Lecture 4

The quest for the origin of the elements
4.1 - The Early Universe

According to the accepted cosmological theories:

• The Universe has cooled during its expansion as

$$T(t) \sim \frac{1}{a(t)} \quad (4.1)$$

• In terms of the time evolved from the Big Bang (for a radiation dominated universe $a \sim t^{1/2}$)

$$T(t) \sim \frac{1.3 \times 10^{10} \text{ K}}{t^{1/2} \text{ [s]}} \quad (4.2)$$

• The particles are in thermal equilibrium. This is guaranteed by reactions which are faster than the expansion rate $(da/dt)/a$. In this case, particles and antiparticles are in equilibrium through annihilation, i.e. particles + antiparticles $\leftrightarrow$ photons

• But when $k_B T \ll mc^2$, particles and antiparticles (with mass $m$) annihilate and photons cannot create them back as their energy is below the threshold.

• At $T = 10^{12} \text{ K}$, there was a slight overabundance of matter over antimatter which lead to the violation of baryon/lepton number conservation.

• Also at $T = 10^{12} \text{ K}$, antinucleons have annihilated with nucleons and the remaining nucleons become the breeding material for primordial nucleosynthesis, or Big Bang Nucleosynthesis (BBN).
The Early Universe

At one-hundredth of seconds of the Universe consisted of an approximately equal number of electrons, positrons, neutrinos and photons, and a small amount of protons and neutrons; the ratio of protons to photons is assumed to have been about $10^{-9}$. The energy density of photons can be calculated from

$$\rho_\gamma = \int E_\gamma \, dn_\gamma$$

where the density of states is given by

$$dn_\gamma = \frac{g_\gamma}{2\pi^2} \frac{K_\gamma^2}{\exp\left(\frac{E_\gamma}{kT}\right) - 1} \, dK_\gamma$$

and $g_\gamma = 2$ is the number of spin polarizations 1 for the photon while $E_\gamma = \hbar K_\gamma c$ is the photon energy (momentum).

Performing the integration gives

$$\rho_\gamma = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T^4$$

which is the familiar blackbody result.
At large temperatures, when $k_B T \gg m_i$, the mass $m_i$ of the particles (electrons, neutrinos, nucleons) are irrelevant and the energy density associated with these particles can also be described by the black-body formula. A straightforward calculation for the density, and number density accounting for all particle degrees of freedom $g_i$ yields

$$\rho_i = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T^4 \times \begin{cases} 7/8 & \text{for fermions} \\ 1 & \text{for bosons} \end{cases}$$

$$n_i = \frac{\zeta(3) k_B^3}{\pi^2 \hbar^3 c^3} T^3 \times \begin{cases} 3/4 & \text{for fermions} \\ 1 & \text{for bosons} \end{cases}$$

as well as the known relation between density and pressure for matter

$$p_i = \frac{\rho_i}{3}$$

In Eq. (4.7) $\zeta(3) = 1.202$ is the Riemann zeta function. The difference for fermions and bosons is because of their different statistical distribution functions.
The Early Universe

As we have seen in Eqs. (2.13) and (2.14), the density of scales with time as \( \rho \sim 1/t^2 \), which together with Eq. (4.6) yields

\[
t \sim \frac{\text{const.}}{T^2} \quad (4.9)
\]

with the precise expression being

\[
t = \left( \frac{90 \hbar^3 c^3}{32 \pi^3 G g^*} \right)^{1/2} \frac{1}{k_B T^2} \quad (4.10)
\]

where \( g^* \) is total the number of degrees of freedom of the particles. Thus, the relation between time and temperature in the early universe depends strongly on the kind of particles present in the plasma. Calculations of time and temperature in the early universe are shown below.

<table>
<thead>
<tr>
<th>T(K)</th>
<th>a/a₀</th>
<th>t(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>~10 MeV</td>
<td>10^{11}</td>
<td>1.9 \times 10^{-11}</td>
</tr>
<tr>
<td>~1 MeV</td>
<td>10^{10}</td>
<td>1.9 \times 10^{-10}</td>
</tr>
<tr>
<td>~100 keV</td>
<td>10^{9}</td>
<td>2.6 \times 10^{-9}</td>
</tr>
<tr>
<td>~10 keV</td>
<td>10^{8}</td>
<td>2.7 \times 10^{-8}</td>
</tr>
</tbody>
</table>

For a Universe dominated by relativistic, or radiation-like, particles \( \rho \propto 1/a^4 \) and the above expressions yield

\[
a \sim T^{-1} \sim t^{1/2}.
\]

