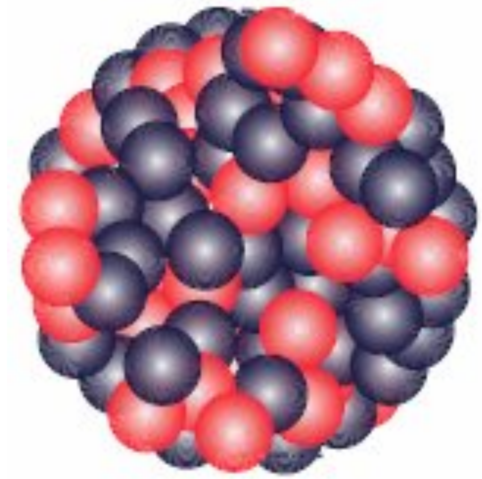
8 - Nuclei

8.1 - The nucleus



The atomic nucleus consists of **protons** and **neutrons**.

Protons and neutrons are called **nucleons**.

A nucleus is characterized by:

- **A**: Mass Number = number of nucleons
- **Z**: Charge Number = number of protons ← Determines the element
- **N**: Neutron Number ← Determines the isotope

Of course A=Z+N

Usual notation:

Mass number A ← ^{12}C

↓

Element symbol – defined by charge number
C is Carbon and Z = 6

So this nucleus is made of 6 protons and 6 neutrons

8.1.1 - Nuclear physics

a. Nucleons

size: ~1 fm

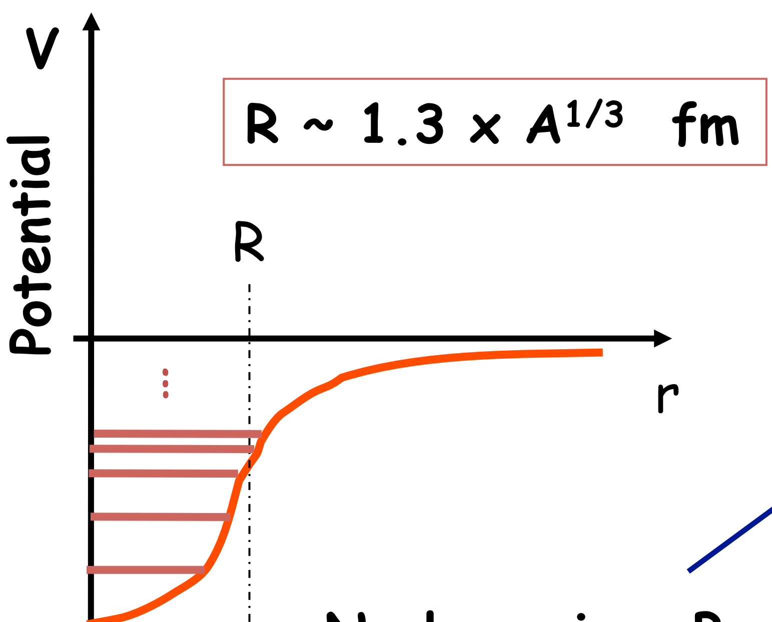
	Mass	Spin	Charge
Proton	938.272 MeV/c^2	1/2	+e
Neutron	939.565 MeV/c^2	1/2	0

b. Nuclei

nucleons attract each other via the strong force (range ~ 1 fm)

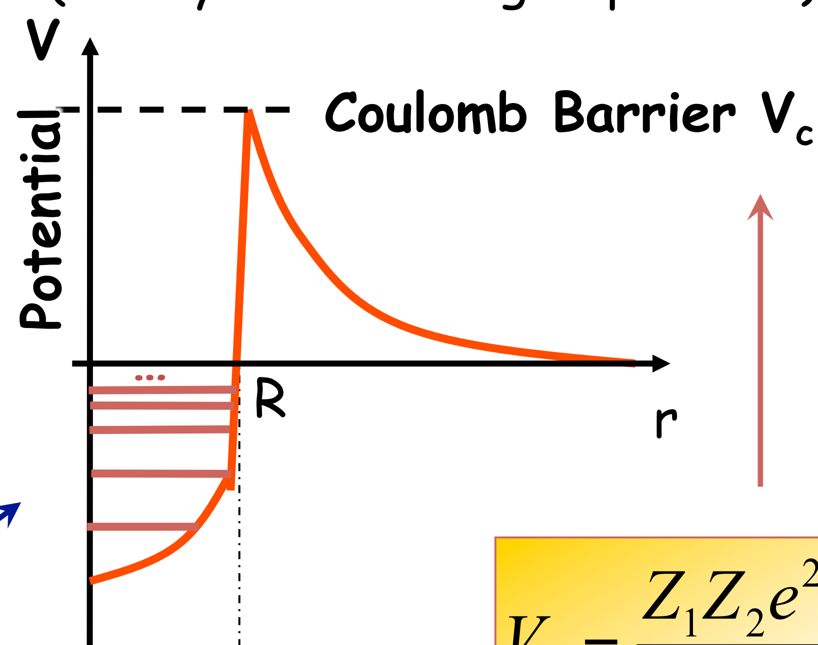
a bunch of nucleons bound together create a potential for an additional :

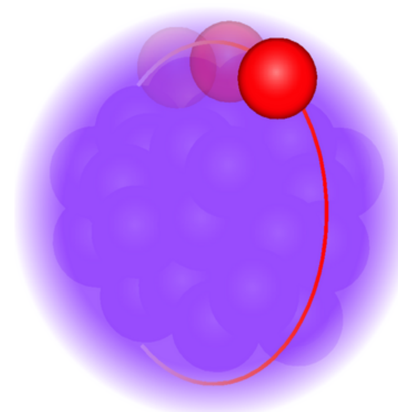
neutron



proton

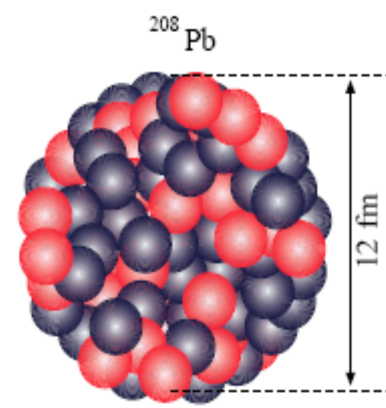
(or any other charged particle)



