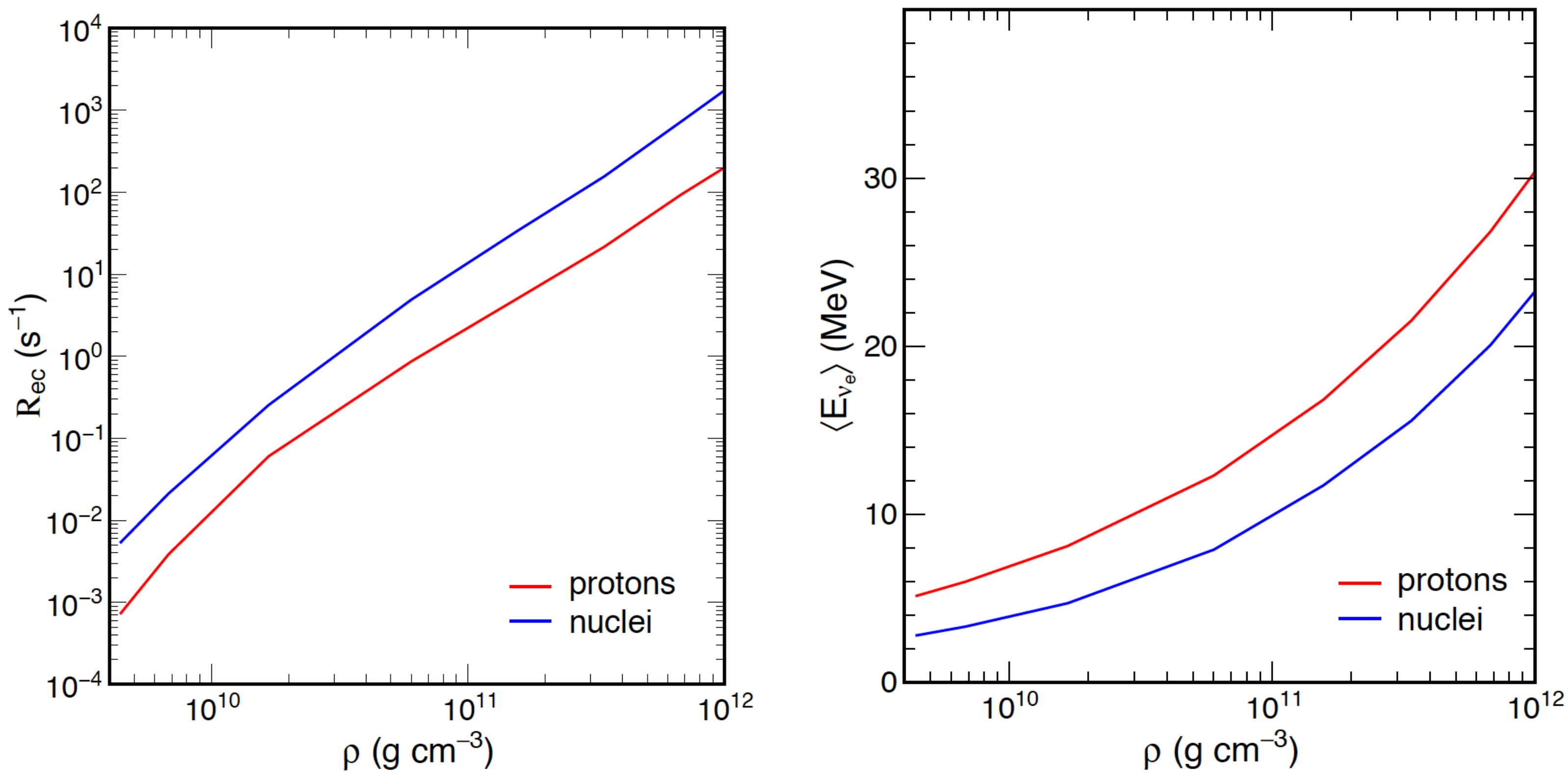
$$\sigma_{\nu p} = 4.5 \times 10^{-44} E_\nu^2 \quad \text{cm}^2 \qquad (14.4)$$

where E_ν is the energy of the emitted neutrino in MeV. The rate of electron capture on protons is then given by

$$r = \sigma_{\nu p} N_A Y_p = 0.016 \rho_7 E_\nu^2 Y_p \ [\text{sec}^{-1}] \qquad (14.5)$$

Neutrino scattering and electron capture

The capture cross section for nuclei present in the core is smaller than for protons (due to the larger Q-value for nuclei). But the abundance of protons is very low. Thus, the electron capture rate is dominated by capture on nuclei.