The quest for the origins of the elements, C. Bertulani, RISP, 2016
At \( t \sim 0.01 \text{ s} \), the temperature is \( T \sim 10^{11} \text{ K} \), and \( k_B T \sim 10 \text{ MeV} \), which is much larger than the electron mass. Neutrinos, electrons and positrons are easily produced and destroyed by means of weak interactions (i.e., interactions involving neutrinos)

\[
\begin{align*}
\text{i) } & \quad n + \nu_e \leftrightarrow p + e^- \\
\text{ii) } & \quad n + e^+ \leftrightarrow p + \bar{\nu}_e \\
\text{iii) } & \quad n \leftrightarrow p + e^- + \bar{\nu}_e
\end{align*}
\]

(4.11)

As long as the weak reactions are fast enough, the neutron-to-proton ratio is given by

\[
[n / p] = \frac{\text{number of neutrons}}{\text{number of protons}} = \frac{N_n(T)}{N_p(T)} = \exp \left[ -\frac{\Delta mc^2}{k_B T} \right]
\]

(4.12)

where \( m(n) = 939.5 \text{ MeV} \), \( m(p) = 938.3 \text{ MeV} \), and \( \Delta m = 1.294 \text{ MeV} \).

At \( T = 10^{11} \text{ K} \), \( k_B T = 8.62 \text{ MeV} \), yielding \( n/p = 0.86 \).

This temperature is far above the temperature of nucleosynthesis, but the \( n/p \) ratio already begins to drop.
4.1.2 - Time $\sim 0.1$ sec

At $t \sim 0.1$ s, the temperature is $T \sim 3 \times 10^{10}$ K, and $k_B T \sim 2.6$ MeV. Neutrinos, electrons and positrons are still in equilibrium according to Eq. (4.11). The lifetime for destruction of a neutron by means of these reactions can be calculated from

$$\lambda(T) = n_\nu \langle \sigma v \rangle_\nu$$

(4.13)

This is not an easy calculation, as it requires the knowledge of $\sigma_v$ for neutrino induced interactions. A detailed calculation yields

$$\lambda(T) = \frac{0.76}{\text{sec}} \left( \frac{k_B T}{\text{MeV}} \right)^5$$

(4.14)

At $T \sim 3 \times 10^{10}$ K, this yields a neutron destruction lifetime of $0.01$ sec. Thus, the weak rates drop very fast, as $T^5$. At some $T$ the weak rates are so slow that they cannot keep up with Universe expansion rate. The Hubble rate at this epoch is found to be

$$H(T) = \frac{0.67}{\text{sec}} \left( \frac{k_B T}{\text{MeV}} \right)^2$$

(4.15)
4.1.3 - Decoupling

When the Hubble expansion rate is equal to the neutron destruction rate, i.e., when Eqs. (4.14) and (4.15) are equal, we find $k_B T \sim 1$ MeV.

As temperature and density decreases beyond this point, the neutrinos start behaving like free particles. Below $10^{10}$ K they cease to play any major role in the reactions. That is, they decouple and matter becomes transparent to the neutrinos.

At $k_B T \sim 1$ MeV (twice the electron mass), the photons also stop producing positron/electron pairs

$$\gamma \leftrightarrow e^+ + e^- \quad (4.16)$$

The $e^- e^+$-pairs begin to annihilate each other, leaving a small excess of electrons. However, the thermal energies are still high enough to destroy any formed nuclei. At this point

$$[n/p] = \exp \left[ -\frac{\Delta mc^2}{kT} \right] \sim 0.25 \quad (4.17)$$

As the temperature drops, neutrino-induced reactions continue creating more protons. In the next 10 secs the $n/p$ ratio will drop to about $0.17 \sim 1/6$. And after that the neutron percentage continues to decrease because of neutron beta-decay. When nucleosynthesis starts the $n/p$ ratio is $1/7$. 

The quest for the origins of the elements, C. Bertulani, RISP, 2016
4.1.4 - Baryon to photon ratio

Neutrons and protons were created at an earlier stage of the universe, when quarks and gluons combined to form them. As with the other particles, very energetic photons produced baryons (nucleons) and antibaryon pairs and were also produced by the inverse reactions. As the temperature decreased, baryons annihilated each other and more photons were created. One does not know why a small number of baryons remained. The resulting baryon/photon ratio at this time was

$$\frac{\rho_b}{\rho_\gamma} = \eta \sim 10^{-9} \quad (4.18)$$

Usually $\eta$ is considered the only parameter of BBN. However, BBN is also sensitive to two other parameters: the neutron lifetime and the number of light neutrino families.

(a) neutron lifetime $\tau_{1/2}(n)$ - An increase in $\tau_{1/2}(n)$ theoretically implies a decrease of all weak rates which convert protons and neutrons. The freeze-out would happen at a higher temperature and would lead to a larger $n/p$ ratio. As all neutrons essentially end up in $^4$He, the $^4$He abundance grows if $\tau_{1/2}(n)$ is increased.

