Lecture 4

The quest for the origin of the elements

4.1 - The Early Universe

According to the accepted cosmological theories:

- The Universe has cooled during its expansion as

$$T(t) \sim \frac{1}{a(t)} \qquad (4.1)$$

- In terms of the time evolved from the Big Bang (for a radiation dominated universe $a \sim t^{1/2}$)

$$T(t) \sim \frac{1.3 \times 10^{10}\ K}{t^{1/2}\ [s]} \qquad (4.2)$$

- The particles are in thermal equilibrium. This is guaranteed by reactions which are faster than the **expansion rate (da/dt)/a.** In this case, particles and antiparticles are in equilibrium through annihilation, i.e. particles + antiparticles ←→ photons
- But when $k_B T \ll mc^2$, particles and antiparticles (with mass m) annihilate and photons cannot create them back as their energy is below the threshold.
- At $T = 10^{12}$ K, there was a slight overabundance of matter over antimatter which lead to the violation of baryon/lepton number conservation.
- Also at $T = 10^{12}$ K, antinucleons have annihilated with nucleons and the remaining nucleons become the breeding material for primordial nucleosynthesis, or **Big Bang Nucleosynthesis (BBN)**.

The Early Universe

At one-hundredth of seconds of the Universe consisted of an approximately equal number of electrons, positrons, neutrinos and photons, and a small amount of protons and neutrons; the ratio of protons to photons is assumed to have been about 10^{-9}. The energy density of photons can be calculated from

$$\rho_\gamma = \int E_\gamma \, dn_\gamma \qquad (4.3)$$

where the density of states is given by

$$dn_\gamma = \frac{g_\gamma}{2\pi^2} \frac{\kappa_\gamma^2}{\exp\left(E_\gamma / kT\right) - 1} d\kappa_\gamma \qquad (4.4)$$

and $g_\gamma = 2$ is the number of spin polarizations 1 for the photon while $E_\gamma = \hbar\kappa_\gamma c$ is the photon energy (momentum).

Performing the integration gives

$$\rho_\gamma = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T^4 \qquad (4.5)$$

which is the familiar blackbody result.

The Early Universe

At large temperatures, when $k_B T >> m_i$, the mass m_i of the particles (electrons, neutrinos, nucleons) are irrelevant and the energy density associated with these particles can also be described by the black-body formula. A straightforward calculation for the density, and number density accounting for all particle degrees of freedom g_i yields

$$\rho_i = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T^4 \times \begin{cases} 7/8 & \text{for fermions} \\ 1 & \text{for bosons} \end{cases} \qquad (4.6)$$

$$n_i = \frac{\varsigma(3) k_B^3}{\pi^2 \hbar^3 c^3} T^3 \times \begin{cases} 3/4 & \text{for fermions} \\ 1 & \text{for bosons} \end{cases} \qquad (4.7)$$

as well as the know relation between density and pressure for matter

$$p_i = \frac{\rho_i}{3} \qquad (4.8)$$

In Eq. (4.7) $\zeta(3) = 1.202$ is the Riemann zeta function. The difference for fermions and bosons is because of their different statistical distribution functions.

The Early Universe

As we have seen in Eqs. (2.13) and (2.14), the density of scales with time as ***ρ ~ 1/t²***, which together with Eq. (4.6) yields

$$t \sim \frac{\text{const.}}{T^2} \qquad (4.9)$$

with the precise expression being

$$t = \left(\frac{90 \hbar^3 c^3}{32 \pi^3 G g^*} \right)^{1/2} \frac{1}{k_B^2 T^2} \qquad (4.10)$$

where g* is total the number of degrees of freedom of the particles. Thus, the relation between time and temperature in the early universe depends strongly on the kind of particles present in the plasma. Calculations of time and temperature in the early universe are shown below.

	T(K)	a/a_0	t(sec)
$\sim$10 MeV	10^{11}	1.9×10^{-11}	0.0108
$\sim$1 MeV	10^{10}	1.9×10^{-10}	1.103
$\sim$100 keV	10^{9}	2.6×10^{-9}	182
$\sim$10 keV	10^{8}	2.7×10^{-8}	19200

For a Universe dominated by relativistic, or radiation-like, particles $\rho \propto 1/a^4$ and the above expressions yield $a \sim T^{-1} \sim t^{1/2}$.

4.1.1 - Time ~ 0.01 sec

At $t \sim 0.01$ s, the temperature is $T \sim 10^{11}$ K, and $k_B T \sim 10$ MeV, which is much larger that the electron mass. Neutrinos, electrons and positrons are easily produced and destroyed by means of weak interactions (i.e., interactions involving neutrinos)

$$\begin{aligned} &\text{i)} \quad n + \nu_e \leftrightarrow p + e^- \\ &\text{ii)} \quad n + e^+ \leftrightarrow p + \bar{\nu}_e \\ &\text{iii)} \quad n \leftrightarrow p + e^- + \bar{\nu}_e \end{aligned} \qquad (4.11)$$

As long as the weak reactions are fast enough, the neutron-to-proton ratio is given by

$$[n/p] = \frac{\text{number of neutrons}}{\text{number of protons}} = \frac{N_n(T)}{N_p(T)} = \exp\left[-\frac{\Delta mc^2}{k_B T}\right] \qquad (4.12)$$

where $m(n) = 939.5$ MeV, $m(p) = 938.3$ MeV, and $\Delta m = 1.294$ MeV.
At $T = 10^{11}$ K, $k_B T = 8.62$ MeV, yielding $n/p = 0.86$.

This temperature is far above the temperature of nucleosynthesis, but the n/p ratio already begins to drop.

4.1.2 - Time ~ 0.1 sec

At $t \sim 0.1$ *s*, the temperature is $T \sim 3 \times 10^{10}$ *K*, and $k_B T \sim 2.6$ *MeV*. Neutrinos, electrons and positrons are still in equilibrium according to Eq. (4.11). The lifetime for destruction of a neutron by means of these reactions can be calculated from

$$\lambda(T) = n_\nu \langle \sigma v \rangle_\nu \qquad (4.13)$$

This is not an easy calculation, as it requires the knowledge of σ_ν for neutrino induced interactions. A detailed calculation yields

$$\lambda(T) = \frac{0.76}{\sec} \left(\frac{k_B T}{\text{MeV}} \right)^5 \qquad (4.14)$$

At $T \sim 3 \times 10^{10}$ *K*, this yields a neutron destruction lifetime of *0.01 sec*. Thus, the weak rates drop very fast, as T^5. At some T the weak rates are so slow that they cannot keep up with Universe expansion rate. The Hubble rate at this epoch is found to be

$$H(T) = \frac{0.67}{\sec} \left(\frac{k_B T}{\text{MeV}} \right)^2 \qquad (4.15)$$

4.1.3 - Decoupling

When the Hubble expansion rate is equal to the neutron destruction rate, i.e., when Eqs. (4.14) and (4.15) are equal, we find ***$k_B T$ ~ 1 MeV***.

As temperature and density decreases beyond this point, the neutrinos start behaving like free particles. Below ***10^{10} K*** they cease to play any major role in the reactions. That is, they decouple and matter becomes transparent to the neutrinos.

At ***$k_B T$ ~ 1 MeV*** (twice the electron mass), the photons also stop produce positron/electron pairs

$$\gamma \leftrightarrow e^+ + e^- \qquad (4.16)$$

The $e^- e^+$-pairs begin to annihilate each other, leaving a small excess of electrons.

However, the thermal energies are still high enough to destroy any formed nuclei. At this point

$$[n/p] = \exp\left[-\frac{\Delta mc^2}{kT}\right] \sim 0.25 \qquad (4.17)$$

As the temperature drops, neutrino-induced reactions continue creating more protons. In the next ***10 secs*** the ***n/p*** ratio will drop to about ***0.17 ~ 1/6***. And after that the neutron percentage continues to decrease because of neutron beta-decay. When nucleosynthesis starts the ***n/p*** ratio is ***1/7***.

4.1.4 - Baryon to photon ratio

Neutrons and protons were created at an earlier stage of the universe, when quarks and gluons combined to form them. As with the other particles, very energetic photons produced baryons (nucleons) and antibaryon pairs and were also produced by the inverse reactions. As the temperature decreased, baryons annihilated each other and more photons were created. One does not know why a small number of baryons remained. The resulting baryon/photon ratio at this time was

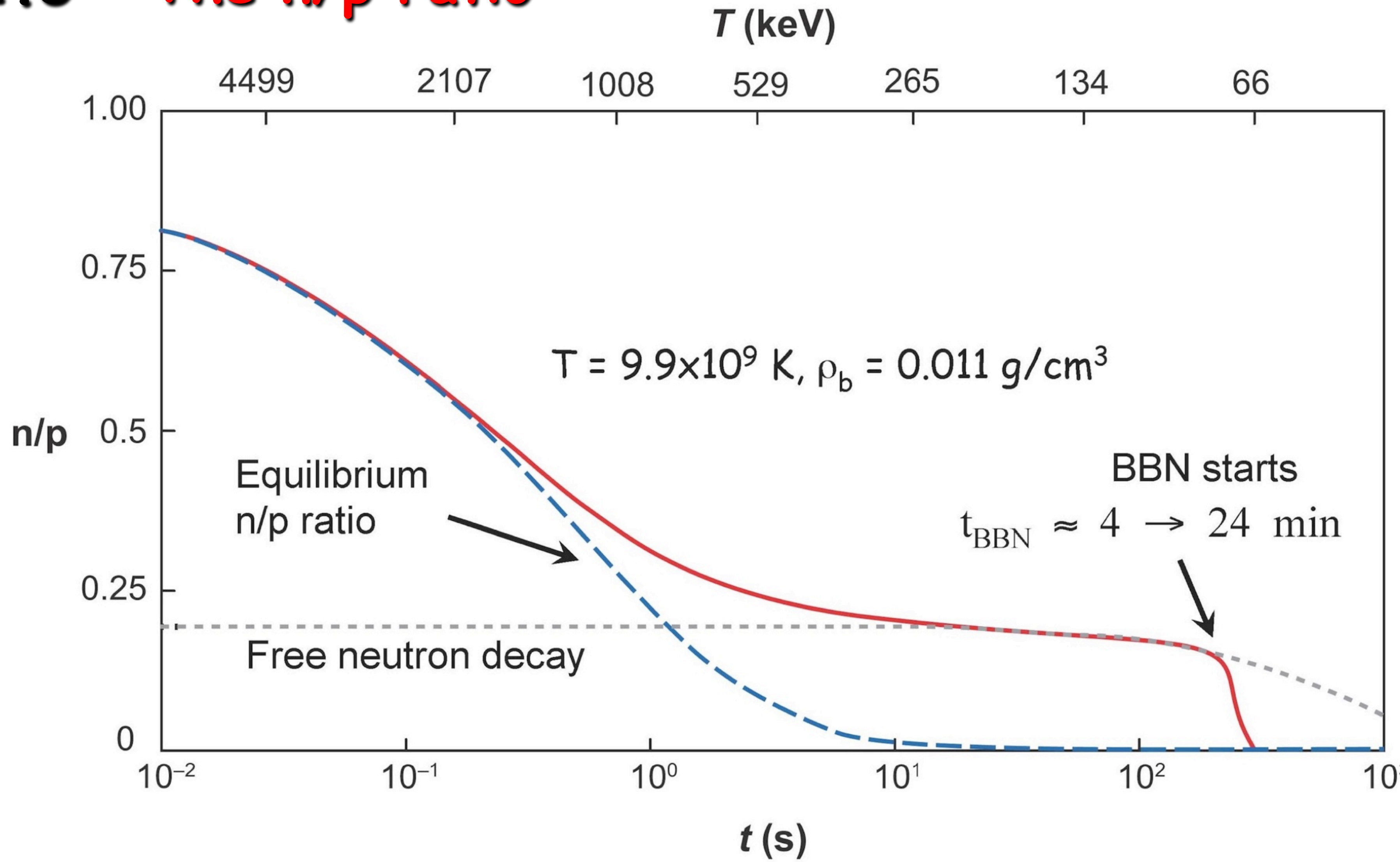
$$\frac{\rho_b}{\rho_\gamma} = \eta \sim 10^{-9} \qquad (4.18)$$

Usually η is considered the only parameter of BBN. However, BBN is also sensitive to two other parameters: the neutron lifetime and the number of light neutrino families.

(a) **neutron lifetime $\tau_{1/2}(n)$** – An increase in $\tau_{1/2}(n)$ theoretically implies a decrease of all weak rates which convert protons and neutrons. The freeze-out would happens at a higher temperature and would lead to a larger ***n/p*** ratio. As all neutrons essentially end up in ^{4}He, the ^{4}He abundance grows if $\tau_{1/2}(n)$ is increased.

(b) **number of neutrino families** - The energy density of the early Universe depends on the number of neutrino families (n_ν): the larger n_ν, the larger and the faster the expansion rate of the Universe $H = (da/dt)/a$. An increase in H leads to an earlier freeze-out and hence more ^4He abundance.

4.1.5 - The n/p ratio



Neutron-to-proton (***n/p***) ratio as a function of time and temperature. The dashed curve is given by ***$\exp(-\Delta m/k_B T)$***. The dotted curve is the free-neutron decay curve, ***$\exp(-t/\tau_n)$***. The solid curve indicates the resulting ***n/p*** ratio as a combination of the two processes. BBN starts at ***$t \sim 4$ min***.

4.2 - Big Bang Nucleosynthesis (BBN)

A summary of the BBN when the temperature of the universe allowed deuteron to be formed without being immediately destroyed by photons is:

1 . The light elements (deuterium, helium, and lithium) were produced in the first few minutes after the Big Bang.

2 . Elements heavier than ^{4}He were produced in the stars and through supernovae explosions.

3 . Helium and deuterium produced in stars do not to match observation because stars destroy deuterium in their cores.

4 . Therefore, all the observed deuterium was produced around three minutes after the big bang, when $T \sim 10^9$ K.

5. A simple calculation based on the n/p ratio shows that BBN predicts that 25% of the matter in the Universe should be helium.

6. More detailed BBN calculations predict that about 0.001% should be deuterium.

The deuteron bottleneck

As the temperature of the Universe decreased, neutrons and protons started to interact and fuse to a deuteron

$$n + p \rightarrow d + \gamma \qquad (4.19)$$

The binding energy of deuterons is small (***E_B = 2.23 MeV***). The baryon-to-photon ratio, called η, at this time is also very small (***$< 10^{-9}$***). As a consequence, there are many high-energy photons to dissociate the formed deuterons, as soon as they are produced.

The temperature at the start of nucleosynthesis is about ***100 keV***, when we would have expected ***~ 2 MeV***, the binding energy of deuterium. The reason is the very small value of η. The BBN temperature, ***~ 100 keV***, corresponds according to Eq. (4.10) to timescales less than about ***200 sec***. The cross-section and reaction rate for the reaction in Eq. (4.19) is

$$\sigma v \sim 5 \times 10^{-20}\ \mathrm{cm}^3/\mathrm{sec} \qquad (4.20)$$

So, in order to achieve appreciable deuteron production rate we need $\rho \sim 10^{-17}\ cm^{-3}$. The density of baryons today is known approximately from the density of visible matter to be $\rho_0 \sim 10^{-7}\ cm^{-3}$ and since we know that the density ρ scales as $a^{-3} \sim T^3$, the temperature today must be $T_0 = (\rho_0/\rho)^{1/3}\, T_{BBN} \sim 10\ K$, which is a good estimate of the CMB.

The deuteron bottleneck

The bottleneck implies that there would be no significant abundance of deuterons before the Universe cooled to about 10^9 K.