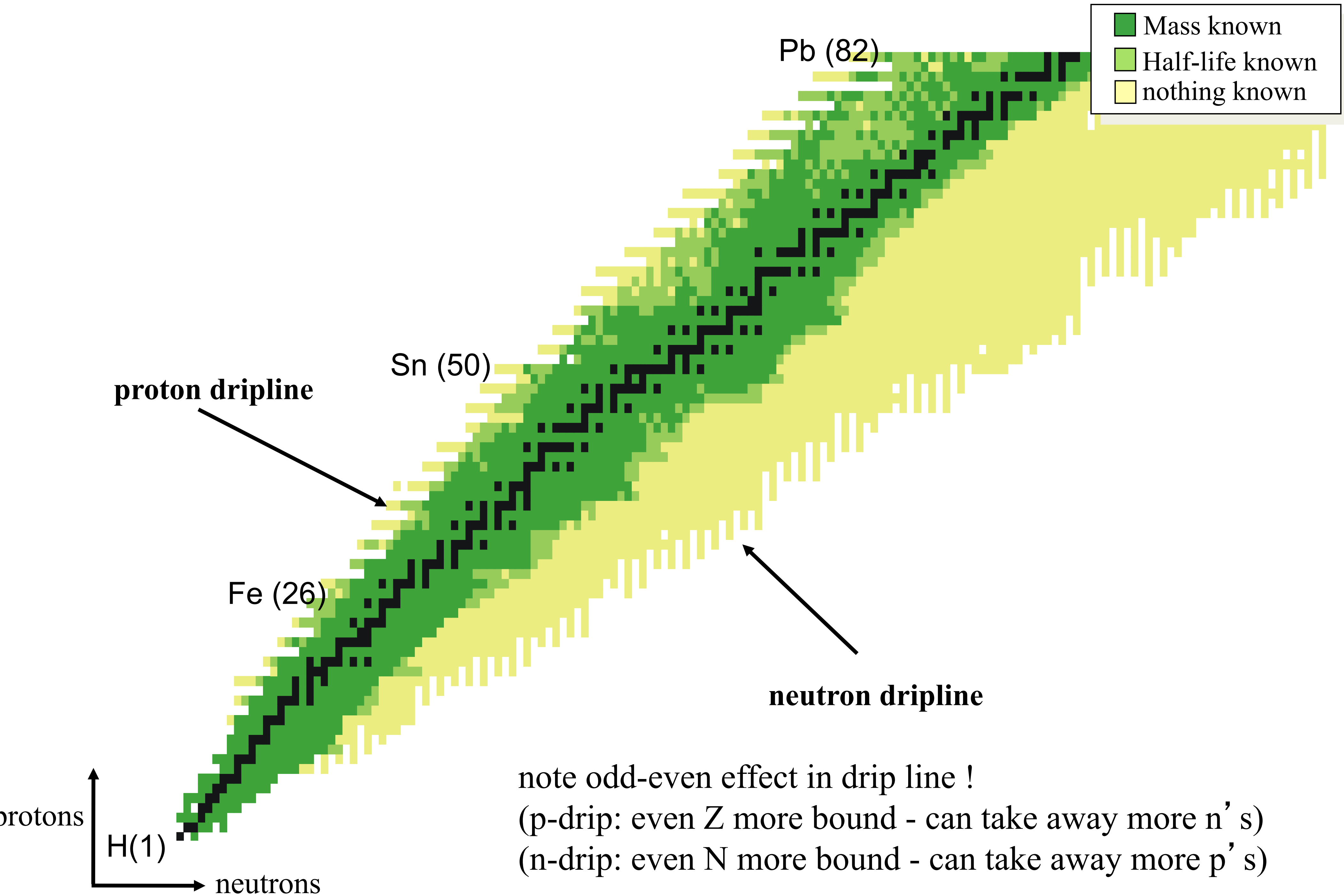


Nucleons in a Box:
Discrete energy levels
in nucleus

$$V_c = \frac{Z_1 Z_2 e^2}{R} \quad (8.1)$$



The Nuclear Chart



note odd-even effect in drip line !
(p-drip: even Z more bound - can take away more n's)
(n-drip: even N more bound - can take away more p's)

8.1.2 - Nuclear Masses and Binding Energies

Energy that is released when a nucleus is assembled from neutrons and protons

$$m(Z,N) = Zm_p + Nm_n - B/c^2 \qquad (8.2)$$

m_p = proton mass, m_n = neutron mass, m(Z,N) = mass of nucleus with Z,N

- B > 0
- With B the mass of the nucleus is determined.
- B is roughly ~ A

Masses are usually tabulated as atomic masses

$$m = m_{nuc} + Z\,m_e + B_e \qquad (8.3)$$

Nuclear Mass
~ 1 GeV/A

Electron Mass
511 keV/Z

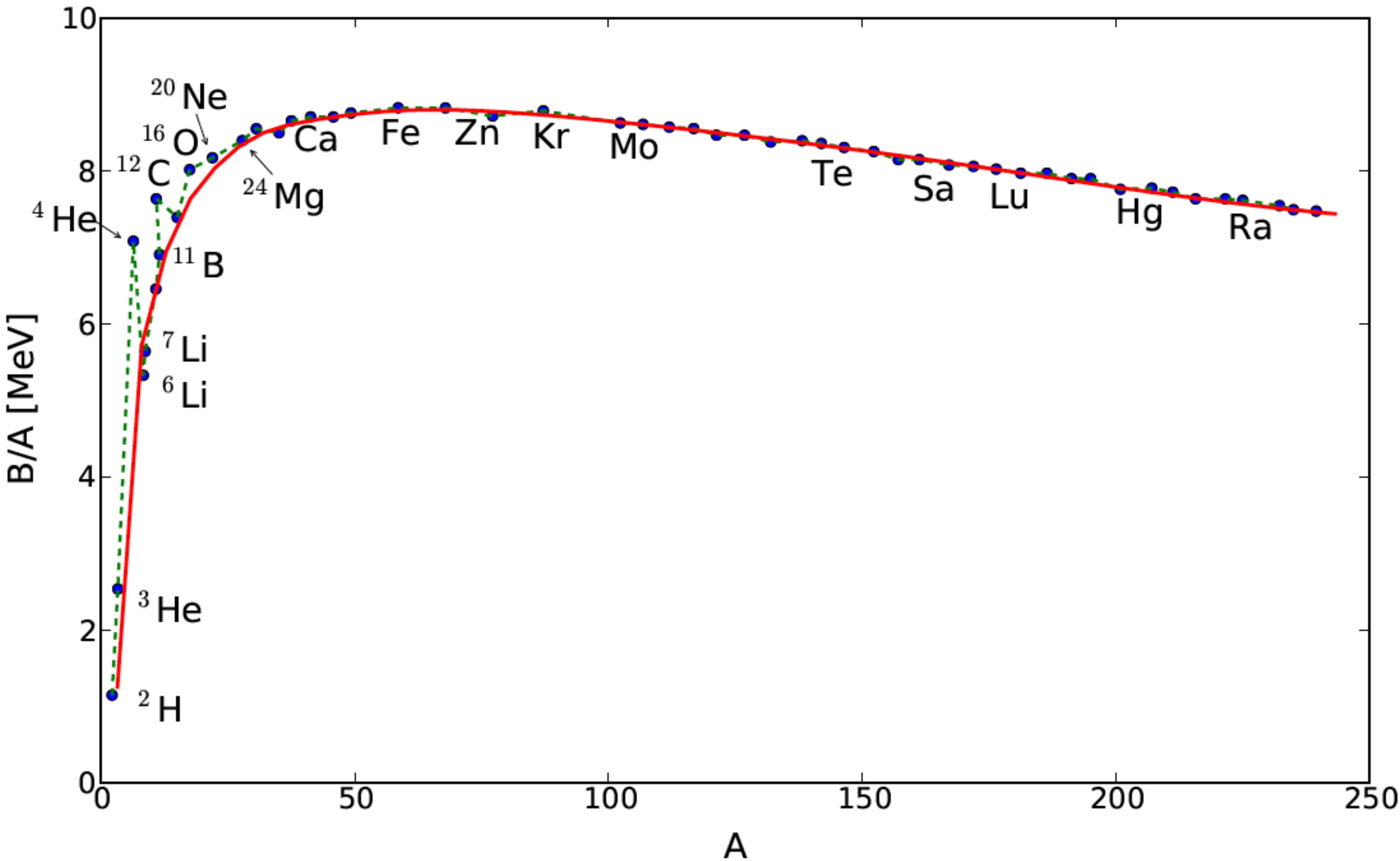
Electron Binding Energy
13.6 eV (H)
to 116 keV (K-shell U) / Z

Most tables give atomic mass excess Δ in MeV:

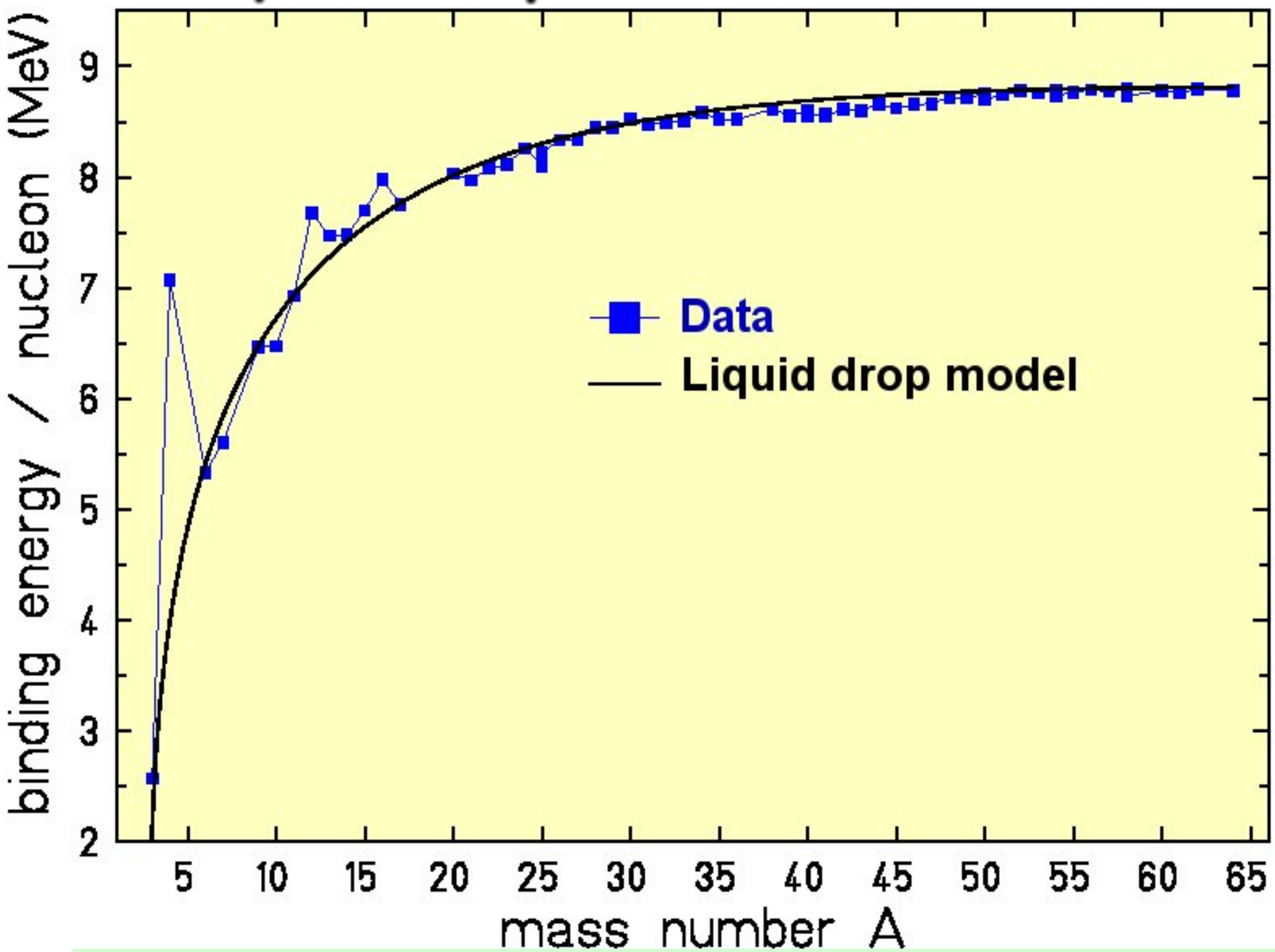
(definition: for ^{12}C: Δ = 0)

$$m = Am_u + \Delta/c^2 \qquad (8.4)$$

Nuclear Masses and Binding Energies



The Liquid Drop Model



Bethe-Weizsäcker formula

$$B(Z,N,A) = a_V A - a_{Surf} A^{2/3} - a_{sym} \frac{(Z-N)^2}{A} + a_{Coul} Z(Z-1) A^{-1/3} \quad (8.5)$$

$$a_V = 15.85\ \text{MeV}, \quad a_{Surf} = 18.34\ \text{MeV}, \quad a_{Symm} = 23.21\ \text{MeV}, \quad a_{Coul} = 0.71\ \text{MeV} \quad (8.6)$$

Understanding the Liquid Drop Model

Assumes incompressible fluid (volume ~ A) and sharp surface)

$$B(Z,A) = a_V A$$ **Volume Term** (8.7)

(8.8) $$- a_s A^{2/3}$$ **Surface Term** ~ surface area (Surface nucleons less bound)

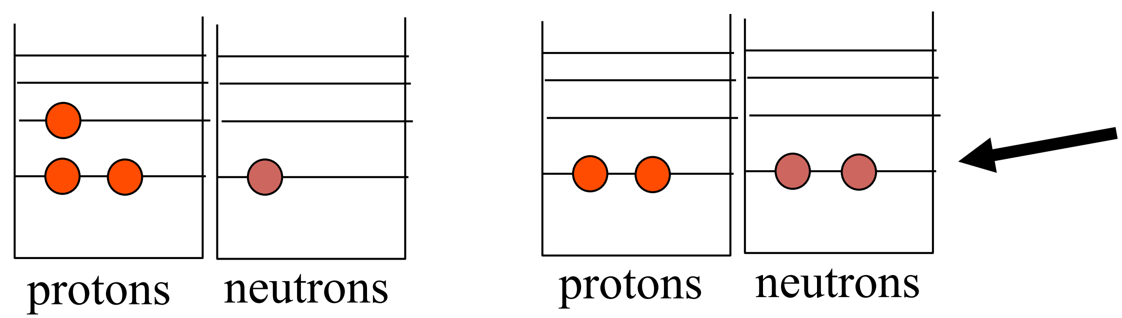
(8.9) $$- a_C \frac{Z^2}{A^{1/3}}$$ **Coulomb term**. Coulomb repulsion leads to reduction uniformly charged sphere has

$$E = \frac{3}{5}\frac{Q^2}{R}$$

$$- a_A \frac{(Z - A/2)^2}{A}$$

Asymmetry term: Pauli principle to protons: symmetric filling of p,n potential boxes has lowest energy (ignore Coulomb)

(8.10)



lower total energy = more bound

(8.11)

$$+ a_p A^{-1/2} \begin{cases} \times 1 & ee \\ \times 0 & oe, eo \\ \times (-1) & oo \end{cases}$$

Pairing term: even number of like nucleons favored

(e=even, o=odd referring to Z, N respectively)

Understanding the Liquid Drop Model

neglect asymmetry term (assume reasonable asymmetry)
neglect pairing and shell corrections (see later) - want to understand average behavior

then
$$B/A = a_V - a_S \frac{1}{A^{1/3}} - a_C \frac{Z^2}{A^{4/3}} \qquad (8.12)$$

const
as strong force
has short range

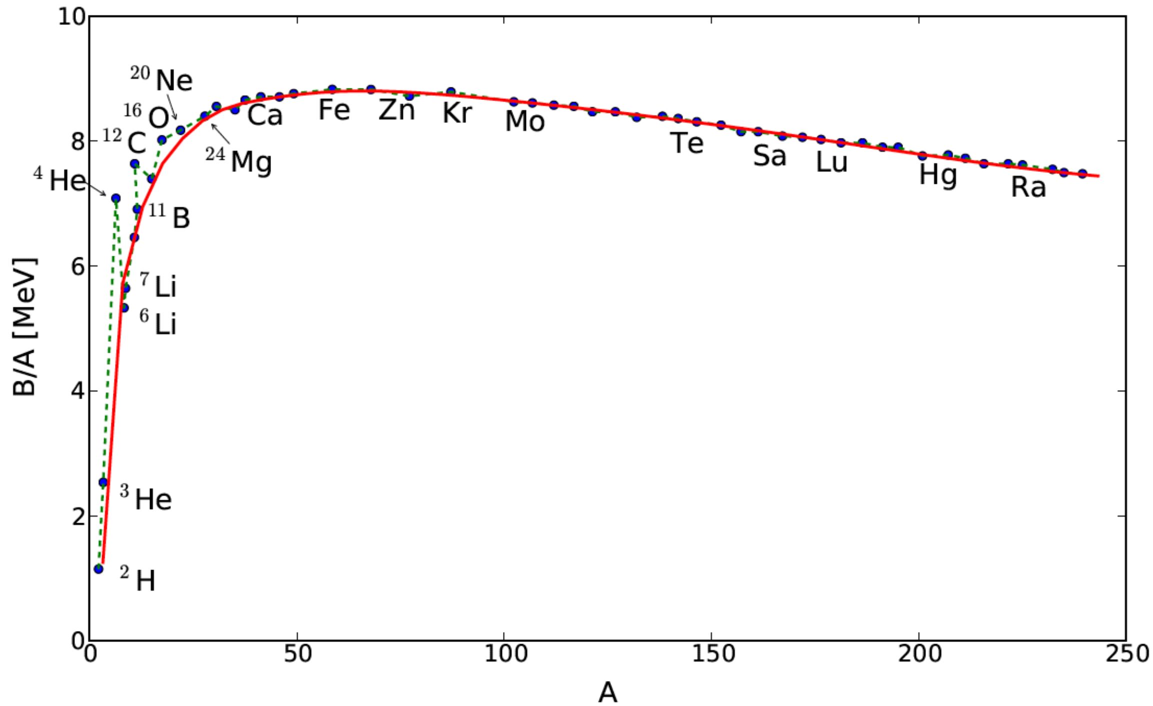
~surface/volume ratio
favors large nuclei