(b) number of neutrino families - The energy density of the early Universe depends on the number of neutrino families ($n_\nu$): the larger $n_\nu$, the larger and the faster the expansion rate of the Universe $H = (da/dt)/a$. An increase in $H$ leads to an earlier freeze-out and hence more $^4$He abundance.
Neutron-to-proton (n/p) ratio as a function of time and temperature. The dashed curve is given by $\exp\left(-\frac{\Delta m}{k_B T}\right)$. The dotted curve is the free-neutron decay curve, $\exp\left(-\frac{t}{\tau_n}\right)$. The solid curve indicates the resulting n/p ratio as a combination of the two processes. BBN starts at $t \sim 4$ min.
4.2 - Big Bang Nucleosynthesis (BBN)

A summary of the BBN when the temperature of the universe allowed deuteron to be formed without being immediately destroyed by photons is:

1. The light elements (deuterium, helium, and lithium) were produced in the first few minutes after the Big Bang.

2. Elements heavier than $^4$He were produced in the stars and through supernovae explosions.

3. Helium and deuterium produced in stars do not match observation because stars destroy deuterium in their cores.

4. Therefore, all the observed deuterium was produced around three minutes after the big bang, when $T \sim 10^9$ K.

5. A simple calculation based on the n/p ratio shows that BBN predicts that 25% of the matter in the Universe should be helium.

6. More detailed BBN calculations predict that about 0.001% should be deuterium.
**The deuteron bottleneck**

As the temperature of the Universe decreased, neutrons and protons started to interact and fuse to a deuteron

\[ n + p \rightarrow d + \gamma \quad (4.19) \]

The binding energy of deuterons is small \( E_B = 2.23 \text{ MeV} \). The baryon-to-photon ratio, called \( \eta \), at this time is also very small \( (< 10^{-9}) \). As a consequence, there are many high-energy photons to dissociate the formed deuterons, as soon as they are produced.

The temperature at the start of nucleosynthesis is about \( 100 \text{ keV} \), when we would have expected \( \sim 2 \text{ MeV} \), the binding energy of deuterium. The reason is the very small value of \( \eta \). The BBN temperature, \( \sim 100 \text{ keV} \), corresponds according to Eq. (4.10) to timescales less than about 200 sec. The cross-section and reaction rate for the reaction in Eq. (4.19) is

\[ \sigma v \sim 5 \times 10^{-20} \text{ cm}^3 / \text{sec} \quad (4.20) \]

So, in order to achieve appreciable deuteron production rate we need \( \rho \sim 10^{-17} \text{ cm}^{-3} \). The density of baryons today is known approximately from the density of visible matter to be \( \rho_0 \sim 10^{-7} \text{ cm}^{-3} \) and since we know that the density \( \rho \) scales as \( a^{-3} \sim T^3 \), the temperature today must be \( T_0 = (\rho_0/\rho)^{1/3} T_{BBN} \sim 10 \text{ K} \), which is a good estimate of the CMB.
The deuteron bottleneck

The bottleneck implies that there would be no significant abundance of deuterons before the Universe cooled to about $10^9$ K.

Other important facts are:

1 - The nucleon composition during BBN was proton-rich.

2 - The most tightly bound light nucleus is $^4$He.

3 - There is no stable nucleus with mass numbers $A = 5$ and $A = 8$.

4 - The early universe was hot but not dense enough to overcome the Coulomb barriers to produce heavier nuclides.

5 - The BBN network is active until all neutrons are bound in $^4$He. As the BBN mass fraction of neutrons was $X_n = N_n / (N_n + N_p) = 1/8$, it follows that the mass fraction of $^4$He after BBN is about $X_{^4\text{He}} = 2X_n = 25\%$. 

BBN Predictions - The Helium abundance

BBN predicts that when the universe had $T = 10^9$ K (1 minute old), protons outnumbered neutrons by 7:1. When $^2$H and He nuclei formed, most of the neutrons formed He nuclei. That is, one expects 1 He nucleus for every 12 H nuclei, or 75% H and 25% He. This is the fraction of He and $^2$H we observe today.

The quest for the origins of the elements, C. Bertulani, RISP, 2016
Standard Big Bang Nucleosynthesis (BBN)

\[ \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}} \left( \rho_\gamma + \rho_{e^\pm} + \rho_b + \rho_\nu \right) \equiv H \]  

\[ \dot{\rho} = -3H(\rho + p) \]  

\[ \mu_e : \text{electron chemical potential} \]

\[ n_b \sum_j Z_j X_j = n_{e^-} - n_{e^+} \equiv \Phi \left( \frac{m_e}{T}, \mu_e \right) \]

\[ \dot{X}_i = \sum_{j,k,l} N_i \left( \Gamma_{kl\rightarrow ij} \frac{{(X_1)}^{N_l}}{N_l !} \frac{{(X_k)}^{N_k}}{N_k !} - \Gamma_{ij\rightarrow kl} \frac{{(X_i)}^{N_i}}{N_i !} \frac{{(X_j)}^{N_j}}{N_j !} \right) \equiv \Gamma_i \]