Other important facts are:

1 - The nucleon composition during BBN was proton-rich.

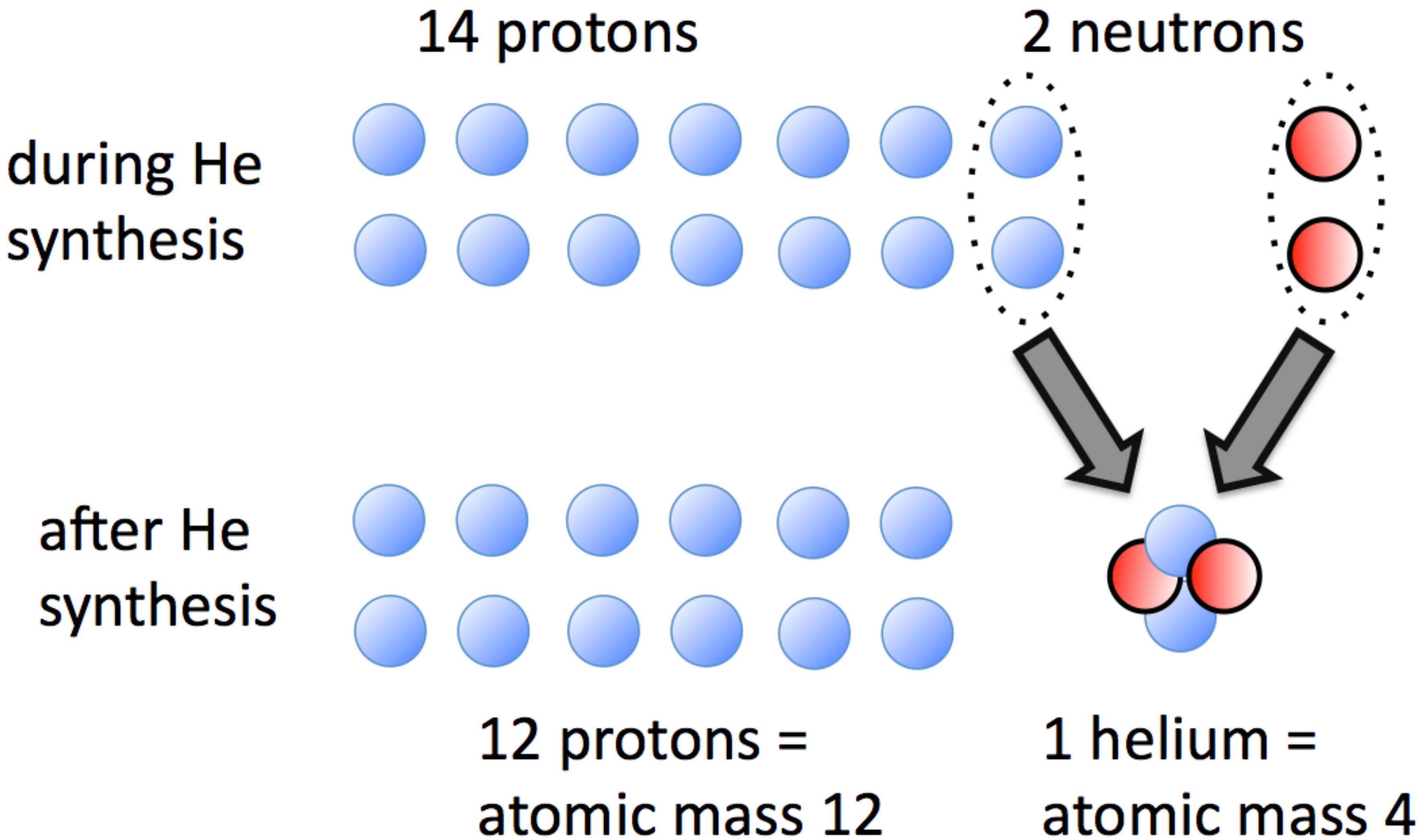
2 - The most tightly bound light nucleus is ^{4}He.

3 - There is no stable nucleus with mass numbers A = 5 and A = 8.

4 - The early universe was hot but not dense enough to overcome the Coulomb barriers to produce heavier nuclides.

5 - The BBN network is active until all neutrons are bound in ^{4}He. As the BBN mass fraction of neutrons was $X_n = N_n /(N_n + N_p) = 1/8$, it follows that the mass fraction of ^{4}He after BBN is about $X_{4He} = 2X_n = 25\%$.

BBN Predictions - The Helium abundance



BBN predicts that when the universe had T = 10^9 K (1 minute old), protons outnumbered neutrons by 7:1. When ^{2}H and He nuclei formed, most of the neutrons formed He nuclei. That is, one expects 1 He nucleus for every 12 H nuclei, or 75% H and 25% He. This is the fraction of He and ^{2}H we observe today.

Standard Big Bang Nucleosynthesis (BBN)

a: scale factor

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\left(\rho_\gamma + \rho_{e^\pm} + \rho_b + \rho_\nu\right)} \equiv H \qquad (4.21)$$

Friedmann Equation

ρ: energy density of relativistic species (m < 1 MeV)

$$\dot{\rho} = -3H(\rho + p) \qquad (4.22)$$

μ_e: electron chemical potential

$$n_b \sum_j Z_j X_j = n_{e^-} - n_{e^+} \equiv \Phi\left(\frac{m_e}{T}, \mu_e\right) \qquad (4.23)$$

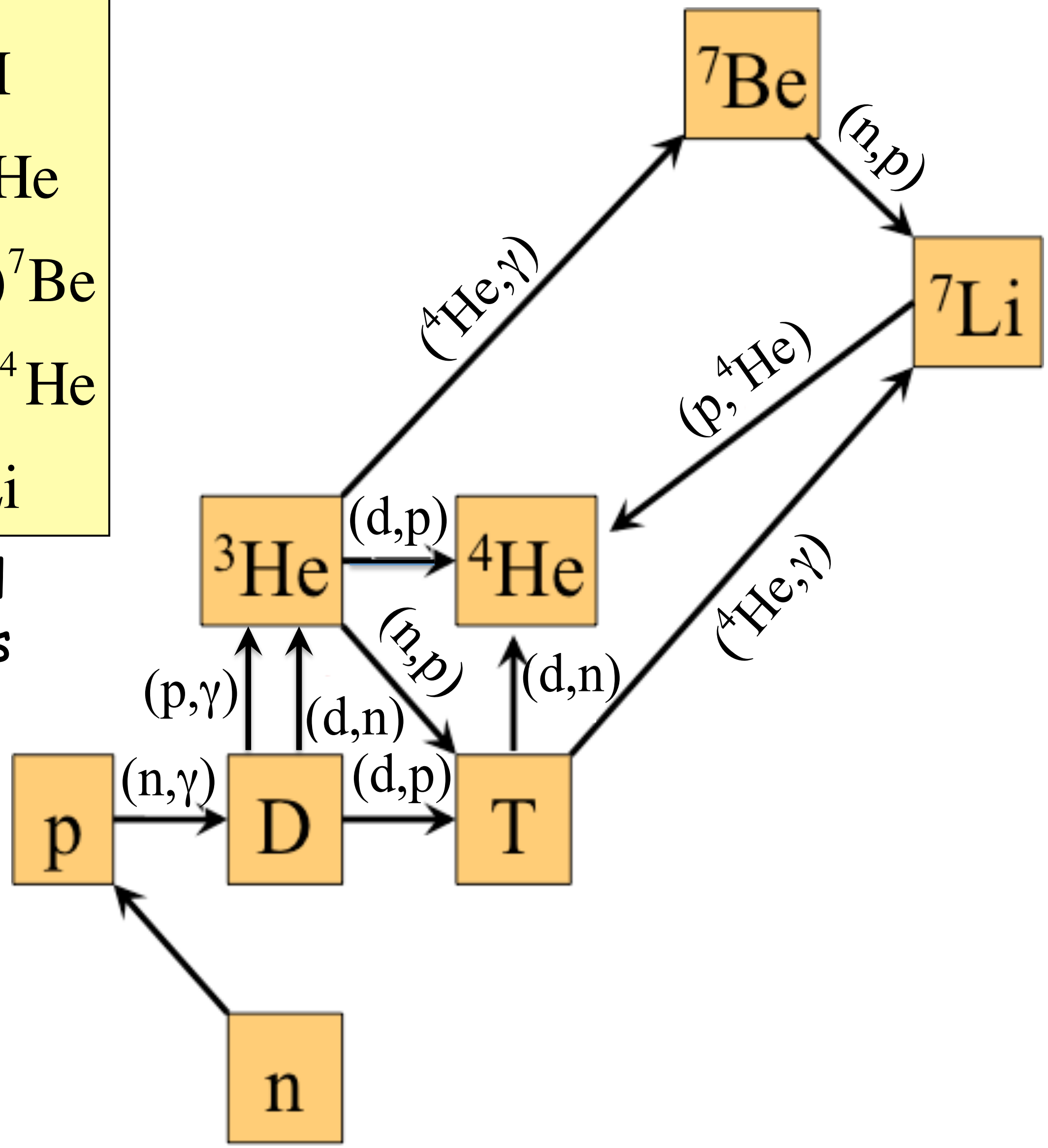
$$\dot{X}_i = \sum_{j,k,l} N_i \left(\Gamma_{kl\to ij} \frac{(X_l)^{N_l}}{N_l!} \frac{(X_k)^{N_k}}{N_k!} - \Gamma_{ij\to kl} \frac{(X_i)^{N_i}}{N_i!} \frac{(X_j)^{N_j}}{N_j!} \right) \equiv \Gamma_i \qquad (4.24)$$

nuclear physics

4.2.1 - The BBN reaction network

After deuterons are produced at $T \sim 10^9$ K, a successive chain of nuclear reactions occur. The most important are

$1: \quad n \to p$	$7: \ ^4He(^3H,\gamma)^7Li$
$2: \quad n(p,\gamma)d$	$8: \ ^3He(n,p)^3H$
$3: \quad d(p,\gamma)^3He$	$9: \ ^3He(d,p)^4He$
$4: \quad d(d,n)^3He$	$10: \ ^4He(^3He,\gamma)^7Be$
$5: \quad d(d,p)^3H$	$11: \ ^7Li(p,^4He)^4He$
$6: \ ^3H(d,n)^4He$	$12: \ ^7Be(n,p)^7Li$



Except for the ^{7}Be electron capture, all reactions are fast. The binding energies of ^{3}He, ^{3}H, ^{4}He are significantly larger than the one of deuterons. Thus these nuclei are not dissociated again.

At $T \sim 10^8$ K BBN terminates because

- the temperature and density are too low
- the Coulomb barriers too high

4.2.2 - BBN Nuclei in Stars

Deuteron

- In stellar processes deuteron is quickly converted to ^{3}He
- Astronomers look at quasars: bright atomic nuclei of active galaxies, ten billion light years away.

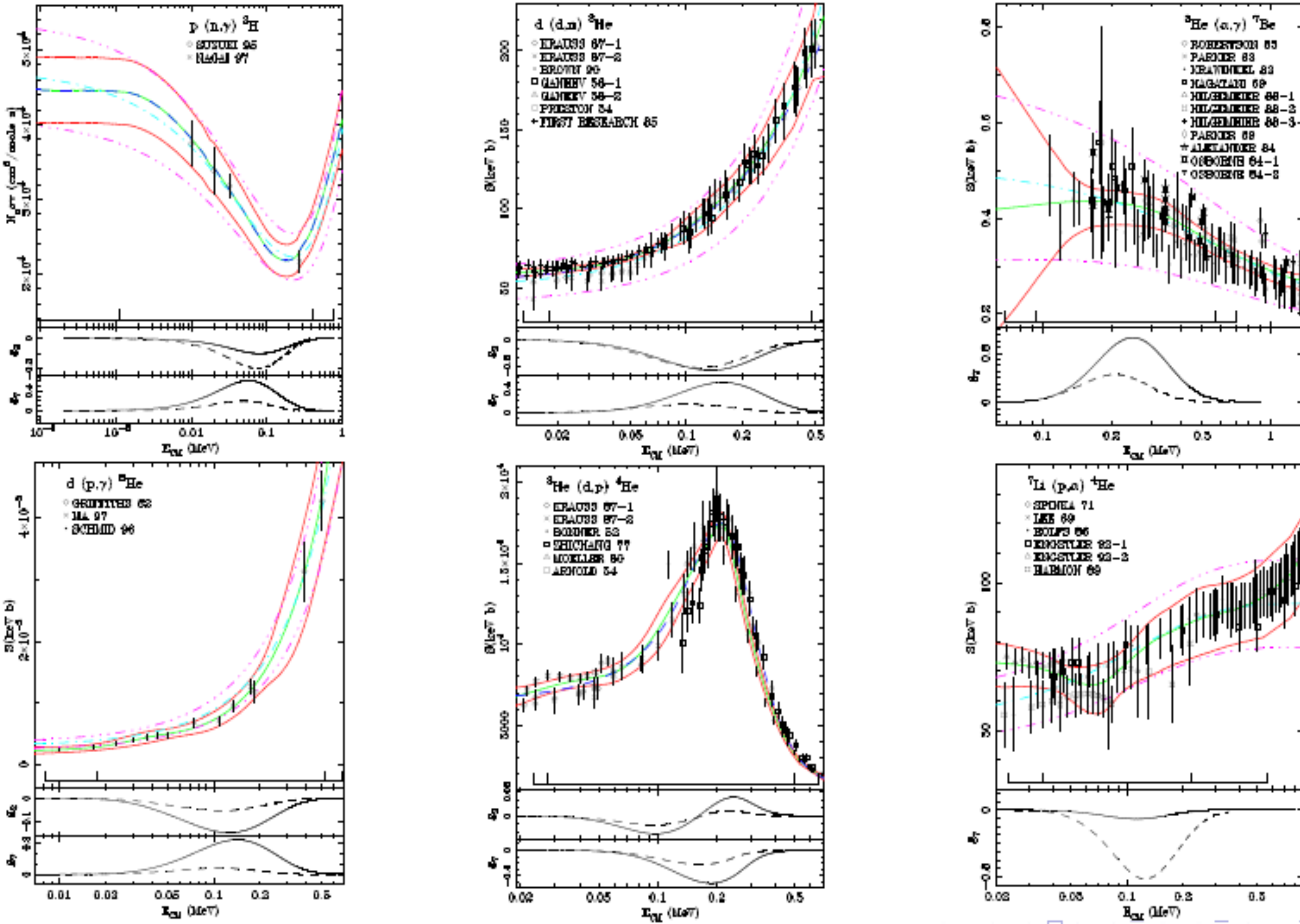
^{3}He

- Stars account for only *0.1%* of all He.
- The ^{3}He abundance in stars is difficult to deduce. Its abundance is increasing in stellar fusion.
- Scientists look to our own galaxy.