Coulomb repulsion has long range
- the more protons the more repulsion
favors small (low Z) nuclei

Maximum around ~ Fe

→ **Fusion** of light elements
release nuclear energy

→ **Fission** of very heavy elements
also release nuclear energy

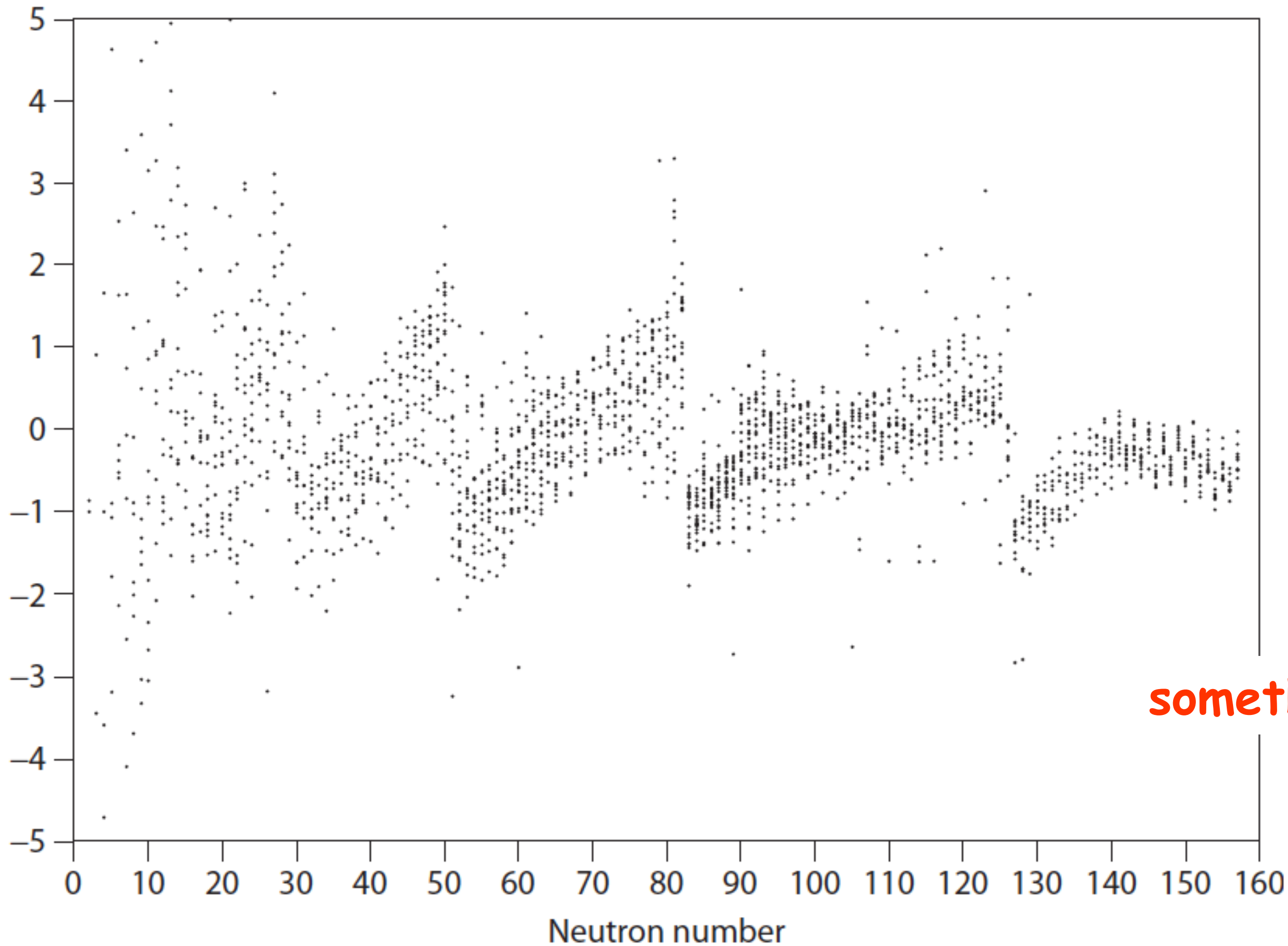


Fitting the Liquid Drop Model to Experimental Data

in MeV/c^2

a_V	a_S	a_C	a_A	a_P
15.85	18.34	0.71	92.86	11.46

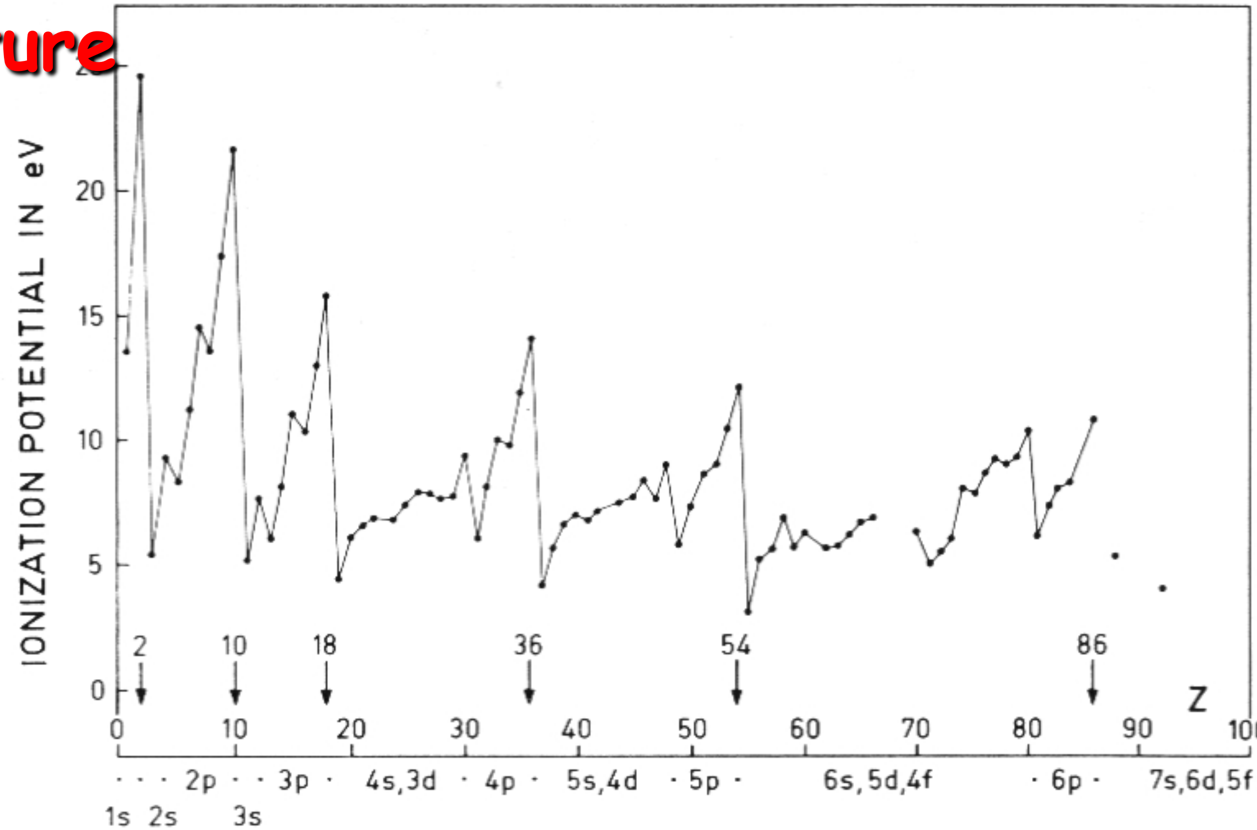
Deviation (in MeV) to experimental masses:



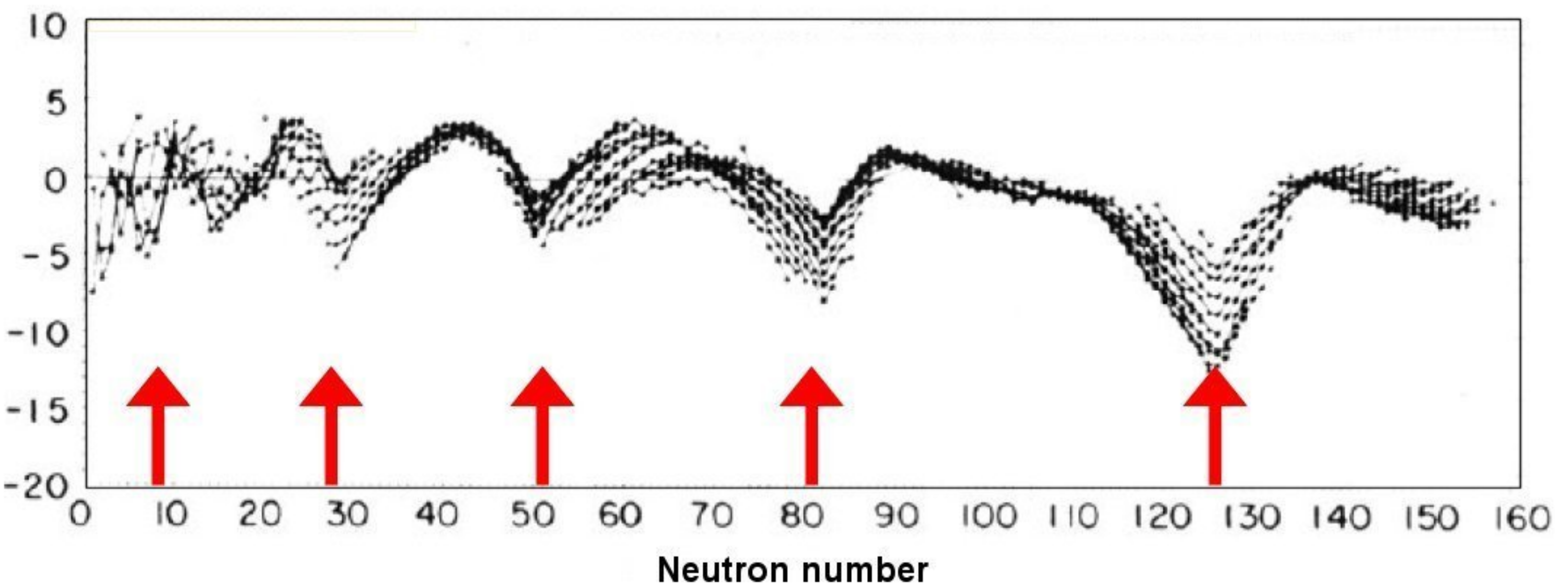
something is missing !

8.1.2 - Shell Structure

Atomic physics



$B_{exp} - B_{LDM}$

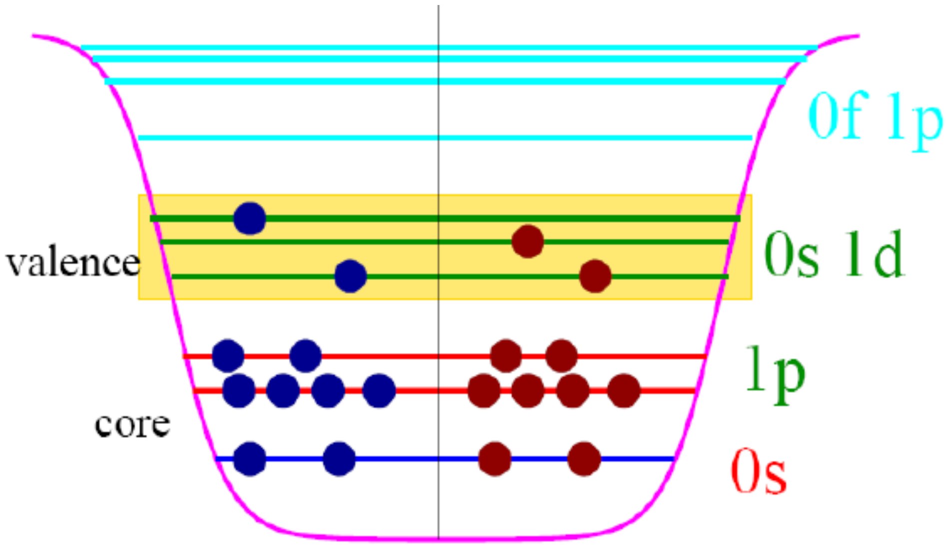


Nuclear physics

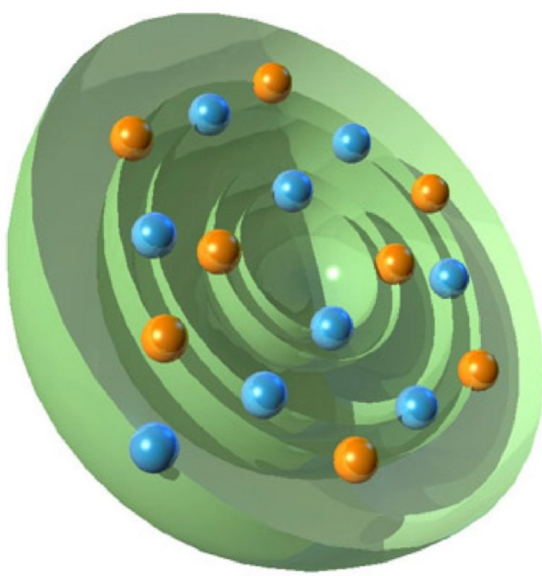
N_{magic} = **2, 8, 20, 28, 50, 82, 126**