- \( \alpha \): scale factor
- \( \rho \): energy density of relativistic species (m < 1 MeV)
- \( \mu_e \): electron chemical potential
- \( \Phi \): nuclear physics
4.2.1 - The BBN reaction network

After deuterons are produced at $T \sim 10^9$ K, a successive chain of nuclear reactions occur. The most important are:

1. $n \rightarrow p$
2. $n(p,\gamma)d$
3. $d(p,\gamma)^3He$
4. $d(d,n)^3He$
5. $d(d,p)^3H$
6. $^3H(d,n)^4He$
7. $^4He(^3H,\gamma)^7Li$
8. $^3He(n,p)^3H$
9. $^3He(d,p)^4He$
10. $^4He(^3He,\gamma)^7Be$
11. $^7Li(p,^4He)^4He$
12. $^7Be(n,p)^7Li$

Except for the $^7Be$ electron capture, all reactions are fast. The binding energies of $^3He$, $^3H$, $^4He$ are significantly larger than the one of deuterons. Thus these nuclei are not dissociated again.

At $T \sim 10^8$ K BBN terminates because:

- the temperature and density are too low
- the Coulomb barriers too high

The quest for the origins of the elements, C. Bertulani, RISP, 2016
4.2.2 - BBN Nuclei in Stars

Deuteron

- In stellar processes deuteron is quickly converted to $^3\text{He}$
- Astronomers look at quasars: bright atomic nuclei of active galaxies, ten billion light years away.

$^3\text{He}$

- Stars account for only 0.1% of all He.
- The $^3\text{He}$ abundance in stars is difficult to deduce. Its abundance is increasing in stellar fusion.
- Scientists look to our own galaxy.

$^7\text{Li}$

- $^7\text{Li}$ can form when “cosmic rays” collide with interstellar gas.
- Observations can be made on old, cool stars in our own galaxy.
- $^7\text{Li}$ is destroyed more than it is created inside of stars.
- Very old stars have low oxygen content, and their outermost layers still contain mostly primordial $^7\text{Li}$.
4.2.3 - Experimental S-factors for BBN reactions
4.2.4 - Time-evolution of BBN - Mass fractions

Mass fractions of light nuclei as a function of time during the BBN.

The quest for the origins of the elements, C. Bertulani, RISP, 2016
Primordial abundances

NOTE: Light elements have been made and destroyed since the Big Bang.

- Some are made in:
  - stars ($^3$He, $^4$He),
  - spallation (scattering) ($^6$, $^7$Li, Be, B)
  - supernova explosions ($^7$Li, $^{11}$B)
- Some are destroyed:
  - d, Li, Be, B are very fragile; they are destroyed in the center of stars
  → observed abundances (at surface of stars) do not reflect the destruction inside

$^4$He : $Y_p = 0.2421 \pm 0.0021$

D : $D/H = (2.78 + 0.44 - 0.38) \times 10^{-5}$

$^7$Li : $Li/H = (1.23 + 0.68 - 0.32) \times 10^{-10}$

$\Omega_B h^2 = 0.0224 \pm 0.0009$
4.2.5 - BBN predictions: Neutron lifetime

\[ \eta_{\text{WMAP}} = 6.2 \times 10^{-10} \]

Helium mass fraction calculated with the BBN model as a function of the baryon-to-photon ratio parameter \( \eta \).
Experiments on Neutron Lifetime

\[ \tau_n \text{ measurements vs. time} \]

885.7 ± 0.8 sec !!!
Helium mass fraction calculated with the BBN model as a function of the number of neutrino families.

\( n_{\text{WMAP}} = 6.2 \times 10^{-10} \)
BBN incredibly successful, except for Lithium problem

SBBN (Standard BBN): one parameter

baryon-to-photon ratio \( \eta \)

\[ \eta = (6.225^{+0.157}_{-0.154}) \times 10^{-10} \]

(WMAP 2010)

<table>
<thead>
<tr>
<th></th>
<th>BBN</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^4\text{He})</td>
<td>0.242</td>
<td>0.242</td>
</tr>
<tr>
<td>(\text{D/H})</td>
<td>(2.62 \times 10^{-5})</td>
<td>(2.78 \times 10^{-5})</td>
</tr>
<tr>
<td>(^3\text{He/H})</td>
<td>(0.98 \times 10^{-5})</td>
<td>((0.9 - 1.3) \times 10^{-5})</td>
</tr>
<tr>
<td>(^7\text{Li/H})</td>
<td>(4.39 \times 10^{-10})</td>
<td>(1.2 \times 10^{-10})</td>
</tr>
</tbody>
</table>
4.3 - Lithium problem

The SBBN model explains very well the abundance of light elements, except for the observed $^6$Li and $^7$Li. While the $^6$Li abundance is difficult to explain because of “astration”, i.e., $^6$Li reprocessing in stars, no theory or model can explain why the observed $^7$Li abundance is so much smaller than predicted.