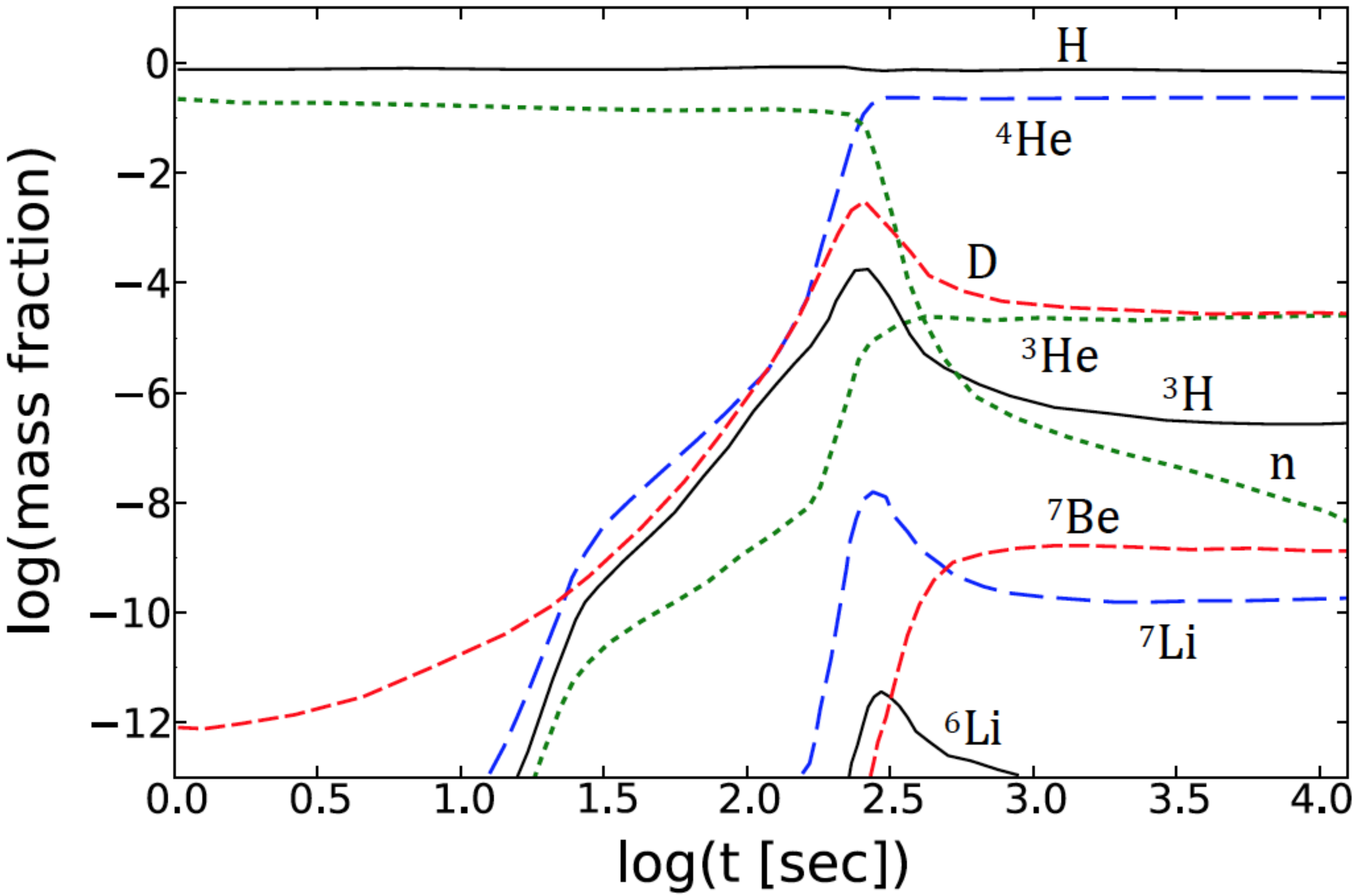
^{7}Li

- ^{7}Li can form when "cosmic rays" collide with interstellar gas.
- Observations can be made on old, cool stars in our own galaxy.
- ^{7}Li is destroyed more than it is created inside of stars.
- Very old stars have low oxygen content, and their outermost layers still contain mostly primordial ^{7}Li.

4.2.3 - Experimental S-factors for BBN reactions



4.2.4 - Time-evolution of BBN – Mass fractions



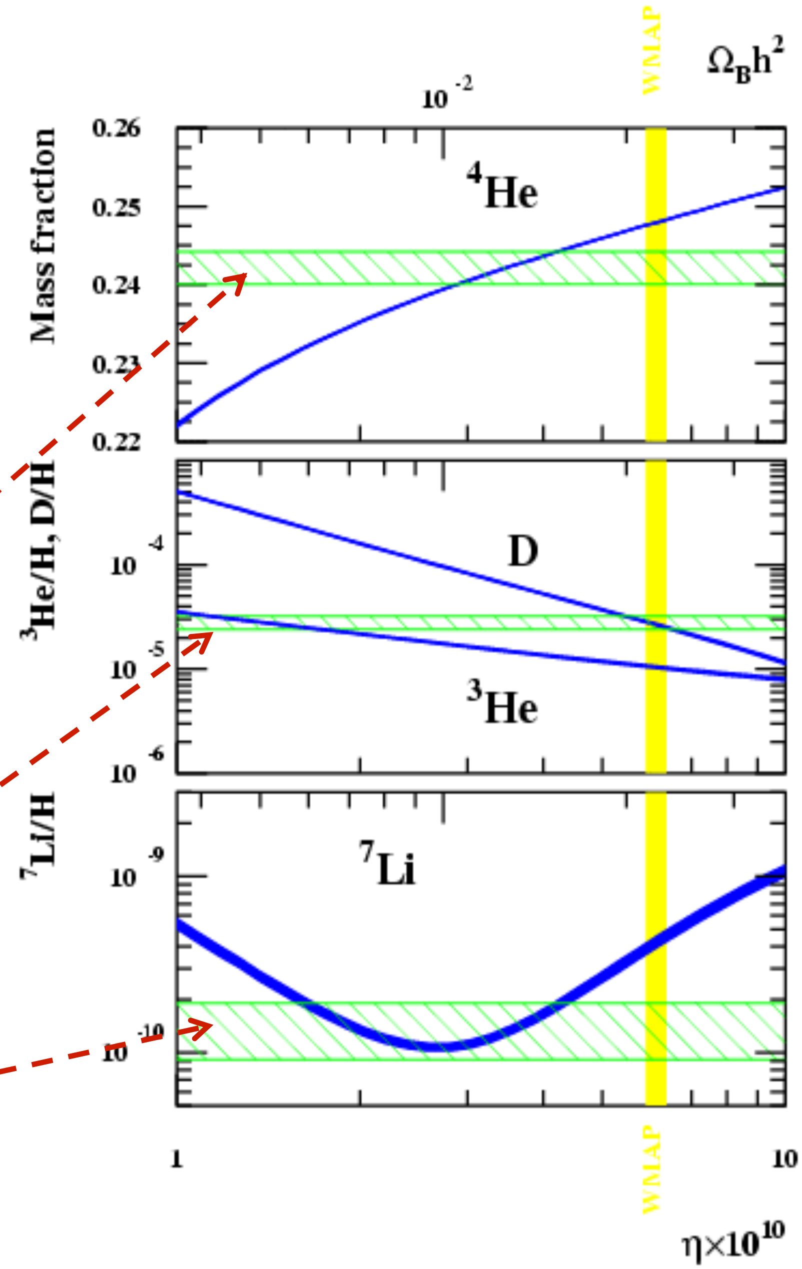
Mass fractions of light nuclei as a function of time during the BBN.

Primordial abundances

NOTE: Light elements have been made and destroyed since the Big Bang.

- Some are made in :
  - stars (^{3}He, ^{4}He),
  - spallation (scattering) (^{6,7}Li, Be, B)
  - supernova explosions (^{7}Li, ^{11}B)
- Some are destroyed:
  - d, Li, Be, B are very fragile; they are destroyed in the center of stars

→ observed abundances (at surface of stars) do not reflect the destruction inside