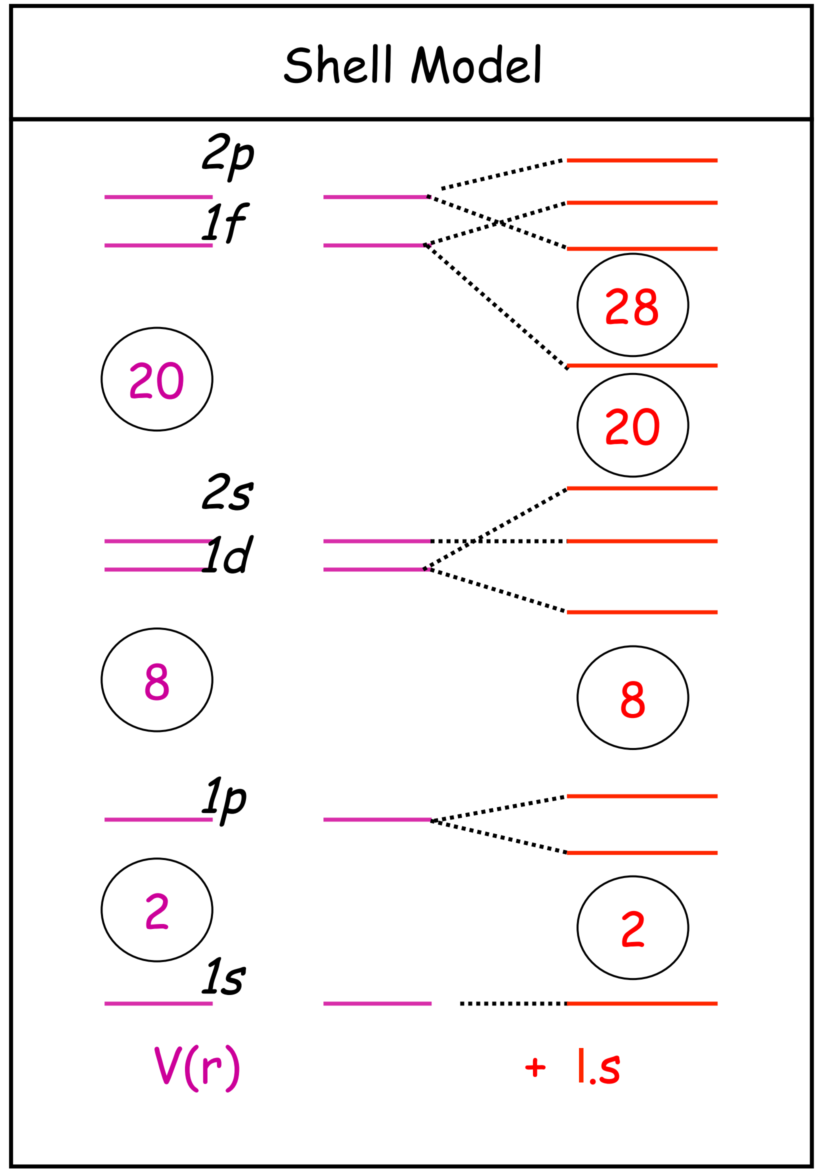
Shell Structure Model

$$H = \sum_{k=1}^{A} \left[\frac{p^2}{2m} + V(r) + V_{SO}(r)\, \boldsymbol{l} \cdot \boldsymbol{s} \right] \quad (8.13)$$



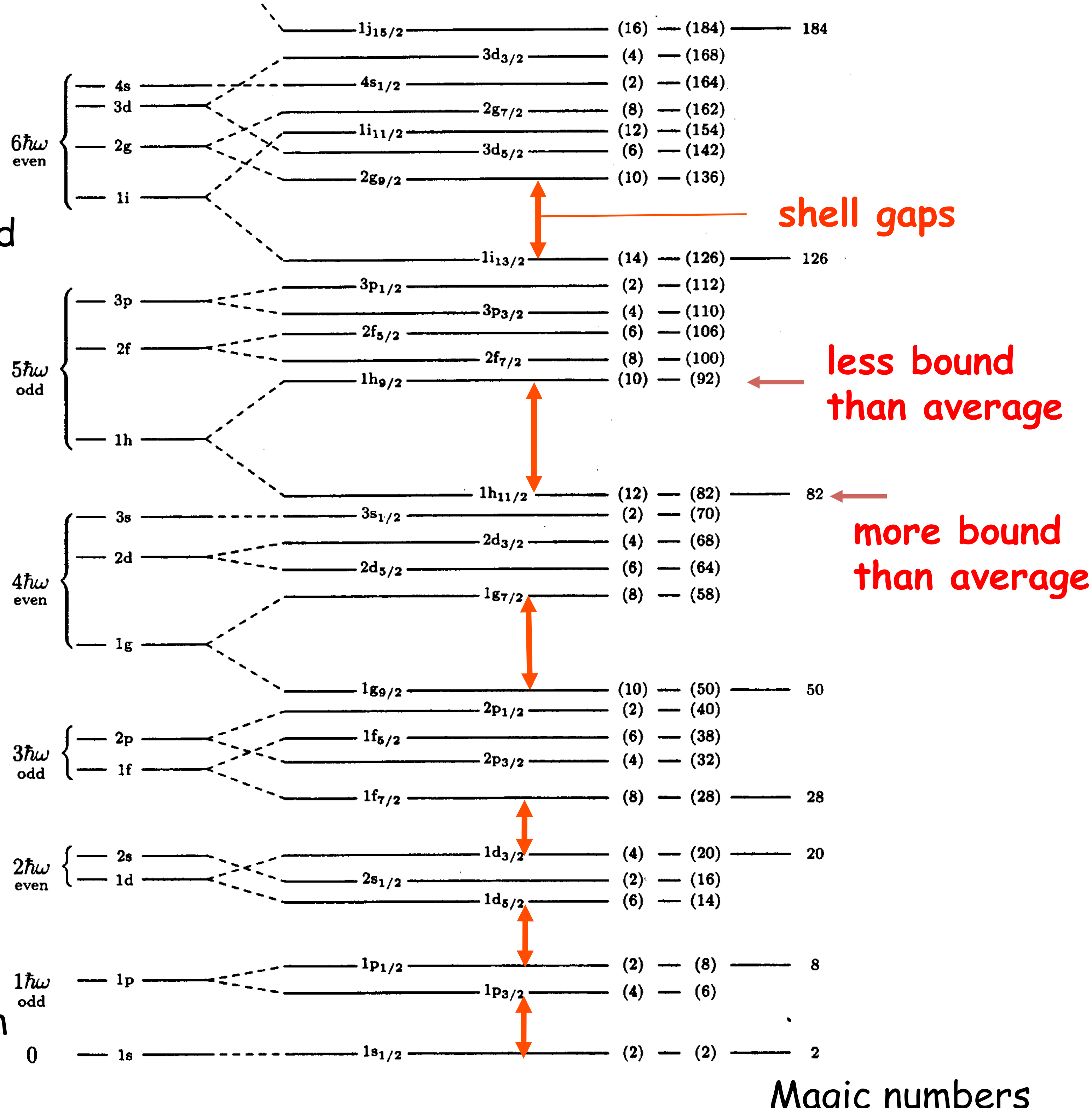
Mayer, Jensen, Nobel Prize, 1963





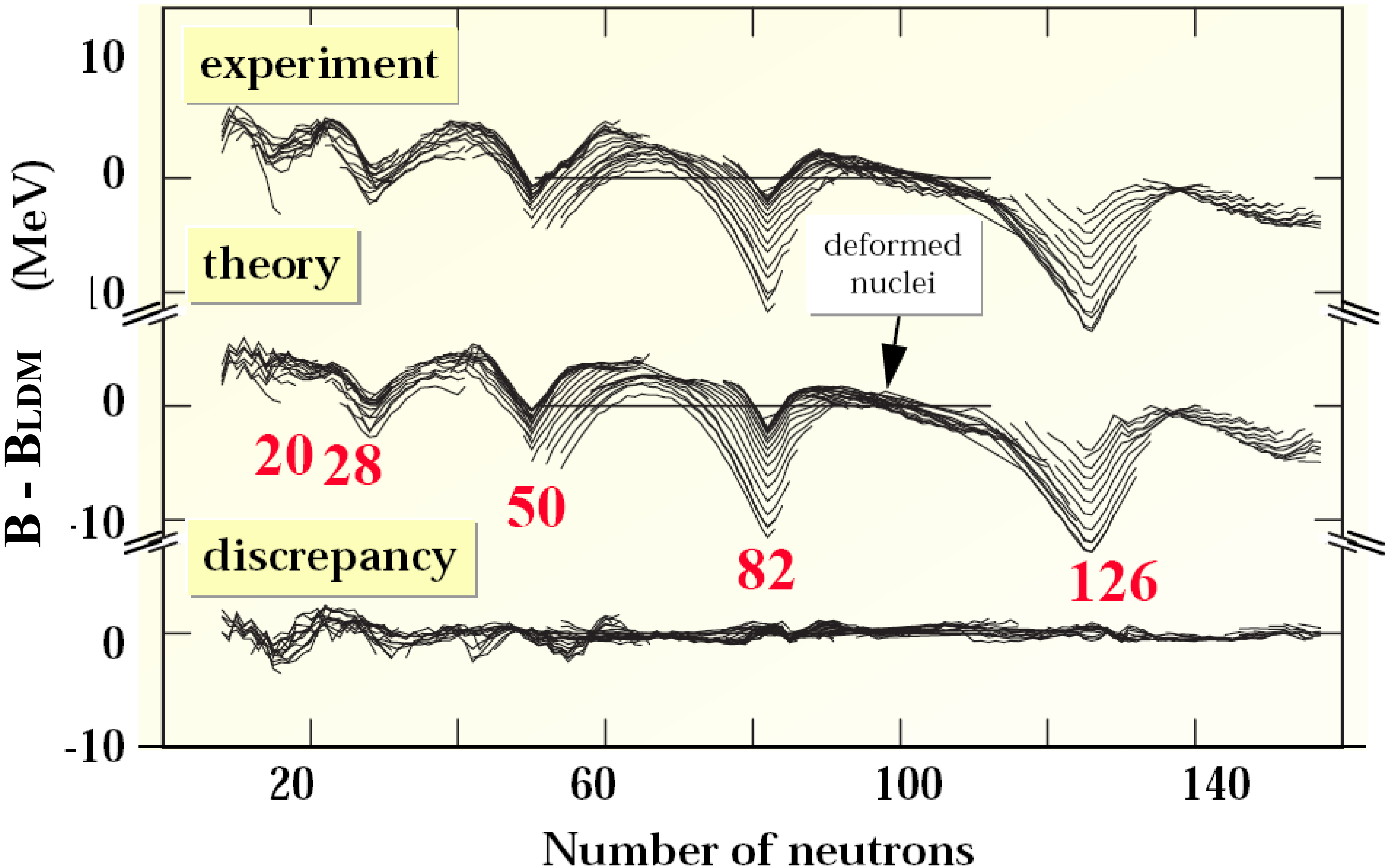
Shell model: (single nucleon energy levels)