For a recent status of the lithium problem, see:
4.3.1 - Problem with nuclear data? Recent data

Recent data

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Comparison to other compilations

Figure 5: Left: Ratio of the $^2$H(d,p)$^3$H reaction rates calculated using THM data to the one obtained from direct data fits (upper panel). The middle and lower panels are similar ratios using rates published in NACRE [Angulo et al. 1999] and Cyburt [Cyburt 2004].

Right: Same as in the left figure but for the $^2$H(d,n)$^3$He reaction. The vertical lines represent the approximated lower and upper temperature limits of interest for big bang nucleosynthesis.

The bare-nucleus cross section for the $^3$He(d,p)$^4$He fusion reaction, at ultra-low energies, is of interest in pure and applied physics and was measured in the energy region of interest for astro-physics by means of several methods both indirect and direct [Aliotta et al. 2000, Geist et al. 1999, Krauss et al. 1987]. For the $^3$He(d,p)$^4$He we used the direct data from [Engstler et al. 1988, Krauss et al. 1987, Bonner et al. 1952, Zhicang et al. 1977, Geist et al. 1999, Moeller et al. 1980, Erramli et al. 2005, Schroeder et al. 1989, Aliotta et al. 1999]. The THM experiment was performed by measuring the $^3$He($^6$Li,p)$^4$He reaction in quasi-free kinematics. The bare nucleus S(E)-factor was then extracted in the 0 to 1 MeV energy range and fitted following Eq. (7), as reported in [La Cognata et al. 2005]. The S-factor for the reaction $^3$He(d,p)$^4$He is shown in figure 3 with red solid circles for THM data and blue filled triangles for the direct measurements. The solid line is an R-matrix fit to the direct and THM data described in section 2.1.
Comparison to other compilations

Figure 6: Left: Ratio of the $^3\text{He}(d,p)^4\text{He}$ reaction rates calculated using THM data to the one obtained from direct data fits (upper panel). The middle and lower panels are similar ratios using rates published in Smith, Kawano and Malaney [Smith et al. 1993] (lower panel).

Right: In the left figure, ratio of rates calculated using THM (as discussed in the text) to the one obtained with a fit to the direct data (without TH) of the S-factors (upper panel). The middle and lower panels are similar ratios with rates published in NACRE [Angulo et al. 1999] and Cyburt [Cyburt 2004]. The vertical lines represent the approximated lower and upper temperature limits of interest for big bang nucleosynthesis.

3.5 Reaction rates with TH data

The reaction rates for the four reactions mentioned above (from a compilation of direct and THM data, as reported in the sections above) have been carried out numerically introducing the R-matrix results in Eq. (3). Thus, we fitted the rates with the parametrization displayed in Equation (11). This is the common procedure adopted in previous works (see, e.g., [Smith et al. 1993, Cyburt 2004, Coc et al. 2012]). For the 4 reactions of interest, we have included the experimental errors from measurements, allowing us to evaluate the respective errors in the reaction rates. The numerical results are then fitted with the expression

$$N_A h v_i = \exp \left( a_1 + a_2 \ln T_9 + a_3 T_9 + a_4 T_9^{-1/3} + a_5 T_9^{-1/3} + a_6 T_9^{-2/3} + a_7 T_9 + a_8 T_9^{-4/3} + a_9 T_9^{-5/3} \right),$$

(11)

which incorporates the relevant temperature dependence of the reaction rates during the BBN. The $a_i$ coefficients for the $^2\text{H}(d,p)^3\text{He}$ and the $^2\text{H}(d,n)^3\text{He}$ reactions are given for both THM and...
The reaction rates of 4 of the main reactions of the BBN network in the temperature range \( 0.001 < T_9 < 10 \) have been calculated numerically including the recent THM measurements. The uncertainties of experimental data for direct and THM data have been fully included. The extension of the same methodology to the other reactions forming the BBN reaction network will be examined in a forthcoming paper.

The parameters of each reaction rates as given in Eq. 11 are reported in Tables 4 and 5. The obtained reaction rates are compared with the some of the most commonly used compilations found in the literature. An updated compilation of direct data for the \(^2\text{H}(d,p)^3\text{H}, d(d,n)^3\text{He}, ^3\text{He}(d,p)^4\text{He}, ^7\text{Li}(p,\gamma)^4\text{He} \) reactions has also been made, and relative expressions for the reaction rate are also given. The reaction rates calculated in the present work are used to calculate the BBN abundance for \(^3\text{He}, ^4\text{He}, \text{D} \) and \(^7\text{Li} \). The obtained abundances are in agreement, within the experimental errors, with those obtained using the compilation of direct reaction rates. Moreover, a comparison of our predictions with the observations for primordial abundance of \(^3\text{He}, ^4\text{He} \) and \(\text{D} \) show an agreement, while showing a relevant discrepancy for \(^7\text{Li} \). The present results show the power of THM as a tool for exploring charged particle induced reactions at the energies typical of BBN.