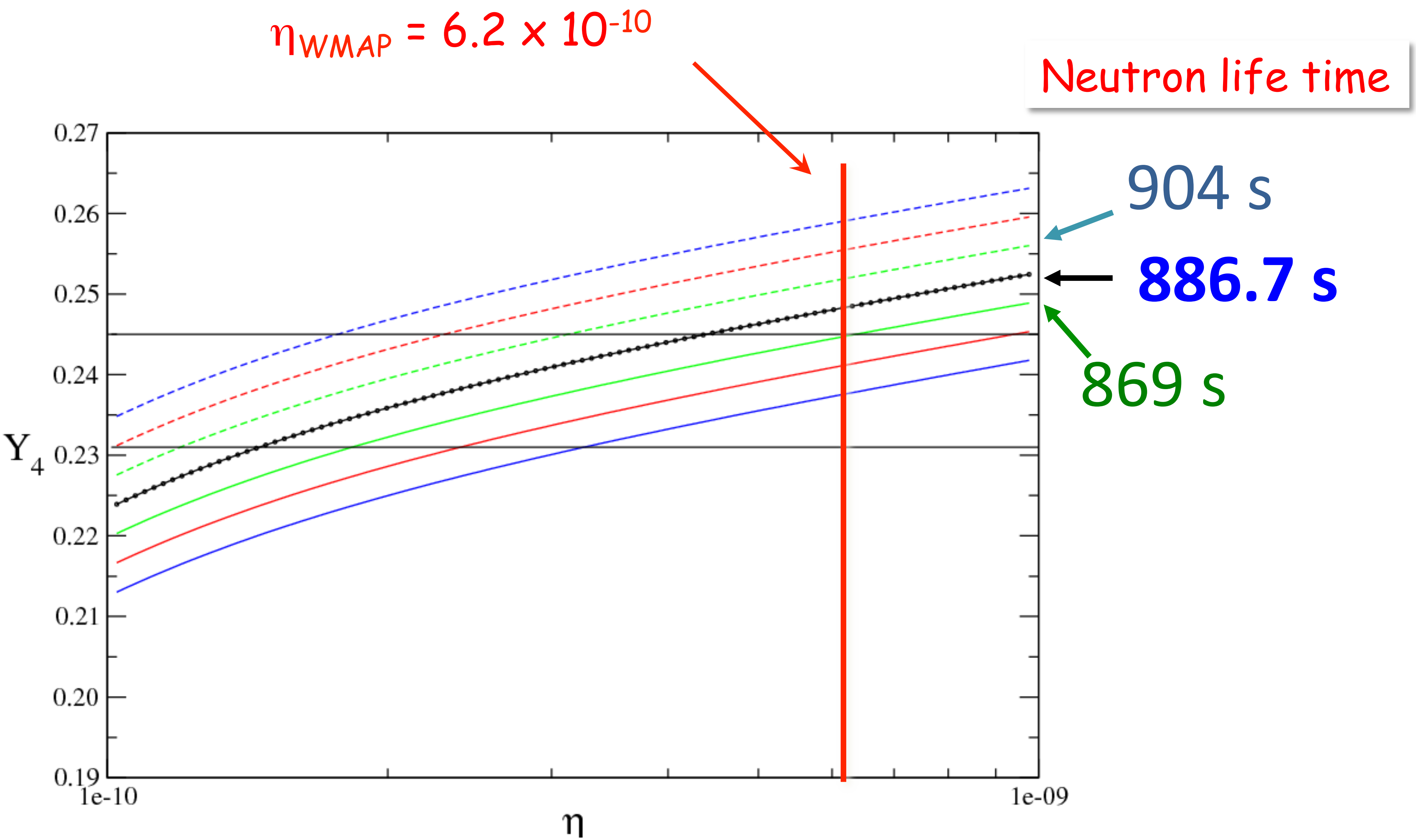
^{4}He : Y_p = 0.2421 ± 0.0021

D : D/H = (2.78 + 0.44 - 0.38) x 10^{-5}

^{7}Li : Li/H = (1.23 + 0.68 - 0.32) x 10^{-10}

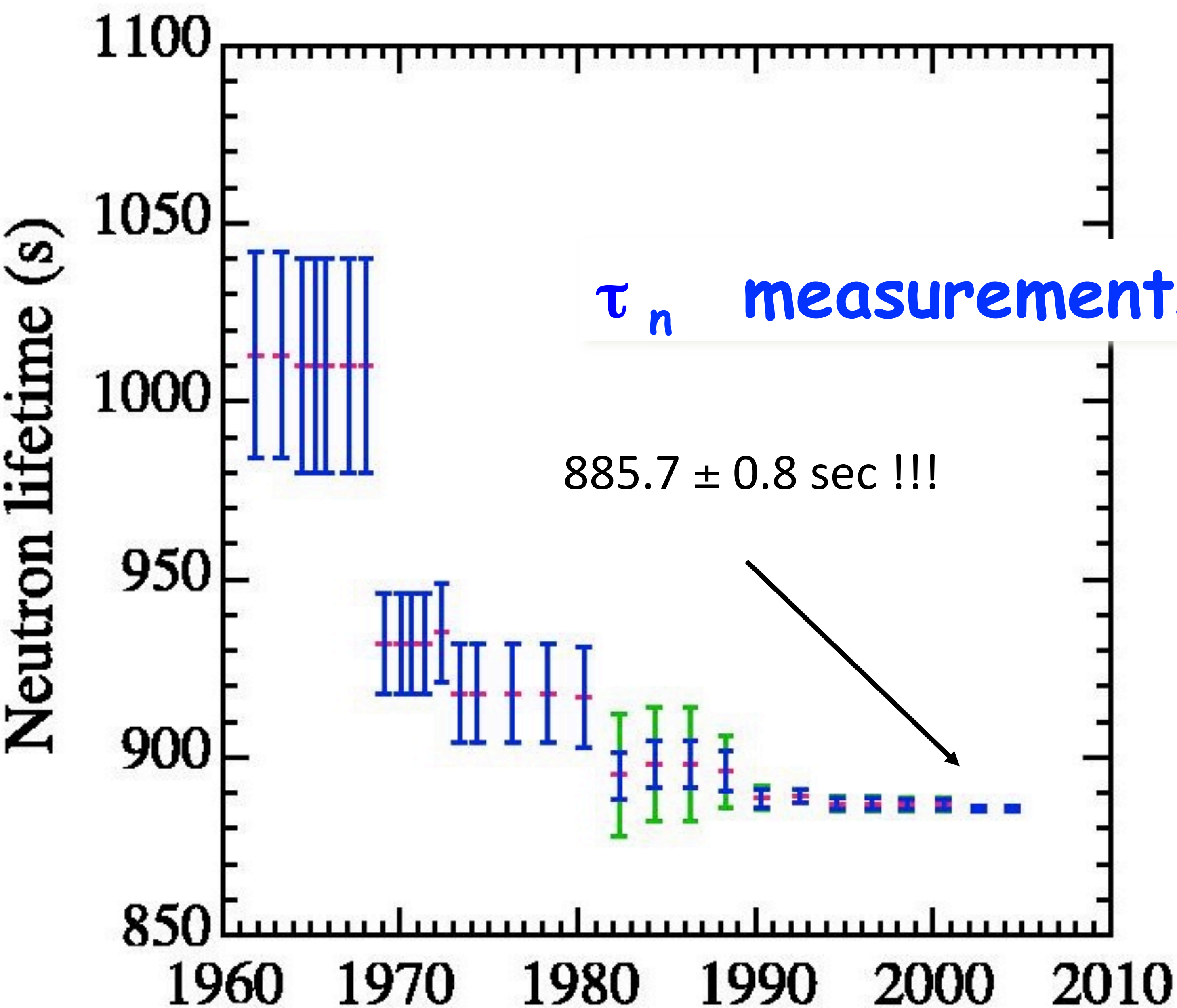
$\Omega_B h^2$ = 0.0224 ± 0.0009

4.2.5 - BBN predictions: Neutron lifetime



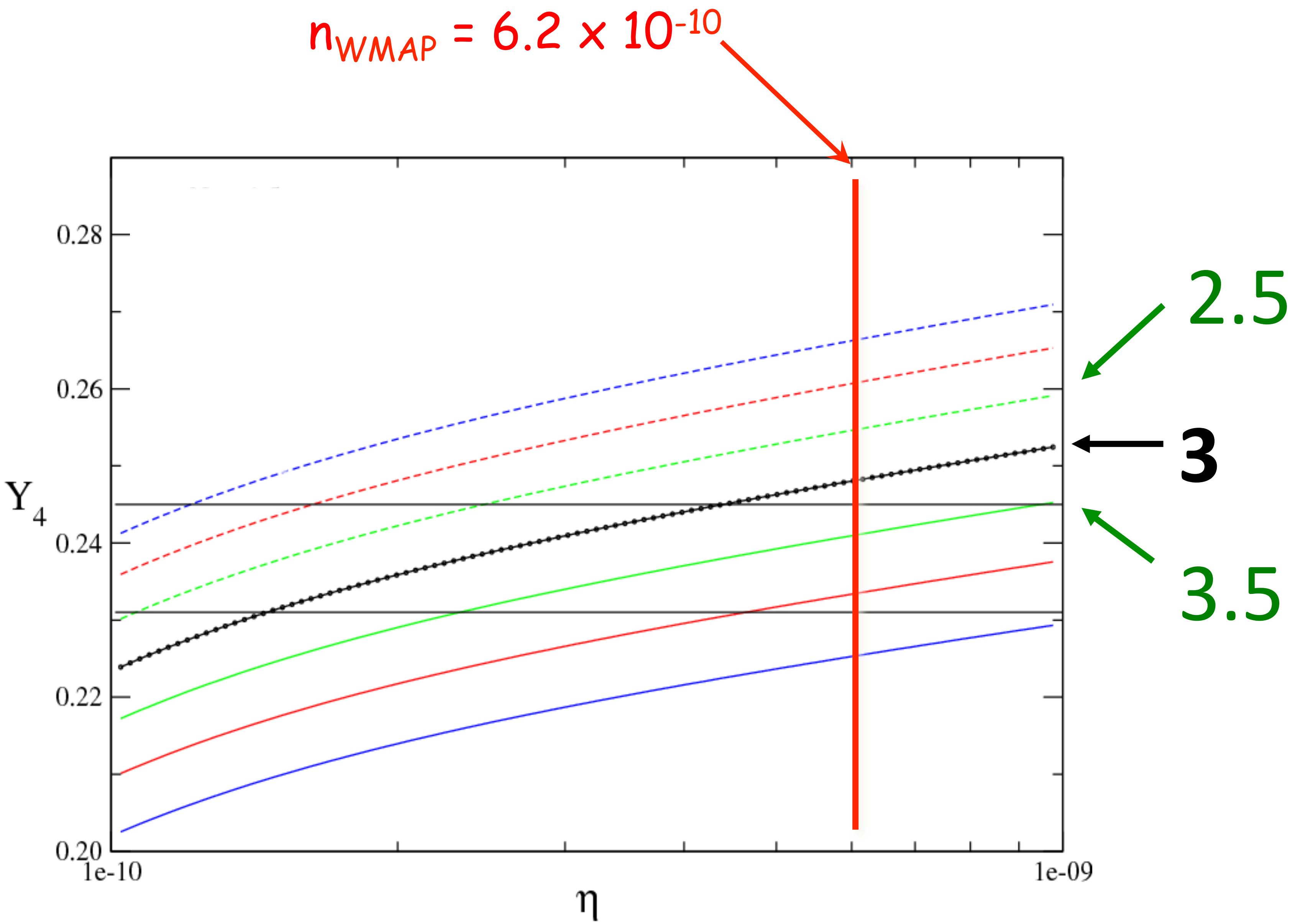
Helium mass fraction calculated with the BBN model as a function of the baryon-to-photon ratio parameter η.

Experiments on Neutron Lifetime



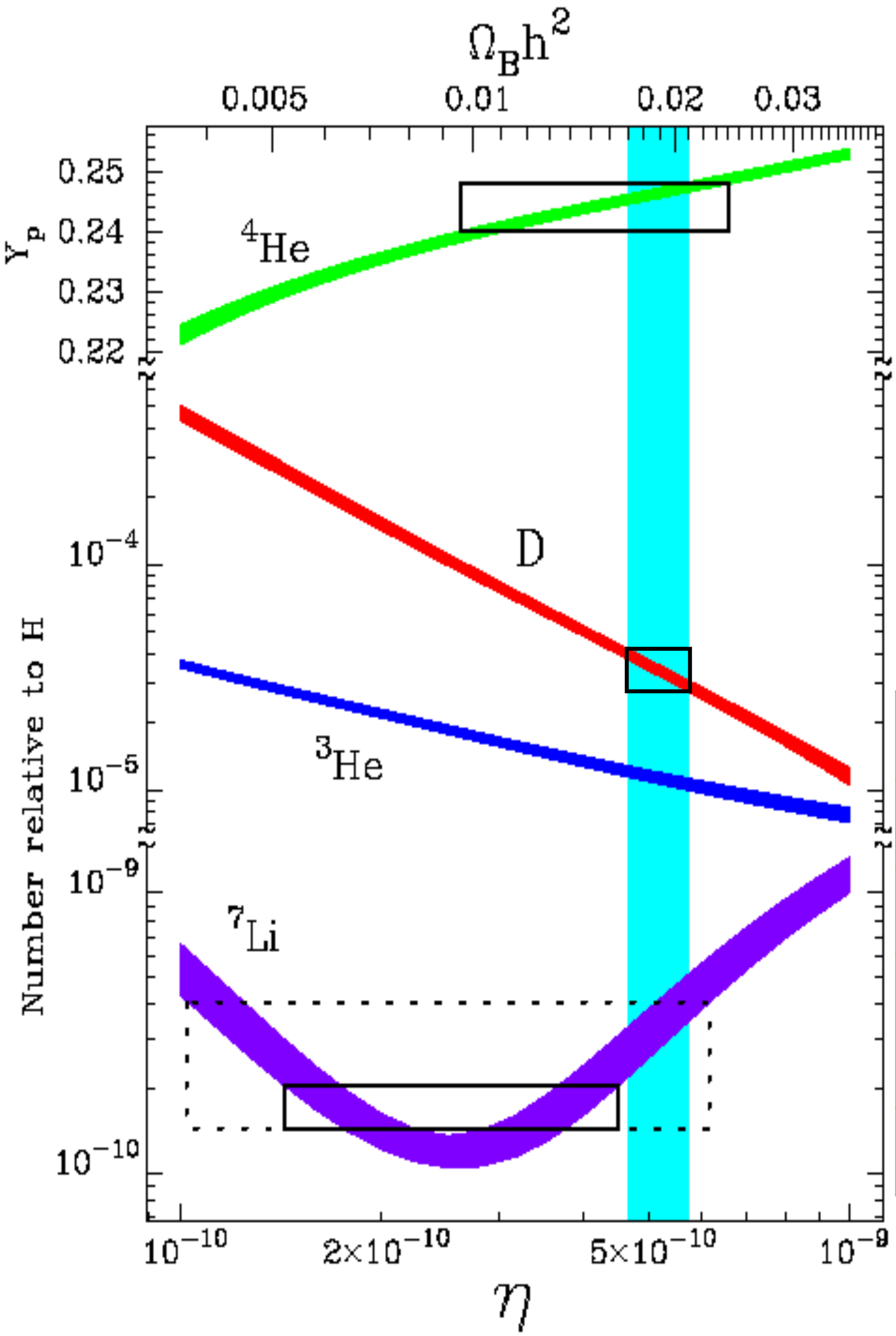
τ_n measurements vs. time

BBN predictions: Neutrino Families



Helium mass fraction calculated with the BBN model as a function of the number of neutrino families.

BBN incredibly successful, except for Lithium problem



SBBN (Standard BBN): one parameter

baryon-to-photon ratio η

$\eta = (6.225^{+0.157}_{-0.154}) \times 10^{-10}$
(WMAP 2010)

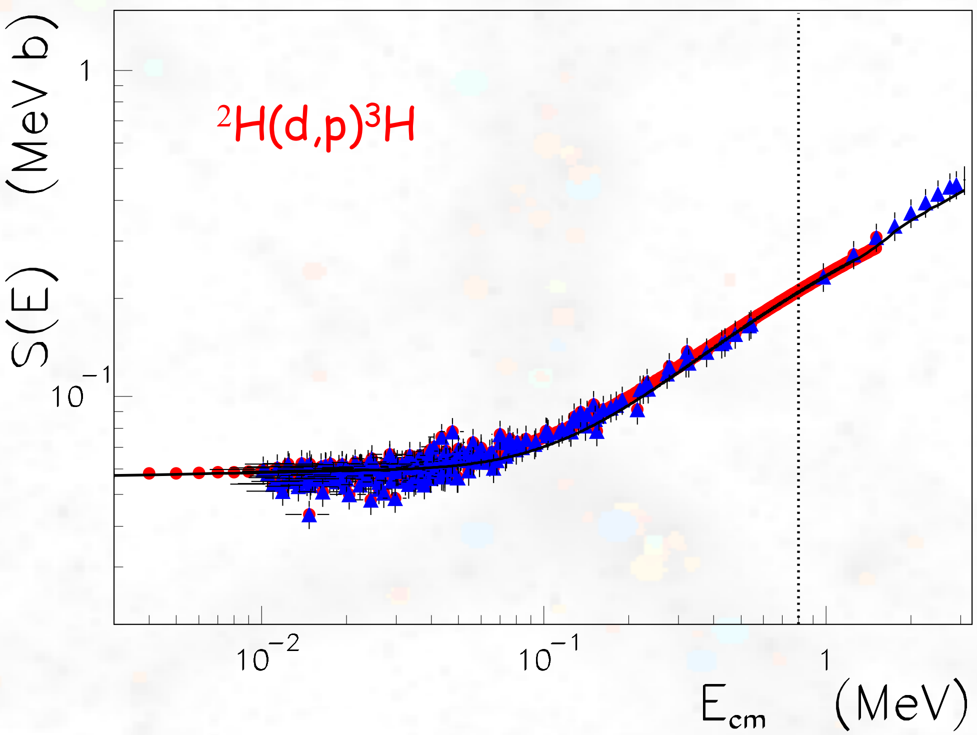
	BBN	Observation
^{4}He	0.242	0.242
D/H	2.62×10^{-5}	2.78×10^{-5}
^{3}He/H	0.98×10^{-5}	$(0.9 - 1.3) \times 10^{-5}$
^{7}Li/H	4.39×10^{-10}	1.2×10^{-10}

4.3 - Lithium problem

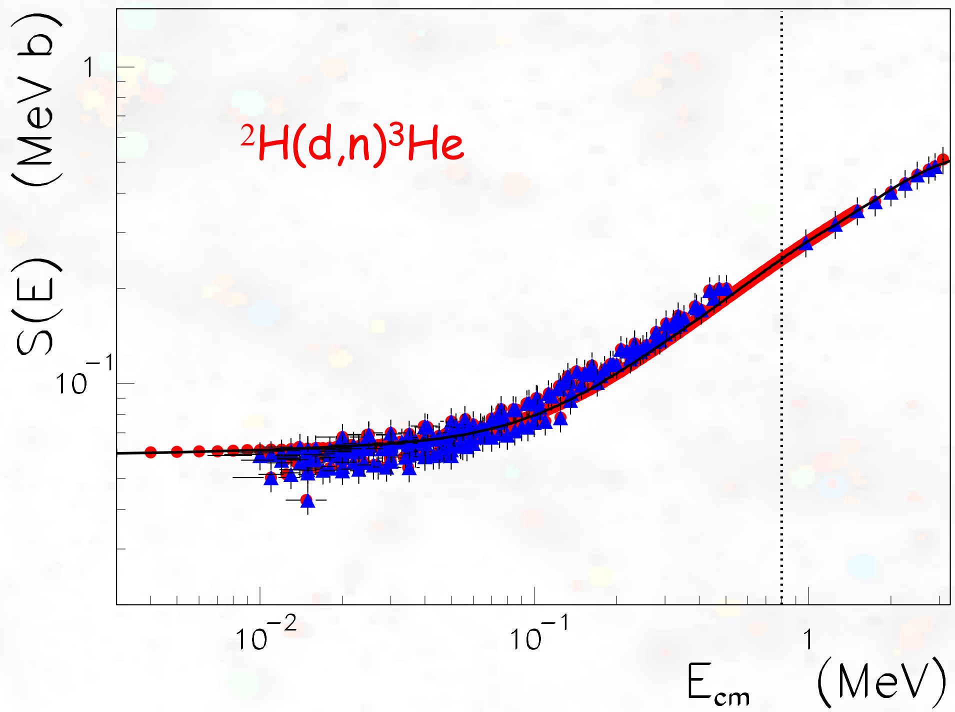
The SBBN model explains very well the abundance of light elements, except for the observed ^{6}Li and ^{7}Li. While the ^{6}Li abundance is difficult to explain because of "astration", i.e., ^{6}Li reprocessing in stars, no theory or model can explain why the observed ^{7}Li abundance is so much smaller than predicted.

For a recent status of the lithium problem, see:
- "Observation of interstellar lithium in the low-metallicity Small Magellanic Cloud", J. C. Howk, N. Lehner,B. D. Fields, G. J. Mathews, Nature 489, 121 (2012).

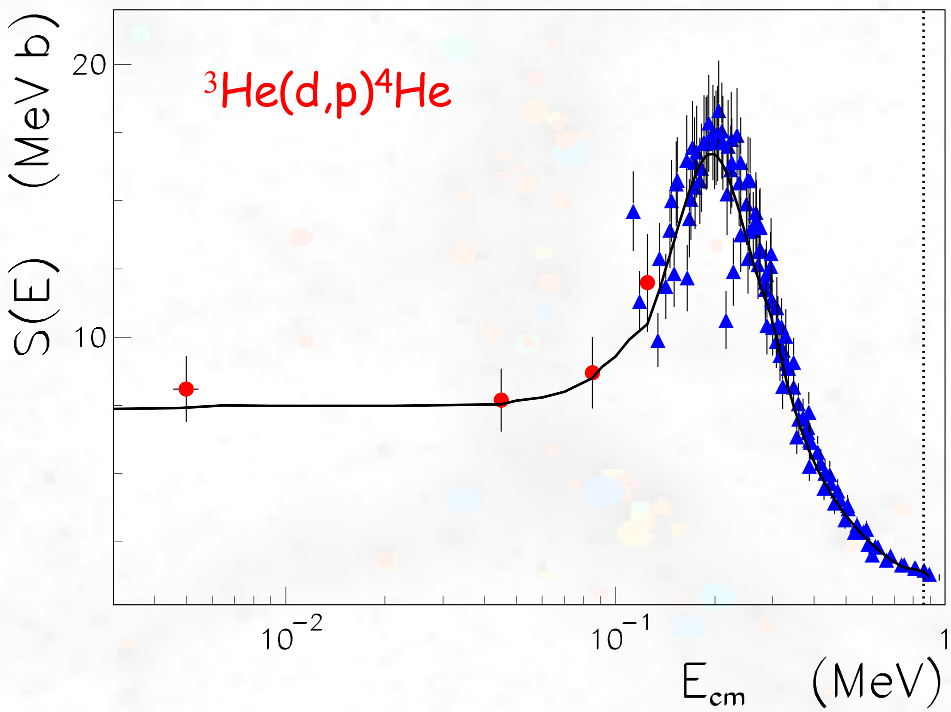
4.3.1 - Problem with nuclear data? Recent data