are not evenly spaced



need to add shell correction term S(Z,N)

Liquid Drop Formula corrected with shell model



8.2 - Nuclear decay

a) Energy generation

nuclear reaction: $A + B \longrightarrow C$

if $m_A + m_B > m_C$ then energy $Q = (m_A + m_B - m_C)c^2$ is generated by reaction

(7.14)

"Q-value" **Q** = Energy generated (>0) or consumed (<0) by reaction

b) Stability

if there is a reaction $A \longrightarrow B + C$ with $Q > 0$ (or $m_A > m_B + m_C$)

then decay of nucleus A is energetically possible.

nucleus A <u>might</u> then not exist (at least not for a very long time)

Nuclear decay

Decay of A in B and C is possible if reaction $A \longrightarrow B + C$ has positive Q-value

BUT: there might be a barrier that prolongs the lifetime

Decay is described by quantum mechanics and is a pure random process, with a constant probability for the decay to happen in a given time interval.

N : Number of nuclei A (parent)
λ : decay rate (decays per second and parent nucleus)

$$dN = -\lambda N dt \quad (8.15)$$

therefore

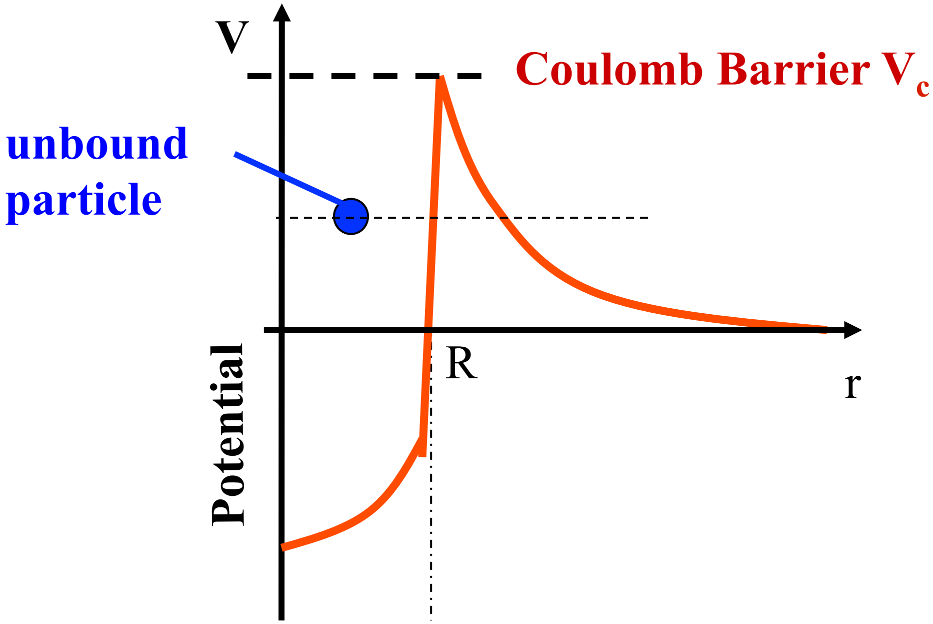
$$N(t) = N(t=0)\, e^{-\lambda t} \quad (8.16)$$

lifetime $\tau = 1/\lambda$

half-life $T_{1/2} = \tau \ln 2 = \ln 2/\lambda$ is time for half of the nuclei present to decay

Decay modes

for anything other than a neutron (or a neutrino) emitted from the nucleus there is a Coulomb barrier



$$V_c = \frac{Z_1 Z_2 e^2}{R}$$

If that barrier delays the decay beyond the lifetime of the universe (~ 14 Gyr) we consider the nucleus as being stable.

Example: for ^{197}Au → ^{58}Fe + ^{139}I has Q ~ 100 MeV ! yet, gold is stable.

→ **not all decays that are energetically possible happen**

most common:

- β decay
- n decay
- p decay
- α decay
- fission

8.2.1 - β decay

p ⟷ n conversion within a nucleus via weak interaction

Modes (for a proton/neutron in a nucleus):

β^+ decay	$p \longrightarrow n + e^+ + \nu_e$	
electron capture	$e^- + p \longrightarrow n + \nu_e$	(8.17)
β^- decay	$n \longrightarrow p + e^- + \overline{\nu}_e$	

Electron capture (or EC) of atomic electrons or, in astrophysics, of electrons in the surrounding plasma

Q-values for decay of nucleus (Z,N)

with nuclear masses **with atomic masses**

$$Q_{\beta+} / c^2 = m_{nuc}(Z,N) - m_{nuc}(Z-1,N+1) - m_e = \mathbf{m(Z,N) - m(Z-1,N+1) - 2m_e}$$

$$Q_{EC} / c^2 = m_{nuc}(Z,N) - m_{nuc}(Z-1,N+1) + m_e = \mathbf{m(Z,N) - m(Z-1,N+1)} \qquad (8.18)$$

$$Q_{\beta-} / c^2 = m_{nuc}(Z,N) - m_{nuc}(Z+1,N-1) - m_e = \mathbf{m(Z,N) - m(Z+1,N-1)}$$

Note: $Q_{EC} > Q_{\beta+}$ by 1.022 MeV

Q-values with Δ values

$$m = Am_u + \Delta / c^2 \qquad (8.19)$$

Q-values for reactions that conserve the number of nucleons can also be calculated directly using the tabulated Δ values instead of the masses

Example: $^{14}C \rightarrow {}^{14}N + e + \nu_e$

$$\begin{aligned} Q / c^2 &= m_{14C} - m_{14N} \\ &= 14m_u + \Delta_{14C} - 14m_u - \Delta_{14N} \\ &= \Delta_{14C} - \Delta_{14N} \quad \text{(for atomic } \Delta\text{'s)} \end{aligned} \qquad (8.20)$$