The core temperatures are kT = 0.1 – 0.8 MeV, the density is $\rho = 10^7 - 10^{10}$ g cm^{-3} and most nuclei are in the iron group region (*A* = 45 – 65). The important processes are

$$e^- + (N,Z) \rightarrow (N+1, Z-1) + \nu_e \qquad (14.6)$$

and β^--decay

$$(N,Z) \rightarrow (N-1, Z+1) + e^- + \bar{\nu}_e \qquad (14.7)$$

Neutrino scattering

The neutrino induced reactions in the core are

$$\nu + \mathrm{A} \leftrightarrow \nu + \mathrm{A} \qquad (14.8)$$

These are elastic process, with no energy, but with momentum transfer. They lead to neutrino trapping in the core.
Another reaction is

$$\nu + \mathrm{e}^- \leftrightarrow \nu' + \mathrm{e}^- \qquad (14.9)$$

which is an inelastic scattering, with energy transfer. This reaction leads to thermalization.

Neutrino inelastic scattering on nuclei, leading to nuclear excitation, also create thermalization

$$\nu + (\mathrm{Z}, \mathrm{A}) \leftrightarrow \nu' + (\mathrm{Z}, \mathrm{A})^* \qquad (14.10)$$

where the star means that the nuclei are excited (absorbed energy).

All these cross sections depend on the square of the neutrino energy, E_ν^2.

The neutrinos will scatter back and forth and the treatment of this behavior is done with transport equations which consider all neutrino types and keep track of neutrino fluxes, energies at all space-time points.

Neutrino transport

Because of elastic scattering, the mean-free path of neutrinos in matter, composed of heavy nuclei with mass fraction X_A and free neutrons with mass fraction X_n, is given by

$$\lambda_\nu = \frac{10^8}{\rho_{12}}\left[\frac{(\mathrm{N}-0.08\mathrm{Z})^2}{6\mathrm{A}}\mathrm{X_A}+\mathrm{X_n}\right]^{-1}\mathrm{E}_\nu^{-2} \quad \mathrm{cm} \qquad (14.11)$$

where E_ν is the neutrino energy in MeV and ρ_{12} the density in 10^{12} g cm^{-3}.

Typical values are $\rho_{12} = 1$ ($N = 50$, $A = 82$, $E_\nu = 20$ MeV). With these values one gets the neutrino mean free path $\lambda_\nu = 0.4$ km. The core radius at that moment is about R ~ 30 km.

Hence, the neutrinos scatter often in the core and their way out should be treated as a diffusion process. The diffusion time scale under these conditions is longer than the supernova core-collapse time scale (of order 1.5 ms for $\rho_{12} = 1$).

Therefore, the neutrinos are effectively trapped in the core during the final collapse.

Core-collapse

With neutrino trapping and thermalization of the core, the neutrinos become degenerate. They are described by a Fermi-Dirac distribution with a Fermi energy $E_\nu^{(F)}$ which, in equilibrium has the value

$$E_\nu^{(F)} = E_e^{(F)} - \left(E_n^{(F)} - E_p^{(F)}\right) \qquad (14.12)$$

The degenerate neutrinos stop the electron capture process. Then a sizable electron fraction and proton fraction survive the collapse.

The inner core, which is effectively in weak equilibrium, collapses as a homologous unit.

When the central density becomes about a factor 2 - 4 larger than nuclear matter density $\rho_{nm} \sim 2 \times 10^{14}$ g/cm^3 the nuclear pressure slows down the infall and finally stops it. The inner core has reached its maximum density. The core rebounds and a shock starts.