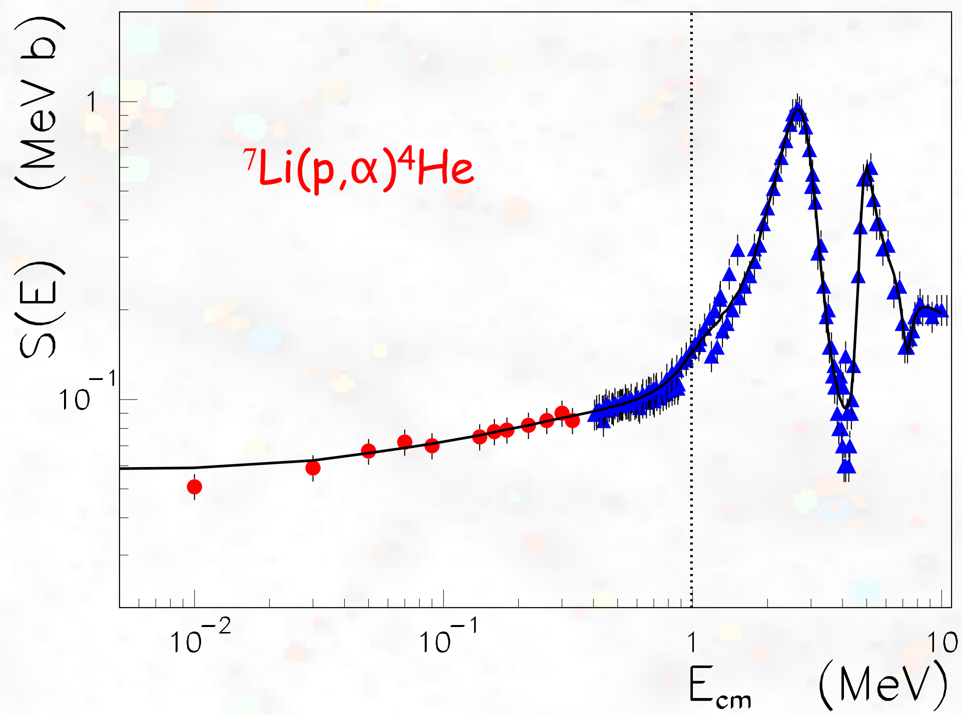
Pizzone, Sparta, CB, et al.,
Astrop. J. 786, 112 (2014)



Recent data

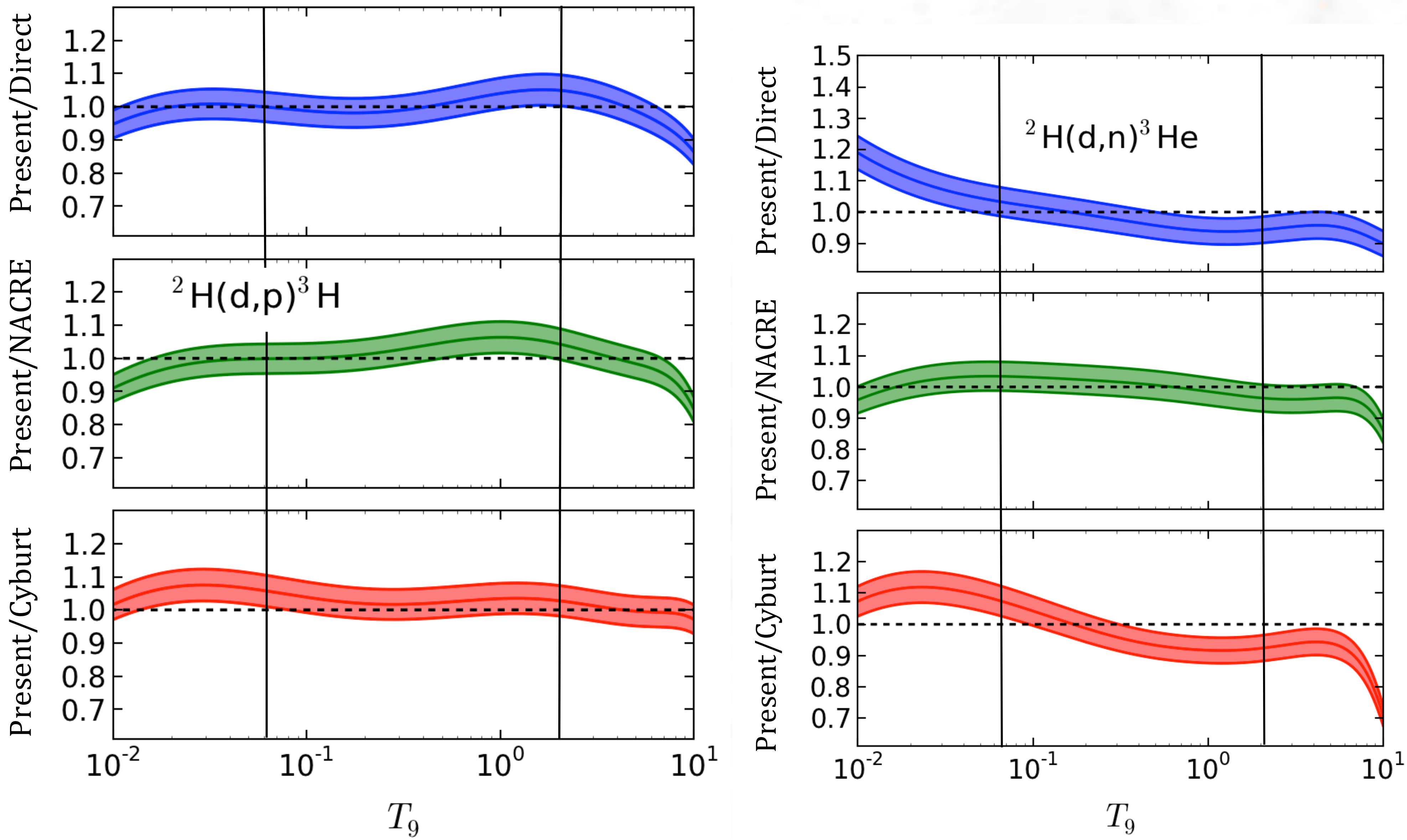


$^3He(d,p)^4He$
S(E) (MeV b)
20
10
10^{-2}
10^{-1}
1
E_{cm} (MeV)

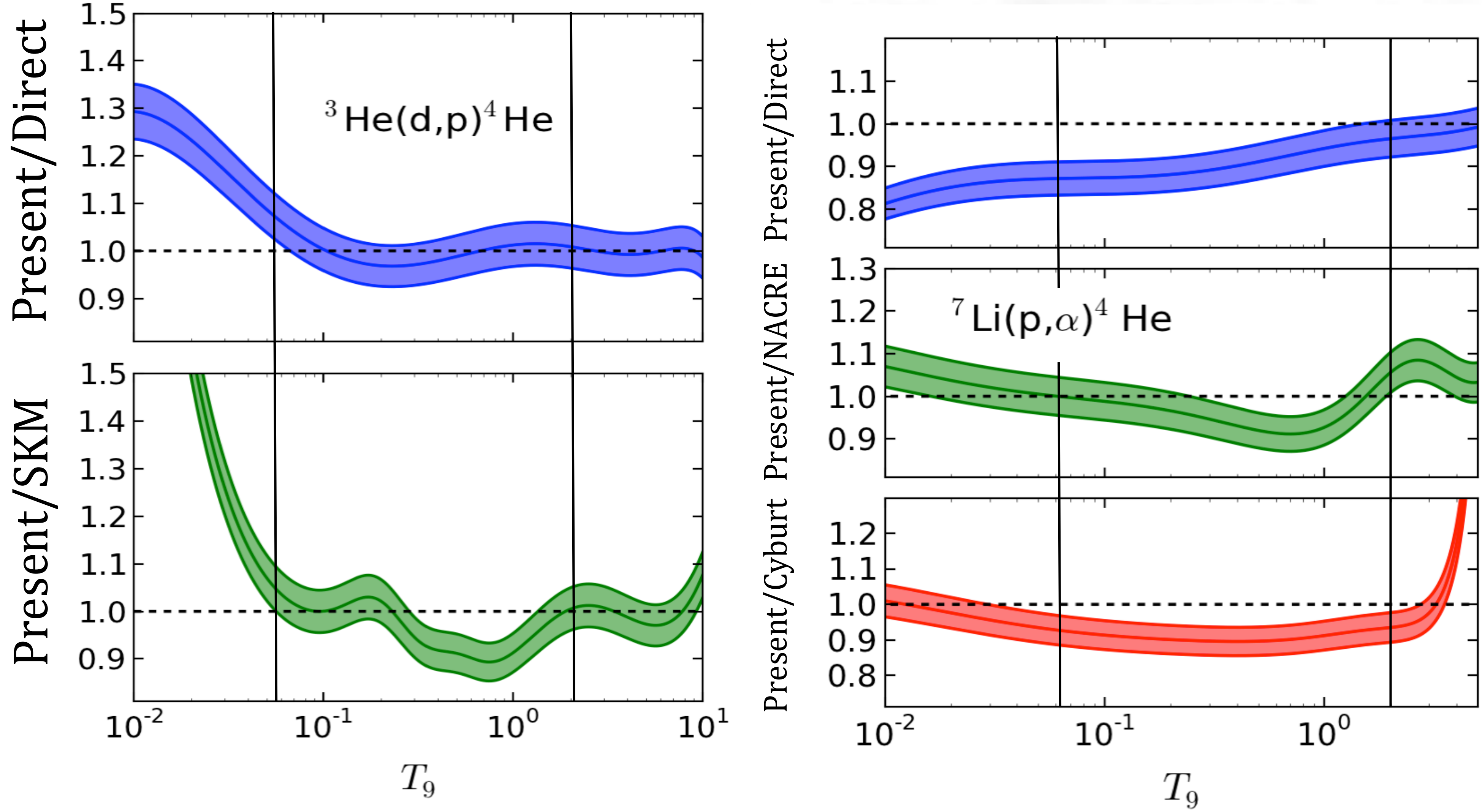


$^7Li(p,\alpha)^4He$
S(E) (MeV b)
1
10^{-1}
10^{-2}
10^{-1}
1
10
E_{cm} (MeV)