Q-values with binding energies B

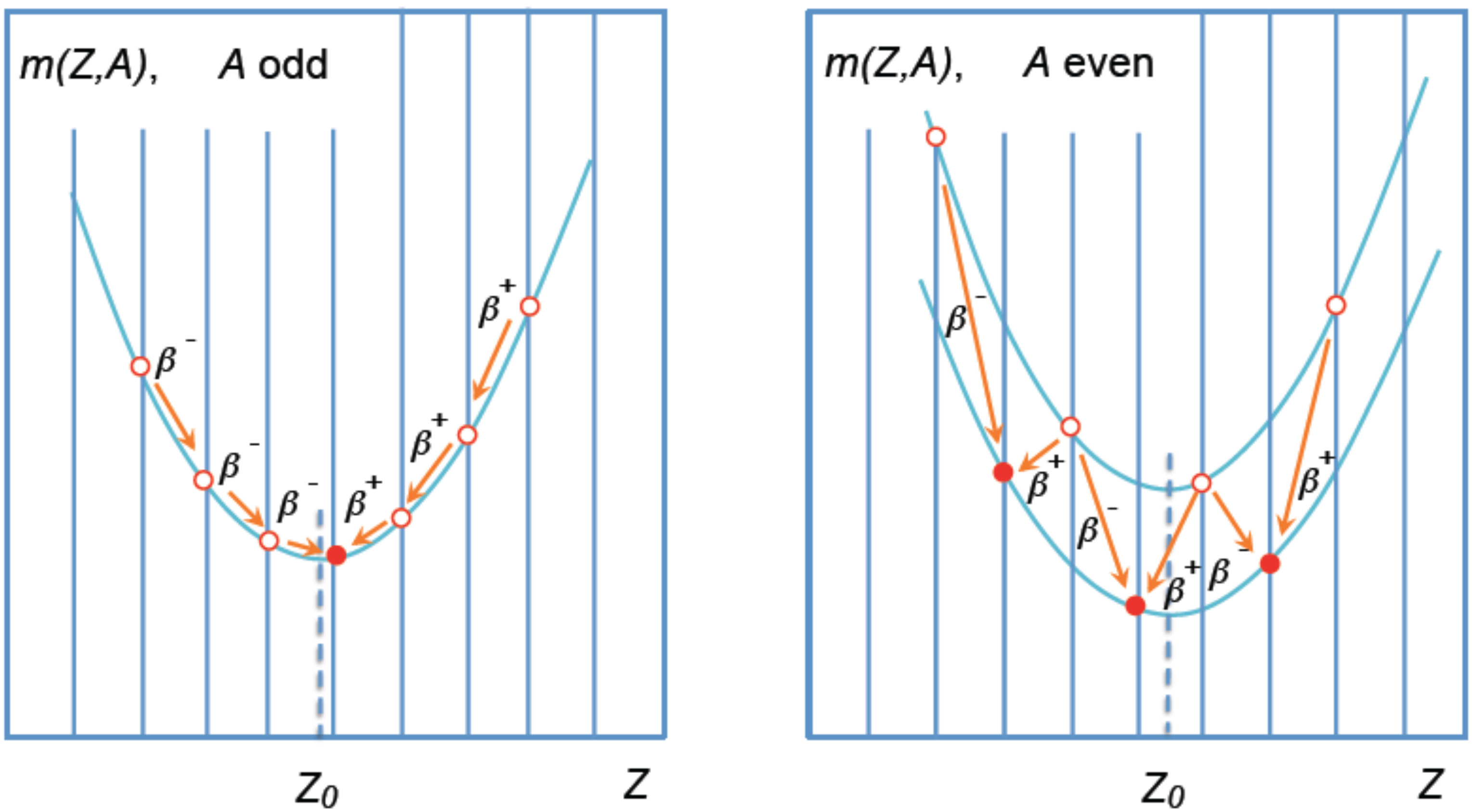
$$m(Z,N) = Zm_p + Nm_n - Bc^2 \qquad (8.21)$$

Q-values for reactions that conserve proton number and neutron number can be calculated using -B instead of the masses

β decay

β decay basically no barrier → if energetically possible it usually happens (except if another decay mode dominates)

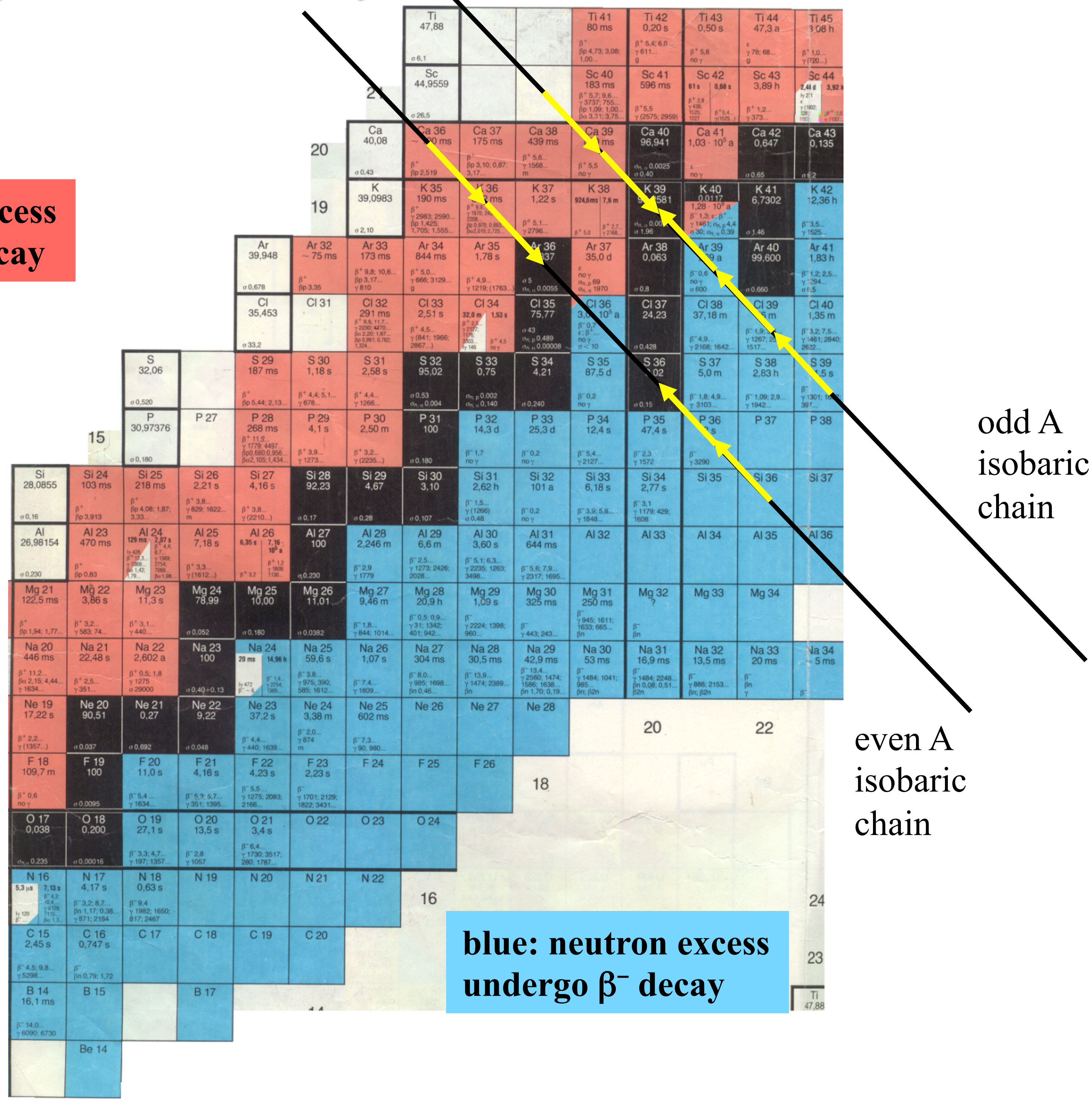
therefore: any nucleus with a given mass number *A* will be converted into the most stable proton/neutron combination with mass number *A* by b decays



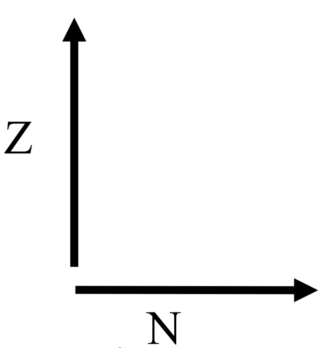
valley of stability

Typical part of the chart of nuclides

red: proton excess
undergo β^+ decay



blue: neutron excess
undergo β^- decay



Typical β-decay half-lives

- very near “stability” : occasionally Millions of years or longer
- more common within a few nuclei of stability: minutes - days
- most exotic nuclei that can be formed: ~ milliseconds