From Table 6 we can see that the primordial abundances calculated using the present reaction rates, agree within the uncertainties with the predictions arising from direct data. The comparison between predicted values and observations clearly confirms the discrepancy for \(^7\text{Li} \) abundance.

<table>
<thead>
<tr>
<th></th>
<th>BBN</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^4\text{He}/\text{H})</td>
<td>0.2485 (+ 0.001 – 0.002) x 10^{-5}</td>
<td>0.256 ± 0.006</td>
</tr>
<tr>
<td>(\text{D}/\text{H})</td>
<td>2.692 (+ 1.77 – 0.07) x 10^{-5}</td>
<td>(2.82 ± 0.26) x 10^{-5}</td>
</tr>
<tr>
<td>(^3\text{He}/\text{H})</td>
<td>0.9441 (+ 0.511 – 0.466) x 10^{-5}</td>
<td>(0.9 - 1.3) x 10^{-5}</td>
</tr>
<tr>
<td>(^7\text{Li}/\text{H})</td>
<td>4.683 (+ 0.335 – 0.292) x 10^{-10}</td>
<td>(1.58 ± 0.31) x 10^{-10}</td>
</tr>
</tbody>
</table>
4.3.2 - Medium effects? Screened big bang

Interactions with photons/electrons of the plasma

Change in the e.m. equation of state due to photon/electron thermal masses

\[ P = P(\rho) \]

Electron density in BBN

mostly due to $\gamma \rightarrow e^+e^-$

$n_e$ (BB) $\sim n_e$ (sun)
but
$n_b$ (sun) $>$ $n_b$ (BB)

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The quest for the origins of the elements, C. Bertulani, RISP, 2016

Baryon density

\[ \rho_b \sim h T_9^3 \]

\[ h \sim 2.1 \times 10^{-5} \]

\[ h \sim 5.7 \times 10^{-5} \]

\[ T_9 \sim 2 \]

\[ T_9 \sim 1 \]
Debye radius x inter-ion distance

\[ R_D \sim \frac{d}{T_9} \]

- \( d \sim 2.1 \times 10^{-5} \text{ cm} \)
- \( d \sim 3.7 \times 10^{-5} \text{ cm} \)

\( \geq 1000 \) particles in Debye sphere

mean-field model OK
If screening effect were much larger

\[
\ln f'_{\text{BBN}} = w \ln f_{\text{BBN}}
\]

\[
\Delta = \frac{Y' - Y}{Y} \times 100
\]


BBN screening of reaction rates by electrons negligible.
4.3.3 - Non-Maxwellian thermal distribution

- Deviation from Boltzmann-Gibbs statistics very popular in plasma physics → turbulence phenomena, systems having memory effects, systems with long range interactions, etc.

- Relevance for cataclysmic stellar systems, e.g. supernovae?

From: hubblesite.org
Standard Boltzmann-Gibbs statistics

Has a maximum when all states have equal probability \( p_i \)

Two simple constraints (normalization and mean value of the energy):

\[
\int_0^\infty p(\varepsilon) \, d\varepsilon = 1 \quad (4.29)
\]

\[
\int_0^\infty \varepsilon p(\varepsilon) \, d\varepsilon = \text{const} \quad (4.30)
\]

Distribution function:

\[
p_i = \frac{e^{-\beta \varepsilon_i}}{\sum_{i=1}^{n} e^{-\beta \varepsilon_i}} \quad (4.31)
\]

\[
\beta = 1 / k_B T
\]

Thermodynamics
**Extensive statistics**

Assumption: particles independent \( \rightarrow \) no correlations

Hypothesis: isotropy of velocity directions \( \rightarrow \) extensivity

+ microscopic interactions short ranged, Euclidean space time, etc.

\[
S = -k_B \sum_{i=1}^{n} p_i \ln p_i
\]

\[ A \hspace{2cm} B \]

\[
p_i^A \hspace{2cm} p_j^B \rightarrow \hspace{2cm} p_{i+j}^{A+B} = p_i^A \cdot p_j^B
\]

\[
S_{A+B} = S_A + S_B
\]

(4.32)

(4.33)

extensive
Non-extensive statistics

Extensive statistics not applicable for long-range interactions

THUS

→ introduce correlations via non-extensive statistics
→ derive corresponding entropy distribution