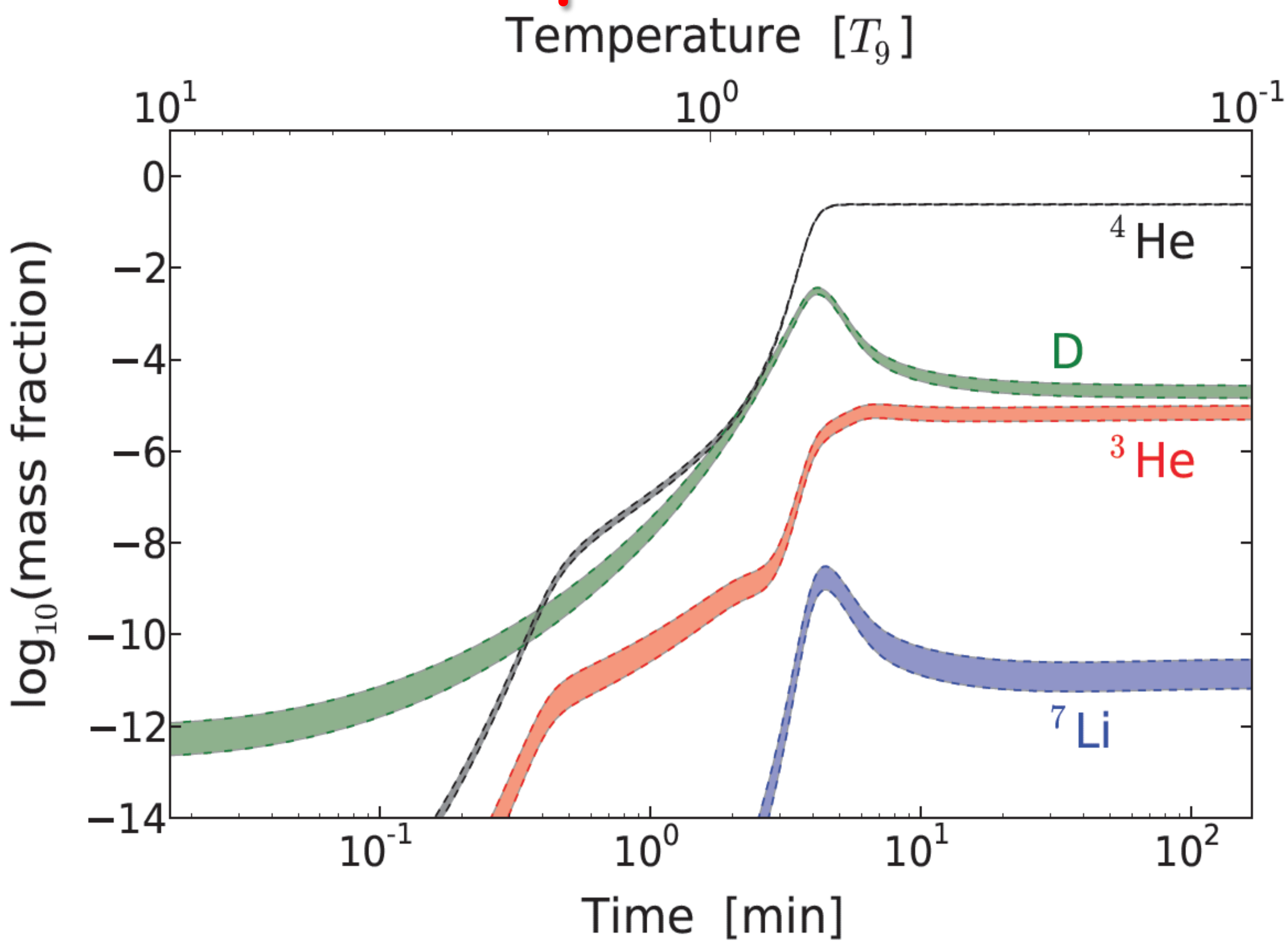
Comparison to other compilations



Comparison to other compilations

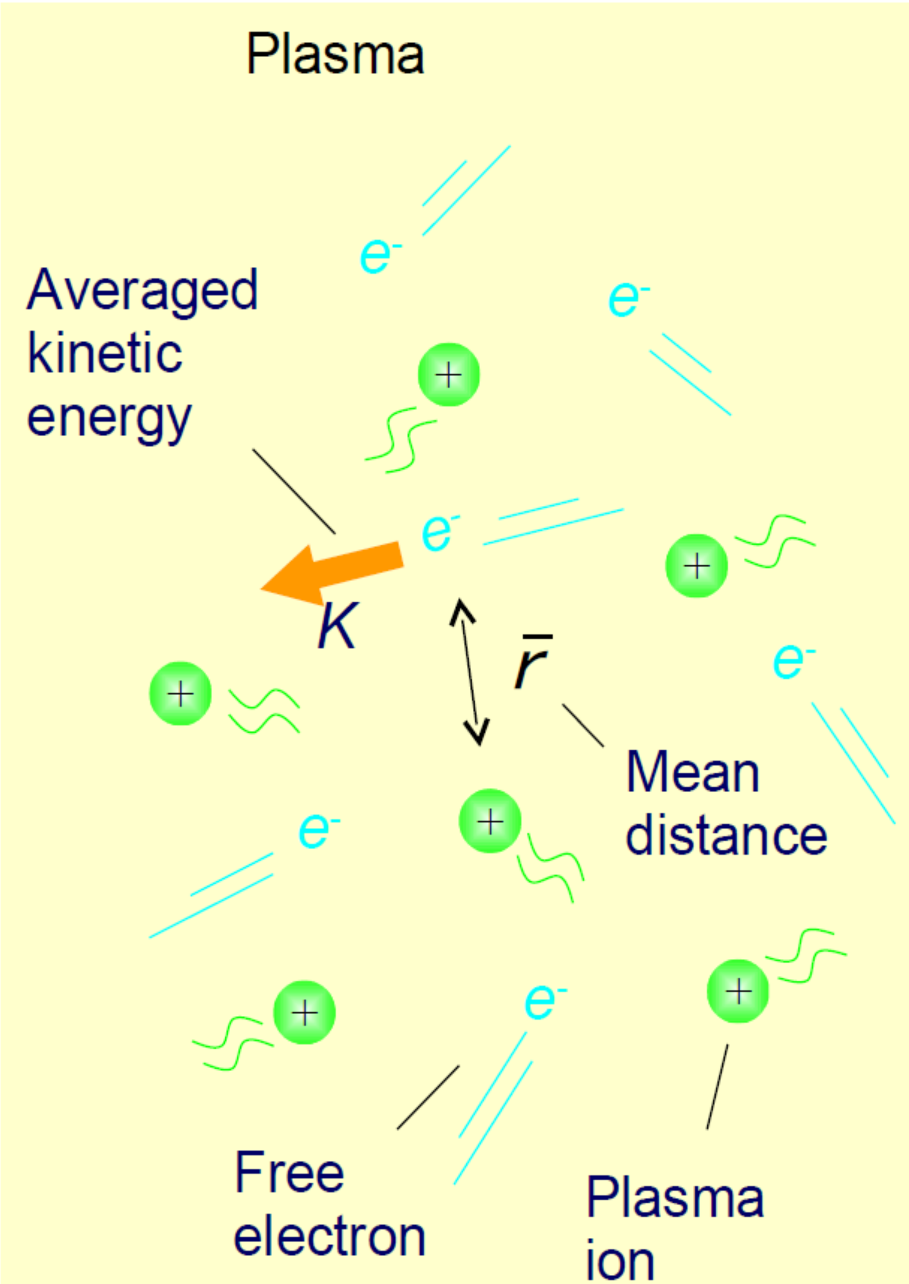


BBN predictions with experimental errors



	BBN	Observation
^{4}He/	0.2485 (+ 0.001 – 0.002)	0.256 ± 0.006
D/H	2.692 (+ 1.77 – 0.07) x 10^{-5}	(2.82 ± 0.26) x 10^{-5}
^{3}He/H	0.9441 (+ 0.511 – 0.466) x 10^{-5}	(0.9 - 1.3) x 10^{-5}
^{7}LI/H	4.683 (+ 0.335 – 0.292) x 10^{-10}	(1.58 ± 0.31) x 10^{-10}

4.3.2 - Medium effects? Screened big bang



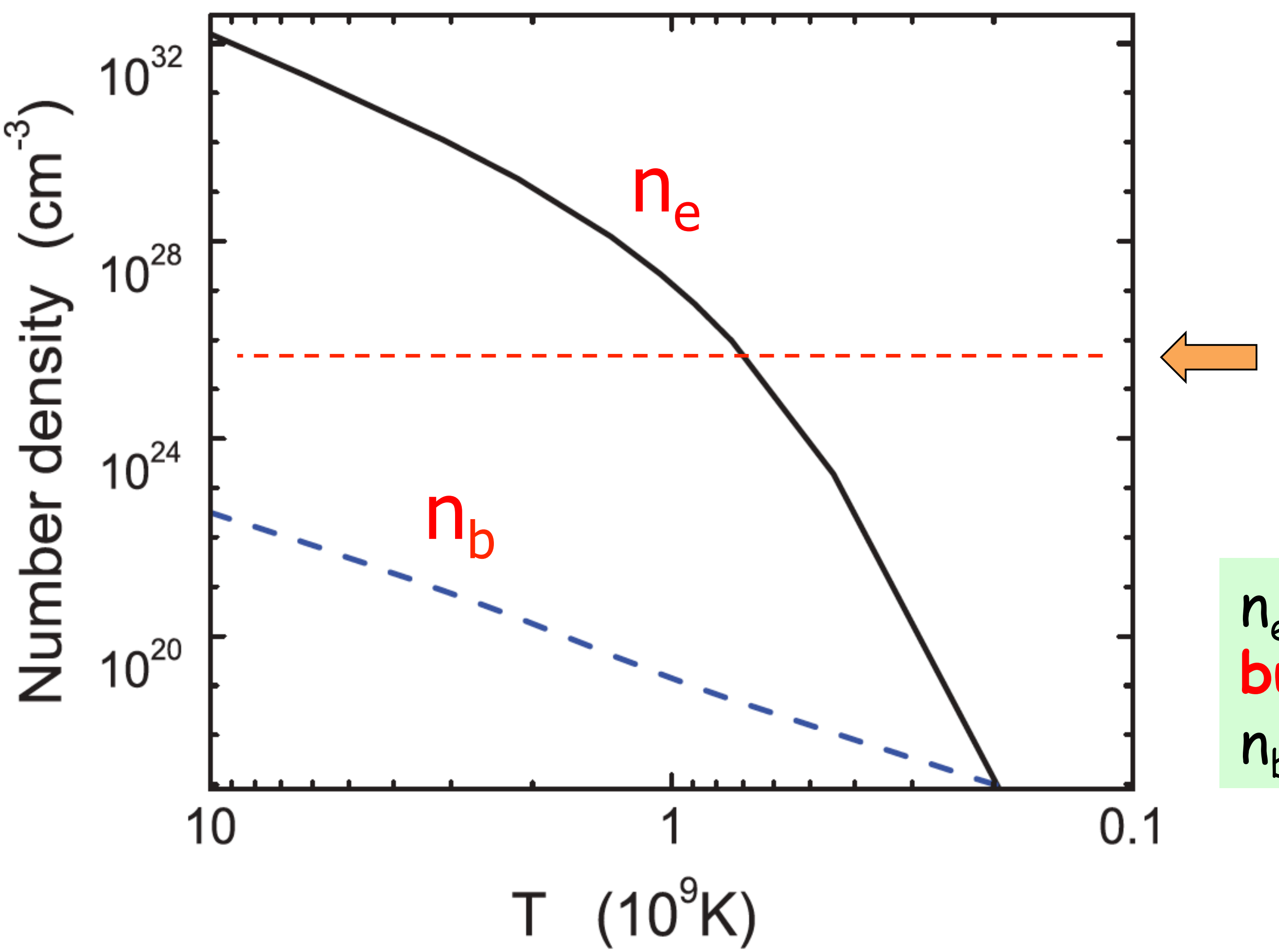
Interactions with photons/ electrons of the plasma

Change in the e.m. equation of state due to photon/electron thermal masses $P = P(\rho)$

Wang, CB, Balantekin, Phys. Rev. C 83, 018801 (2011)

Electron density in BBN

mostly due to $\gamma \rightarrow e^+e^-$

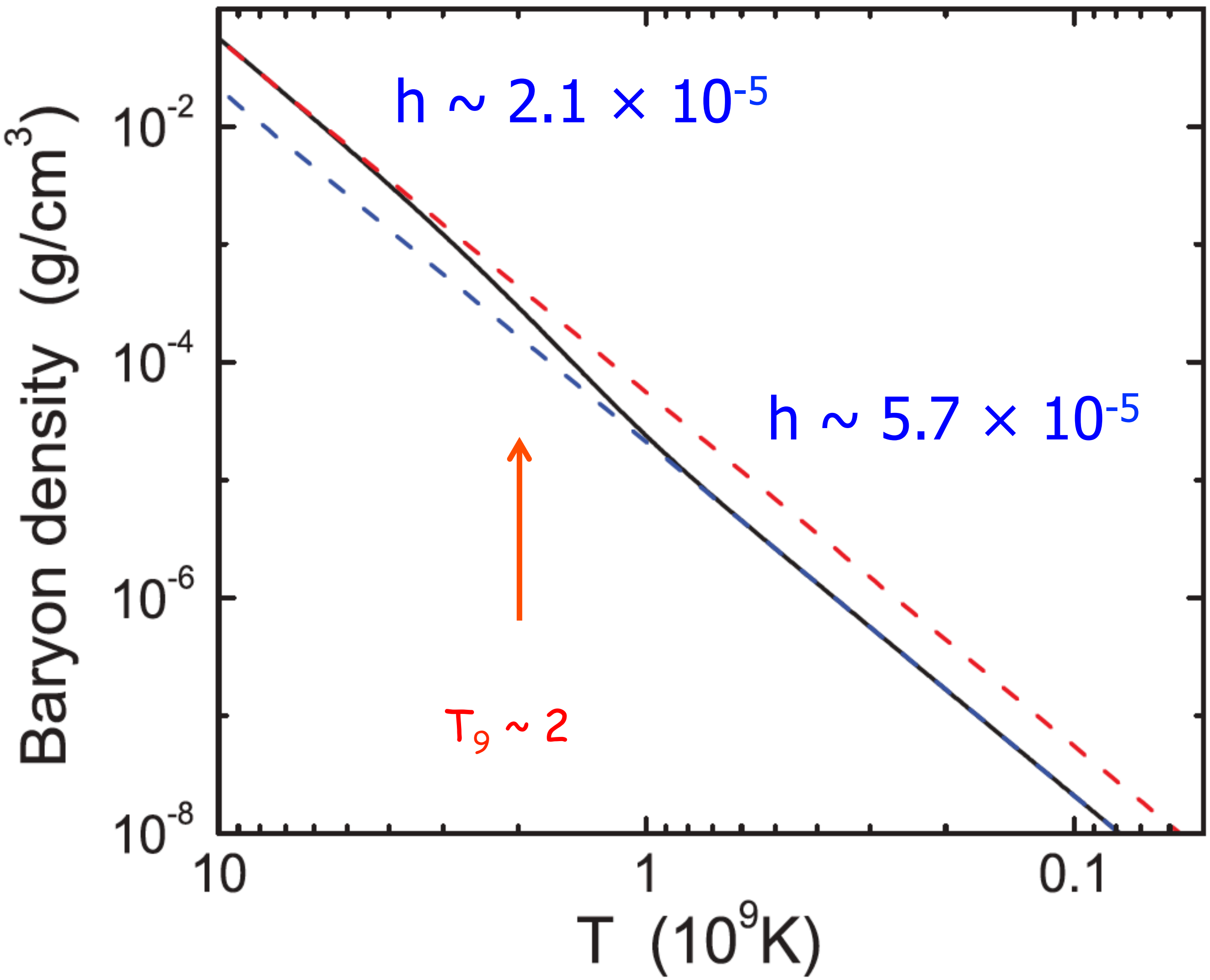


n_e center of sun

$n_e(BB) \sim n_e(sun)$
but
$n_b(sun) > n_b(BB)$

Baryon density

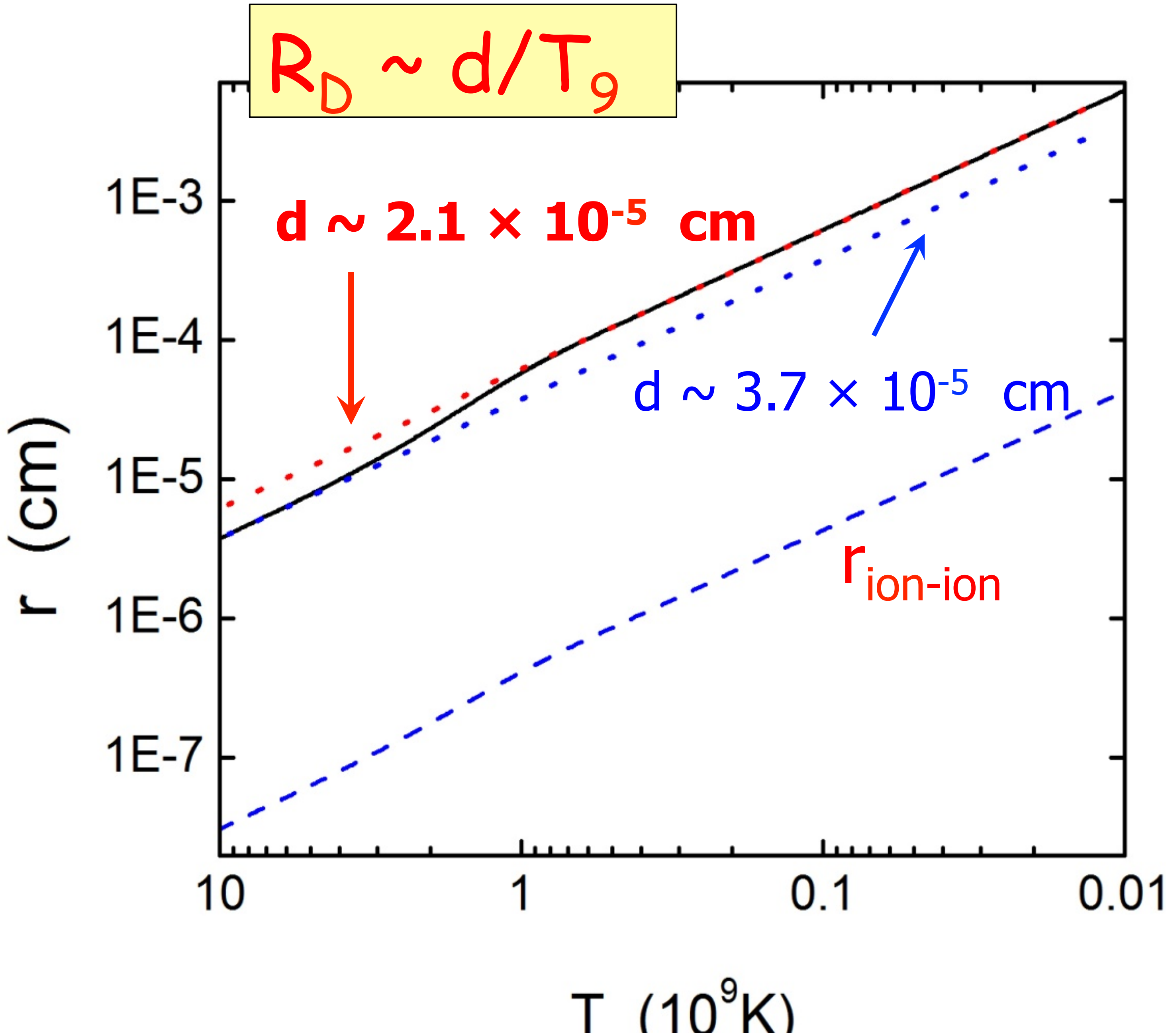
(4.25)