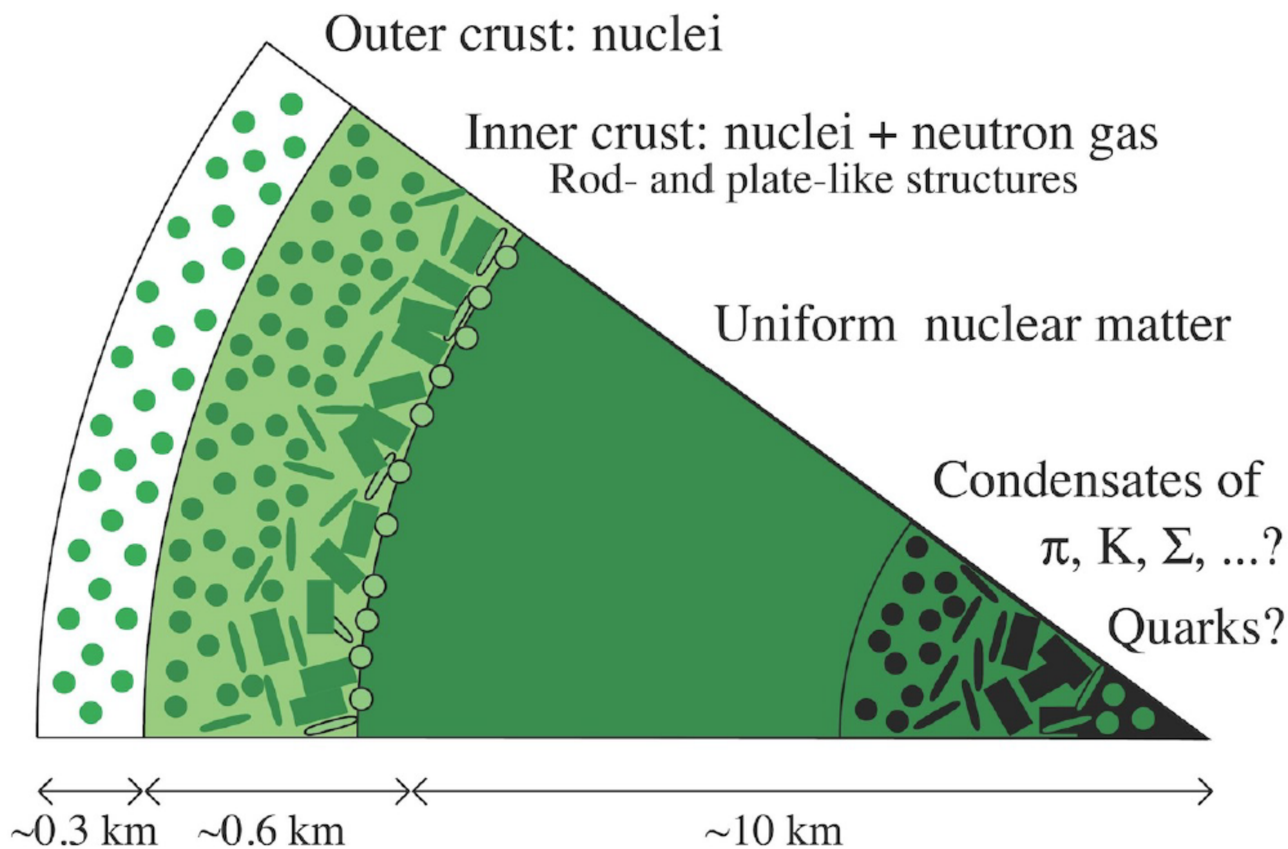
A decisive quantity for this stage of the collapse is the Equation of State (EoS). The matter consists of nuclear and electron components. Neutrinos are only important for the determination of quantities like Y_e and the temperature.

The nuclear compression modulus best accepted value is $K = 9[dP/d\rho] = 230$ MeV.

Core-collapse

Most nucleons reside in heavy nuclei until neutrino trapping occurs. With increasing density, nuclear matter forms in a two-phase system: (a) nuclei surrounded by a low-density gas of alpha-particles and (b) nucleons.