Renyi, 1955 - Tsallis, 1985

\[ S_q = k_B \frac{1 - \sum_{i=1}^{n} p_i^q}{1 - q} \]

\[ S_q (A + B) = S_q (A) + S_q (B) + \frac{(1 - q)}{k_B} S_q (A) S_q (B) \]
Non-extensive statistics

\[ S_q = k_B \left( 1 - \sum_{i=1}^{n} p_i^q \right) / (q - 1) \]  (4.36)

- **q is Tsallis parameter**: in general labels an infinite family of entropies

- \( S_q \) is a natural generalization of Boltzmann-Gibbs entropy which is restored for \( q = 1 \):
  \[ S_q = -k_B \sum_{i=1}^{n} p_i \ln p_i \]
  \[ \lim_{q \to 1} \]
  (4.37)

- BG formalism yields exponential equilibrium distributions, whereas non-extensive statistics yields (asymptotic) **power-law distributions**

- Renyi entropy is related through a monotonic function to the Tsallis entropy (with \( k_B = 1 \))
  \[ S_q^R = \ln \left( \sum_{i=1}^{n} p_i^q \right) / (1 - q) = \ln \left[ 1 - (1 - q)S_q \right] / (1 - q) \]  (4.38)
Maxwell distribution & reaction rates

\[ r_{ij} = \frac{n_in_j}{1 + \delta_{ij}} \langle \sigma v \rangle \]

\[ \sigma = \frac{S(E)}{E} \exp \left[ -\frac{Z_i Z_j}{\hbar v} \right] \]

\[ r_{ij} \sim \int dE S(E) \exp \left[ -\left( \frac{E}{k_B T} \right) + 2\pi \eta(E) \right] \]
Non-Maxwellian distribution

\[
f(E) = \exp\left[-\frac{E}{k_B T}\right] \quad \rightarrow \quad f_q(E) = \left(1 - \frac{q - 1}{k_B T} E\right)^{\frac{1}{q-1}}
\]  

(4.39)

\[
f_q \rightarrow \exp\left[-\frac{E}{k_B T}\right]
\]

(4.40)

\[
q \rightarrow 1
\]

(4.41)

\[
r_{ij} \sim \int dE S(E) M_q(E, T)
\]

(4.42)

Non-Maxwellian rate

\[
M_q(E, T) = N(q, T)\left[1 - \frac{q - 1}{k_B T} E\right]^{\frac{1}{q-1}} \exp[-\eta(E)]
\]  

(4.43)

The quest for the origins of the elements, C. Bertulani, RISP, 2016
The quest for the origins of the elements, C. Bertulani, RISP, 2016

BBN deuterium abundance

due to suppression of D-destruction
The quest for the origins of the elements, C. Bertulani, RISP, 2016

BBN $^4$He abundance

time (min)

$^4$He/H

$^4$He due to suppression of $t(d,n)^4$He, $^3$H(d,p)$^4$He

$q = 0.5$

$q = 2$

$q = 1$
BBN $^{7}$Li abundance

$^{7}$Li/H

$q = 2$
$q = 1$
$q = 0.5$

Temperature ($10^9$ K)

Time (min)
<table>
<thead>
<tr>
<th></th>
<th>BNN</th>
<th>Non-ext. $q=0.5$</th>
<th>Non-ext. $q=2$</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4$He</td>
<td>0.249</td>
<td>0.243</td>
<td>0.141</td>
<td>0.256 ± 0.006</td>
</tr>
<tr>
<td>D/H</td>
<td>2.62</td>
<td>3.31</td>
<td>570</td>
<td>(2.82 ± 0.26) x 10^{-5}</td>
</tr>
<tr>
<td>$^3$He/H</td>
<td>0.98</td>
<td>0.091</td>
<td>69.1</td>
<td>(0.9 - 1.3) x 10^{-5}</td>
</tr>
<tr>
<td>$^7$Li/H</td>
<td>4.39</td>
<td>6.89</td>
<td>356.</td>
<td>(1.58 ± 0.31) x 10^{-10}</td>
</tr>
</tbody>
</table>
4.3.4 - Parallel universes of dark + visible matter

Parallel Universes of Dark Matter

1. Leaves unchanged long distance properties of SM and Gravity

2. No Higgs Mechanism

3. Compatible with Cosmological constraints and BBN

\[ L = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \sum_f \bar{Q}_f \left[ i\gamma^\mu D_\mu - m_f \right] Q_f + \frac{1}{2} (D^\mu \phi^a)(D_\mu \phi^a) - V_{oct}(\phi^a\phi^a) \]

(4.44)

Grey Boson + Matter Field + Scalar

\[ D_\mu = \partial_\mu + ig_M T^a M^a_\mu \]  