$\rho_b \sim h T_9^3$

degrees of freedom changes around $T_9 \sim 1$

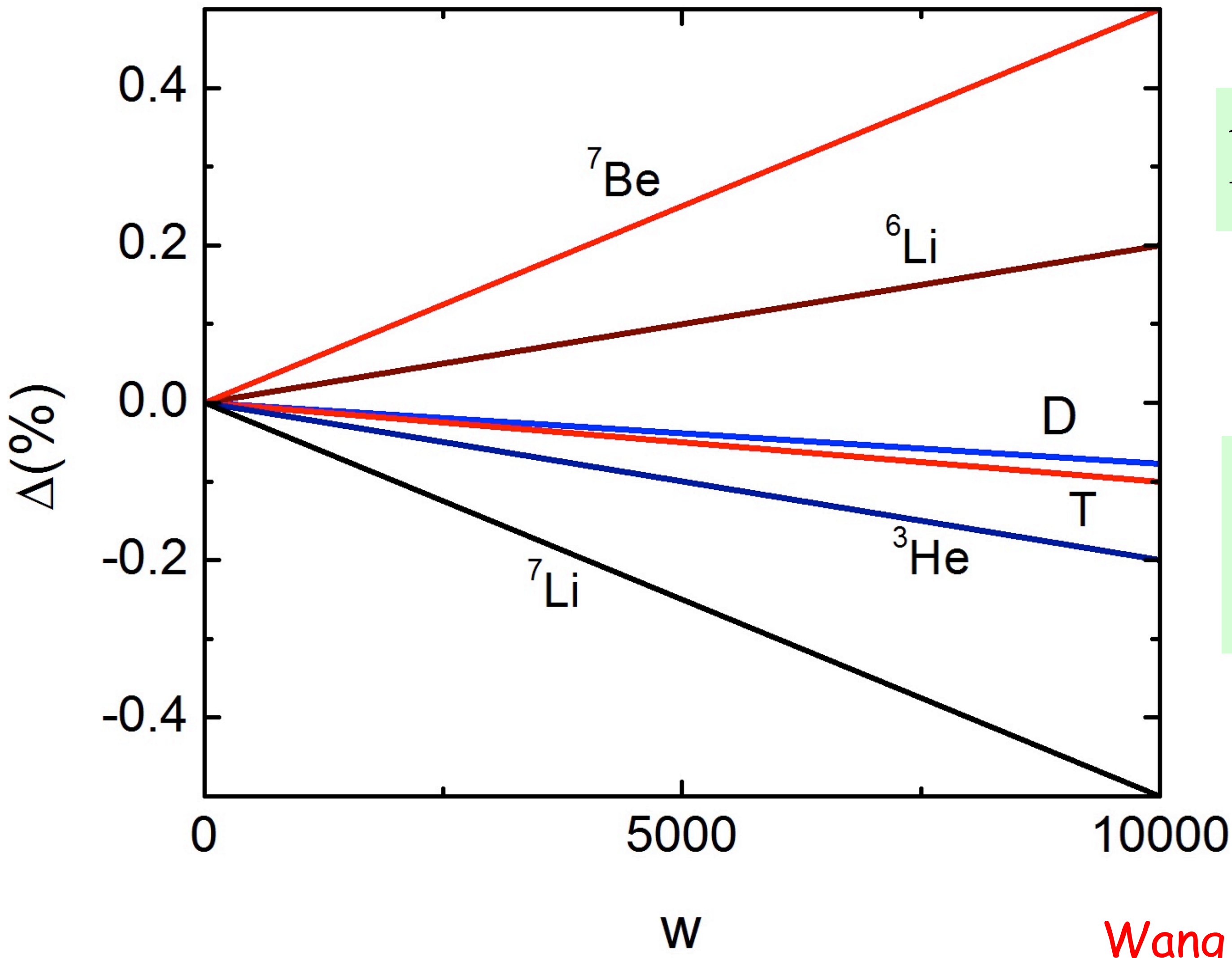
Debye radius × inter-ion distance



≥ 1000 particles in Debye sphere

mean-field model OK

If screening effect were much larger



$$\ln f'_{BBN} = w \ln f_{BBN} \quad (4.26)$$

$$\Delta = \frac{Y' - Y}{Y} \times 100 \quad (4.27)$$

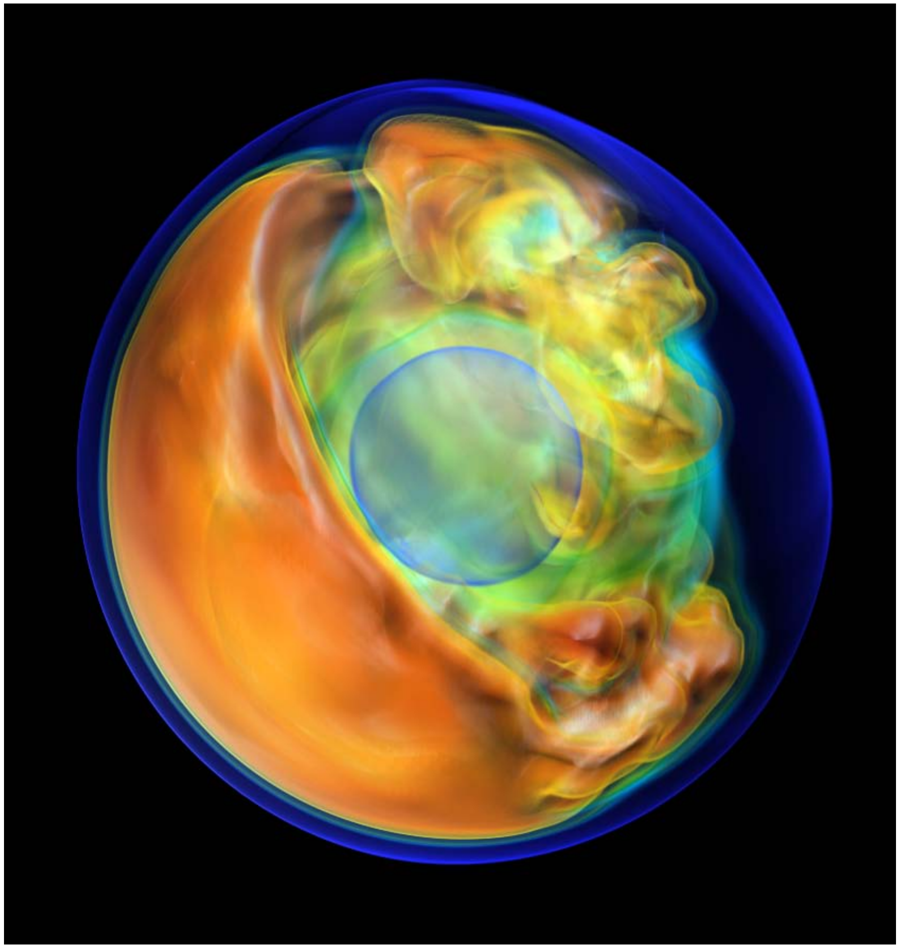
Wang, CB, Balantekin,
Phys. Rev. C 83, 018801 (2011)

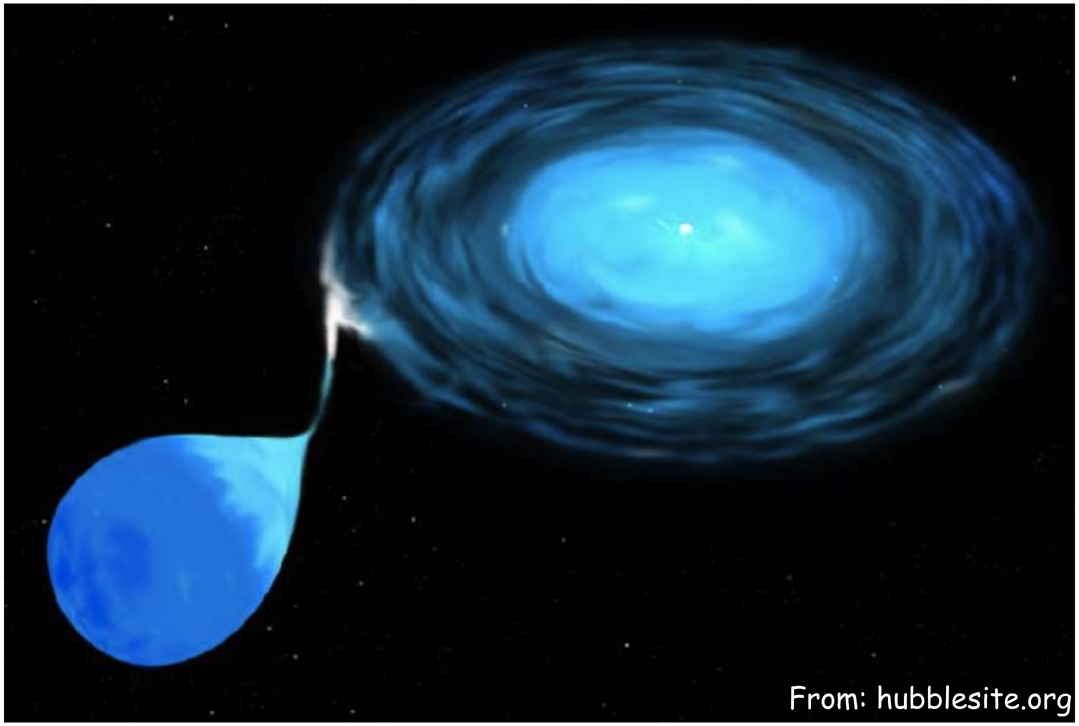
BBN screening of reaction rates by electrons negligible.

4.3.3 - Non-Maxwellian thermal distribution

- Deviation from Boltzmann-Gibbs statistics very popular in plasma physics → turbulence phenomena, systems having memory effects, systems with long range interactions, etc.

- Relevance for cataclysmic stellar systems, e.g. supernovae?





Extensive statistics

Standard Boltzmann-Gibbs statistics

$$S = -k_B \sum_{i=1}^{n} p_i \ln p_i$$

Has a maximum when all states have equal probability p_i (4.28)

Two simple constraints (**normalization** and **mean value of the energy**):

$$\int_0^\infty p(\varepsilon)\, d\varepsilon = 1 \qquad (4.29)$$

$$\int_0^\infty \varepsilon p(\varepsilon)\, d\varepsilon = \text{const} \qquad (4.30)$$

➡ distribution function:

$$p_i = \frac{e^{-\beta\varepsilon_i}}{\sum_{i=1}^{n} e^{-\beta\varepsilon_i}} \qquad (4.31)$$

$$\beta = 1 / k_B T$$

➡ thermodynamics

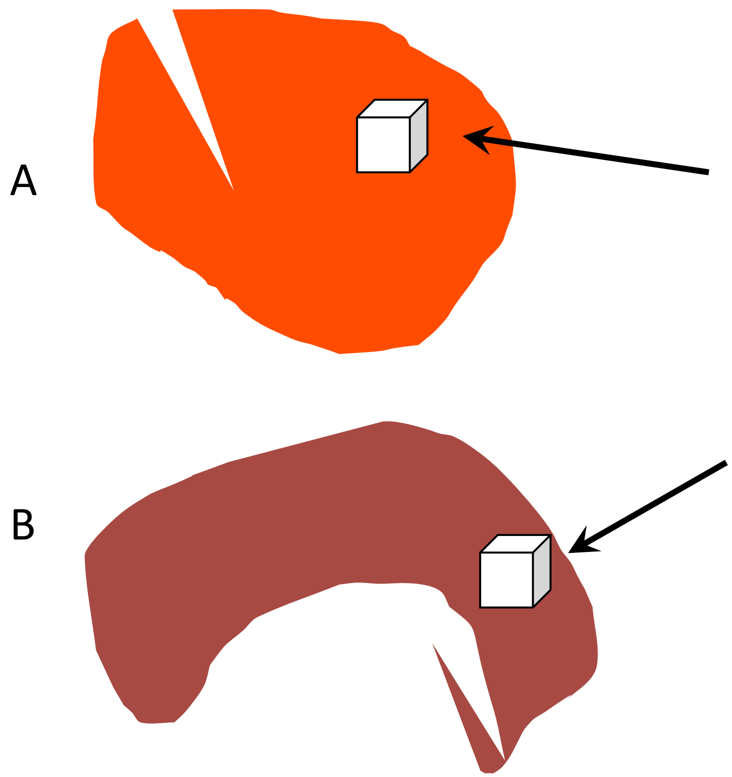
Extensive statistics

$$S = -k_B \sum_{i=1}^{n} p_i \ln p_i$$

Assumption: particles independent → no correlations

Hypothesis: isotropy of velocity directions → extensivity

+ microscopic interactions short ranged, Euclidean space time, etc.



p_i^A

p_j^B

$$p_{i+j}^{A+B} = p_i^A \cdot p_j^B \qquad (4.32)$$

$$S_{A+B} = S_A + S_B \qquad (4.33)$$

extensive

Non-extensive statistics

Extensive statistics not applicable for long-range interactions

THUS

→ **introduce correlations** via non-extensive statistics

→ derive corresponding entropy distribution

Renyi, 1955 - Tsallis, 1985

$$S_q = k_B \frac{1 - \sum_{i=1}^{n} p_i^q}{1 - q} \qquad (4.34)$$

$$S_q(A+B) = S_q(A) + S_q(B) + \frac{(1-q)}{k_B} S_q(A) S_q(B) \qquad (4.35)$$

departure from extensivity

Non-extensive statistics

$$S_q = k_B\left(1 - \sum_{i=1}^{n} p_i^q\right) / (q-1) \quad (4.36)$$

- *q is Tsallis parameter*: in general labels an infinite family of entropies

- S_q is a natural generalization of Boltzmann-Gibbs entropy which is **restored for q = 1**:

$$\lim_{q \to 1} S_q = -k_B \sum_{i=1}^{n} p_i \ln p_i \quad (4.37)$$

- BG formalism yields exponential equilibrium distributions, whereas non-extensive statistics yields (asymptotic) **power-law distributions**

- Renyi entropy is related through a monotonic function to the Tsallis entropy (with k_B = 1)

$$S_q^R = \ln\left(\sum_{i=1}^{n} p_i^q\right) / (1-q) = \ln\left[1-(1-q)S_q\right] / (1-q) \quad (4.38)$$

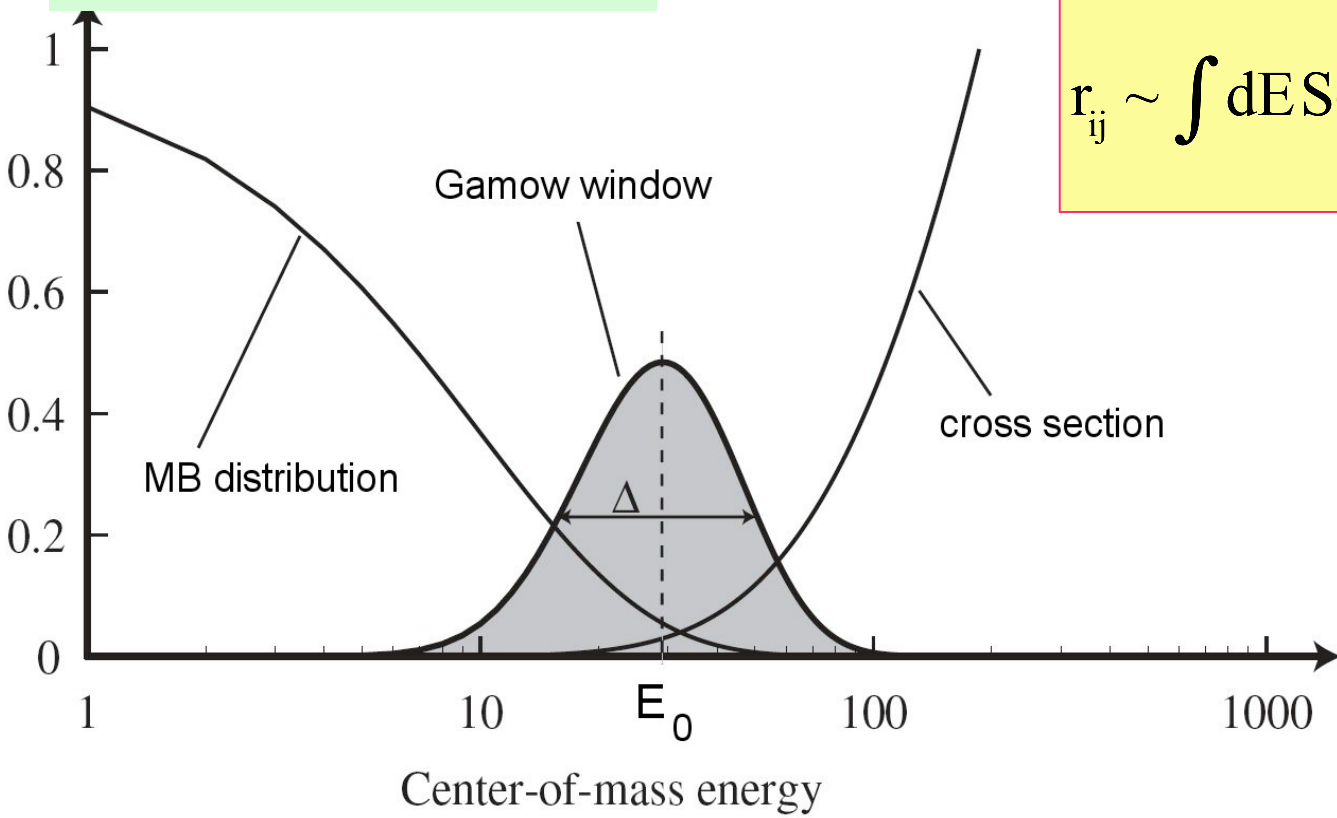
Maxwell distribution & reaction rates

$$r_{ij} = \frac{n_i n_j}{1+\delta_{ij}} \langle \sigma v \rangle$$

CB, Fuqua, Hussein,
Ap. J. 167, 67 (2013)

$$\sigma = \frac{S(E)}{E} \exp\left[-\frac{Z_i Z_j}{\hbar v} \right]$$

$$r_{ij} \sim \int dE\, S(E) \exp\left[-\left(\frac{E}{k_B T} \right) + 2\pi\eta(E) \right]$$