At densities between 10^{13} g/cm^3 and saturation one finds the 'spaghetti', 'lasagna' or 'Swiss cheese' phases, (rods and slabs of nuclear matter, parts of space filled with uniform nuclear matter and holes in between), and finally nuclear matter filling space uniformly. As with a neutron star, the core might look like the figure below (H. Heiselberg, 2002).



Shock wave

When the core reaches nuclear density, it becomes very hard to compress.

Pressure builds up and a pressure wave propagates outward. Near the sonic point close to the surface of the homologous core this pressure wave turns into a shock wave.

In the shock the temperature increases, nuclei are dissociated, costing energy (8 - 9 MeV/nucleon – the average binding or nuclei around iron).

The shock could be energetic enough to expel most of of the star. But simulations prove that this does not really happen. Due to nuclear dissociation, the shock wave will loose energy. At a radius of about 200 km, the shock slows down.

The trapped neutrinos can escape, further reducing the energy of the shock wave. Heavy nuclei are changed to free nucleons, electrons are captured by protons, leading to a burst in electron neutrinos taking away even more energy from the shock wave. The proton-to-nucleon ratio in the core is reduced to the value of the neutron star which is formed as the remnant of the explosion in the center.

It is thought that the neutrino absorption on nucleons right behind the shock wave revives the shock which then moves outwards and expels the rest of the star.

Shock wave revival

The gravitational energy of the collapsed core , ~10^{53} ergs, is radiated away in neutrinos of all types. There is a large luminosity in neutrinos (L > 10^{52} ergs/s) for nearly 10 seconds, before it decreases.

The cross section for neutrino absorption on a nucleon is given by

$$\sigma_{\nu,\mathrm{abs}} \sim 10^{-43} E_\nu^2 \quad \mathrm{cm}^2 \qquad (14.13)$$

where E_ν is the energy of the neutrino in MeV. If L is the neutrino luminosity right behind the shock wave, the energy gain per nucleon at distance R from the center of the star is

$$\frac{dE}{dt} = \frac{L\sigma_{\nu,\mathrm{abs}}}{4\pi R^2} X_n \sim 15 \ \ \mathrm{MeV/s} \qquad (14.14)$$