(4.45)
The quest for the origins of the elements, C. Bertulani, RISP, 2016
The quest for the origins of the elements, C. Bertulani, RISP, 2016

\[ \rho = \frac{\pi^2}{30} g_*(T) T^4 \]  
(4.52)

\[ s = \frac{2\pi^2}{45} g_s(T) T^3 \]  
(4.53)

Do it for \( T \) and \( T' \) + use Friedmann equation + evolve to BBN time

\[ \bar{g}_s(T) = g_* \left[ 1 + N_D \left( \frac{T'}{T} \right)^4 \right] \]  
(4.56)

\[ g_s(T) \bigg|_{T=1\text{MeV}} = 10.75 \]  
(4.57)

BBN + \(^4\text{He}, ^3\text{He}, D\) and \(^7\text{Li}\) constraints

\[ \frac{T'}{T} < \frac{0.78}{N_D^{1/4}} = 0.52 \]  
(4.58)
Baryon asymmetry and DM halo dynamics

\[ \eta = \frac{\text{density of baryons}}{\text{density of photons}} \]

\[ \frac{\text{dark baryons}}{\text{ordinary baryons}} = \frac{\eta'}{\eta} \left( \frac{T'}{T} \right)^3 \approx 1 \]

\[ \eta' = 7\eta \] (4.59) (4.60)

**Acoustic oscillations**

To change CMB:

\[ \frac{T'}{T} \geq 0.6 \] (4.61)

\[ N_D \geq 0.35 \text{ lower bound} \] (4.62)

**DM-DM interactions**

\[ \sigma = \left( g^2 \frac{T}{\Lambda^2} \right)^2 \] (4.63)

\[ \frac{\sigma'}{\sigma} \sim \left( \frac{T'}{T} \right)^2 \]

BBN

\[ \frac{\sigma'}{\sigma} \sim 0.61 \sqrt{N_D} \] (4.64) (4.65)

→ Dark sectors are essentially collisionless

Model is compatible with cosmological, BBN, and CMB constraints
But does not do anything to solve the Lithium problem

The quest for the origins of the elements, C. Bertulani, RISP, 2016
BBN incredibly successful, except for Lithium problem

This has led to a large number of speculations, such as if the MB distribution is valid for the BBN scenario, or if another statistics should be adopted. See, e.g.,

Other possibilities even include electron screening, the effect of dark matter, or parallel universes. See, e.g.,

For more details on current status of the lithium problem, see:
1 - At the end of BBN $^4$He and $^1$H made up 24% and 76% of the total mass respectively. Assuming that all the free neutrons became bound in the $^4$He isotope and that the process was fast compared with the neutron lifetime, estimate using the Boltzmann distribution the temperature at which the n/p ratio was frozen.

2 - From the atomic masses below, calculate the energy released per kilogram of matter in the production of helium assuming once again that 24% by mass is converted (1 a.m.u. = $1.6605 \times 10^{-27}$ kg = 931.494 MeV/c$^2$).

<table>
<thead>
<tr>
<th>Element/Nucleon</th>
<th>Atomic Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4$He</td>
<td>4.002602 u</td>
</tr>
<tr>
<td>$^1$H</td>
<td>1.00794 u</td>
</tr>
<tr>
<td>n</td>
<td>1.008664 u</td>
</tr>
</tbody>
</table>

3 - The mean lifetime for free neutrons is 880 s. The n/p ratio was frozen at 0.2 but the formed deuterons were destroyed by photodisintegration until the mean photon energy was reduced by the decreasing temperature. By that time neutron decay had lowered the n/p ratio to 0.134. Estimate the duration of this time period.
Practice

4 - Estimate the height of the Coulomb barrier for the reaction

$$^4\text{He} + ^3\text{H} \rightarrow ^7\text{Li} + \gamma$$

and the temperature at which the nuclei will have kinetic energies of approximately this value (assume that the nuclear radius is given by $1.2 \times A^{1/3} \text{ fm}$).
Computing practice

1 - Get together in groups of maximum 4 persons.

2 - Use 5 different values of the neutron lifetime within a window of 200 seconds around the accepted experimental value and compute the final BBN 4He abundance as a function of the baryon to photon ratio, \( \eta \).

3 - Use 5 different values of the number of neutrino families within a window of \( \Delta n_\nu = 1 \) around the accepted experimental value and compute the final BBN 4He abundance as a function of the baryon to photon ratio, \( \eta \).

4 - Compute the abundances of 4He, D, 3He, 6Li and 7Li as a function of baryon to photon ratio, \( \eta \).

5 - Compute the abundances of 4He, D, 3He, 6Li and 7Li as a function of time and temperature for the accepted value of the baryon to photon ratio, \( \eta \).

6 - Find the lines within the code where the rates of the BBN reactions are entered. Modify some of the reaction rates (keep track of it, so that you can undo the changes) by a factor 10 or 1/10 and make a table with the results in the final prediction for all elements. Explain why the results changed in terms of the reaction chain.