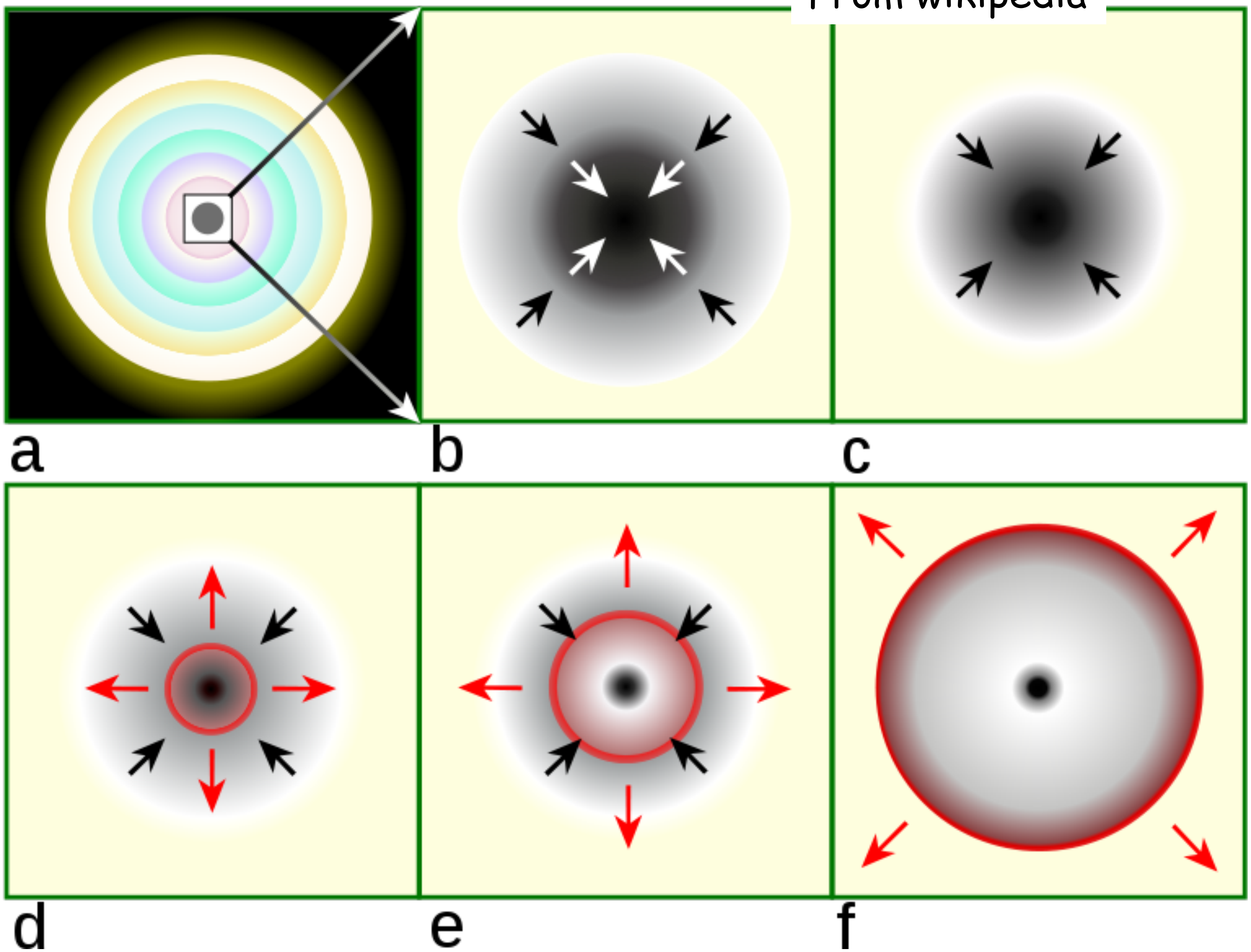
where the right-hand-side is obtained with typical values, R = 200 km, L = 10^{53} ergs/s, E = 10 MeV and X_n = 1. This energy should be compared to the required escape energy for a nucleon from the gravitational force of the mass inside the shock M(R)

$$E_g = \frac{GM(R)m_n}{R} \sim 10 \quad \mathrm{MeV} \qquad (14.15)$$

where we assumed M(R) = 1.5 $M_\odot$ and R = 200 km. Thus it takes about 1 s for the neutrinos to deliver this energy. Hence, the neutrinos behind the shock wave are very effective in pushing it out.

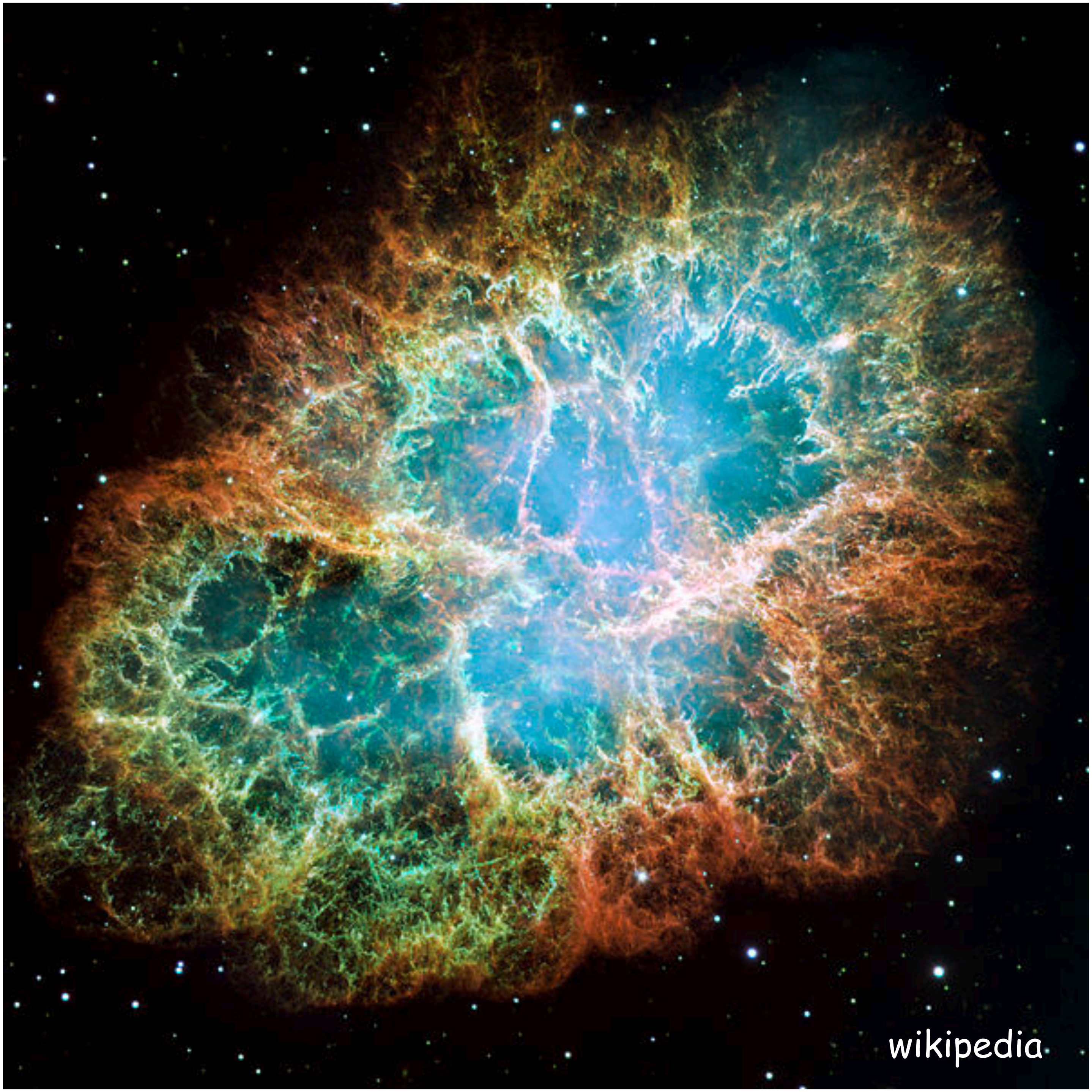
Explosion – summary

From wikipedia



The collapse followed by explosion of stars with $M > 20 - 25\ M_\odot$ probably leaves behind a core which is heavy enough to become to a black hole. Stars stars with $8\ M_\odot < M < 20 - 25\ M_\odot$ probably leave behind a neutron star.

Explosion – summary



Supernova nucleosynthesis

The hydrodynamics and neutrino transport in supernova explosions are difficult to handle. In particular, convection is thought to be of major importance, but its effects are difficult to compute. Convection should bring neutrinos from deeper (hotter) layers to the shock wave and increase the energy transfer.

The effects of magnetic fields are also difficult to handle and should help to drive and create jets from the stellar core, changing appreciably the shock scenario.

Finally, supernovae produce heavy elements by nuclear fusion of iron and other nuclei through the so-called **r-process**. For high temperatures and neutron densities, a rapid capture of neutrons followed by beta decay paves the way to heavy elements (as shown in the figure in the next slide. Highly neutron-rich nuclei are produced which beta-decay to heavy and stable nuclei afterwards. The process has to be fast (rapid) so that a neutron-rich nucleus absorbs another neutron before it beta decays.

The best candidates for the r-process and for the production of heavy elements in the universe are type II supernovae and neutron star mergers. They are thought to produce about half of all the elements beyond iron, up to uranium. Another process thought to produce heavy elements is the **s-process** occurring in large, old red giant stars. The s-process is much slower than the r-process. They cannot produce elements heavier than lead. The description of r-, s-, and rp (rapid proton capture)-processes requires another lecture.

Supernova nucleosynthesis