Non-Maxwellian distribution

$$f(E) = \exp\left[-\frac{E}{k_B T}\right] \quad \rightarrow \quad f_q(E) = \left(1 - \frac{q-1}{k_B T}E\right)^{\frac{1}{q-1}} \qquad (4.39)$$

$$(4.40) \qquad \lim_{q \rightarrow 1} f_q \quad \rightarrow \quad \exp\left[-\frac{E}{k_B T}\right]$$

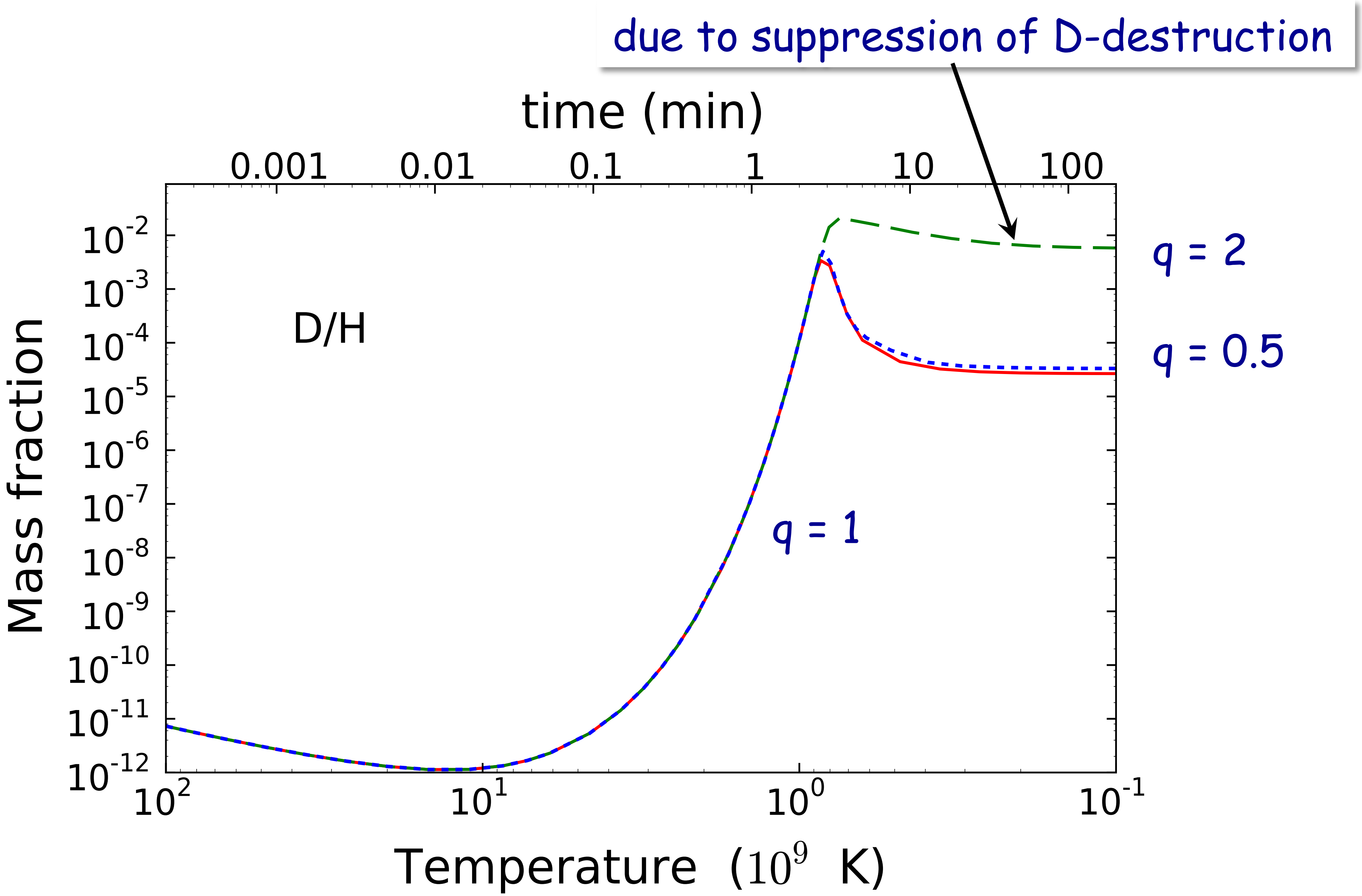
$$0 \le E \le \frac{k_B T}{q-1}, \qquad \text{if } q \ge 1$$
$$0 \le E \le \infty, \qquad \text{if } q \le 1 \qquad (4.41)$$

$$r_{ij} \sim \int dE\, S(E) M_q(E,T) \qquad (4.42)$$

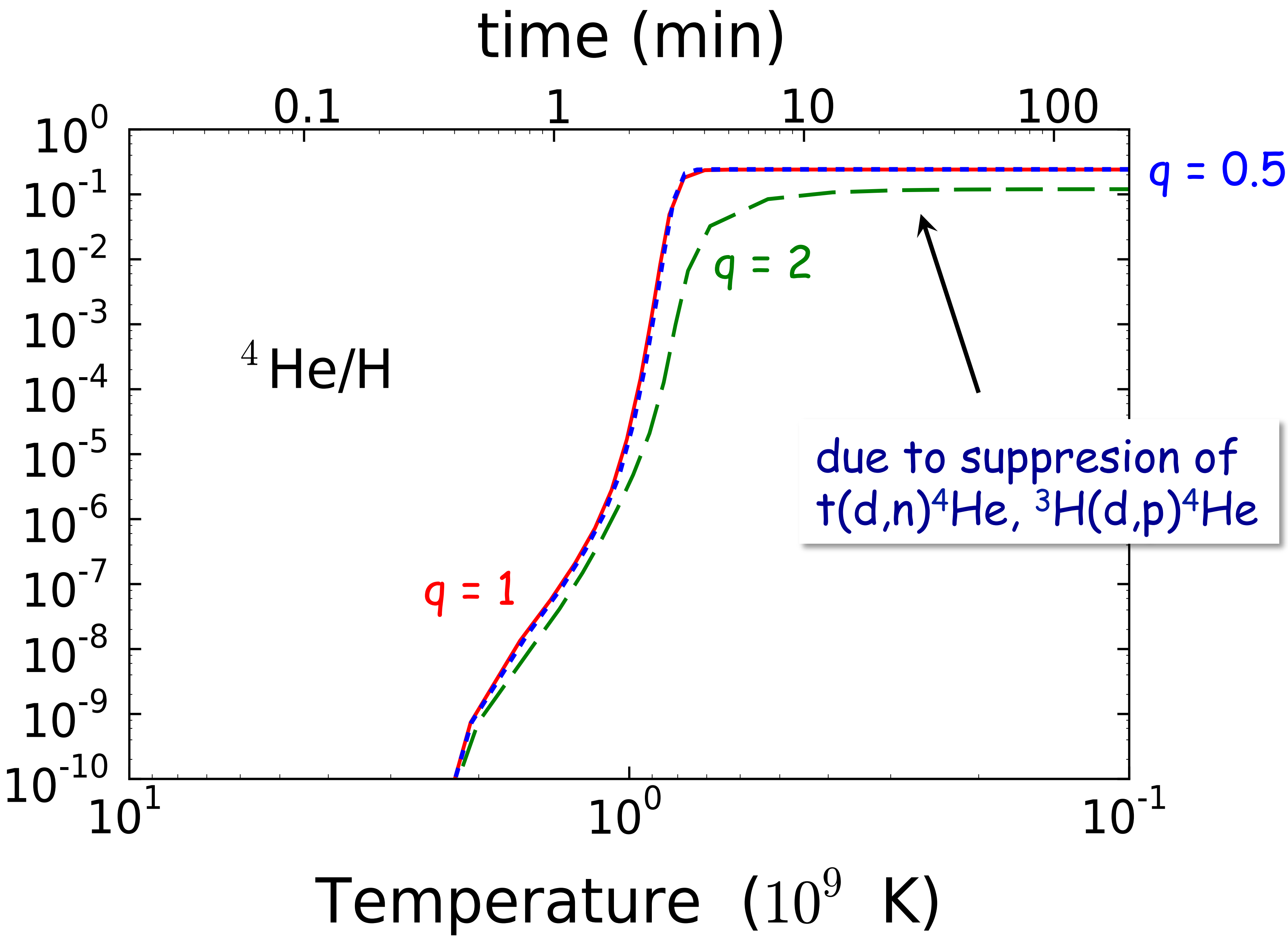
Non-Maxwellian rate

$$M_q(E,T) = N(q,T)\left[1 - \frac{q-1}{k_B T}E\right]^{\frac{1}{q-1}} \exp\left[-\eta(E)\right] \qquad (4.43)$$

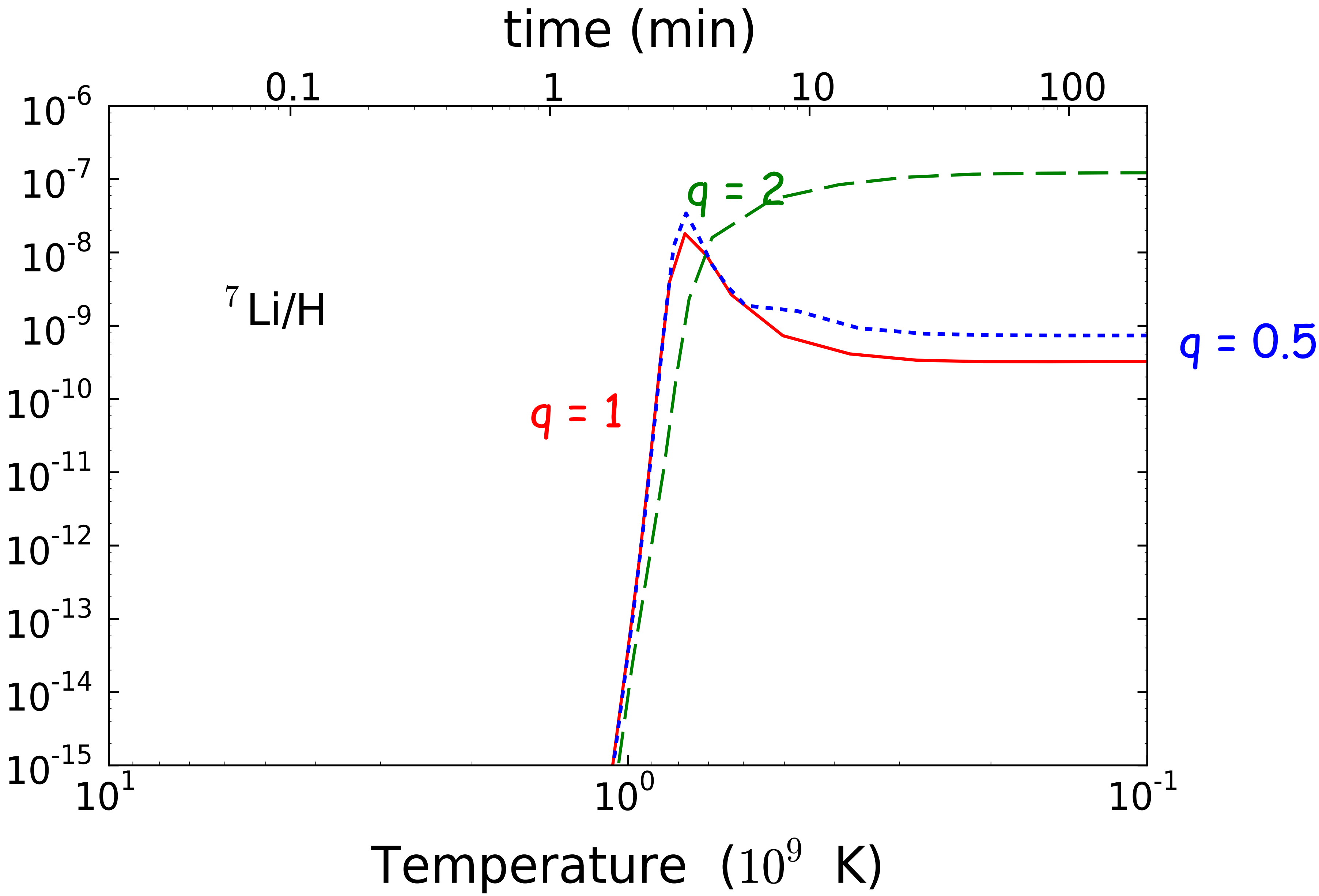
BBN deuterium abundance



BBN ^{4}He abundance



BBN ^{7}Li abundance



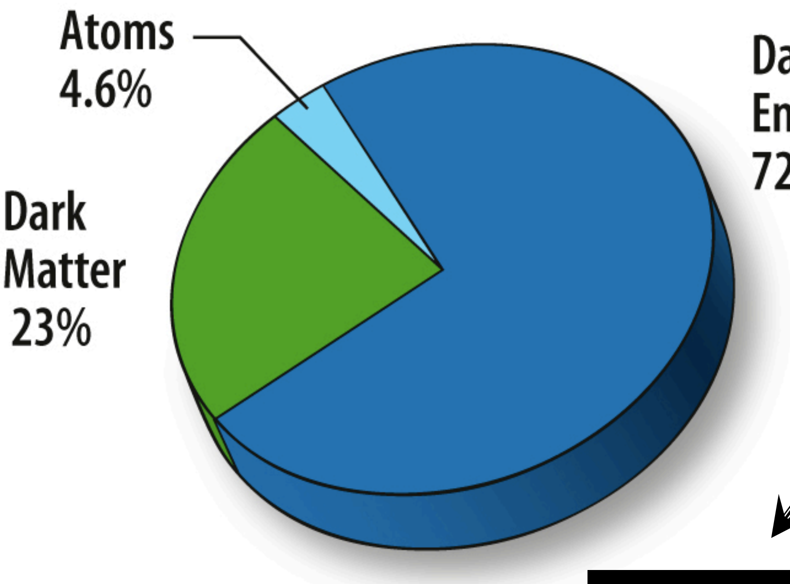
Comparison to observations

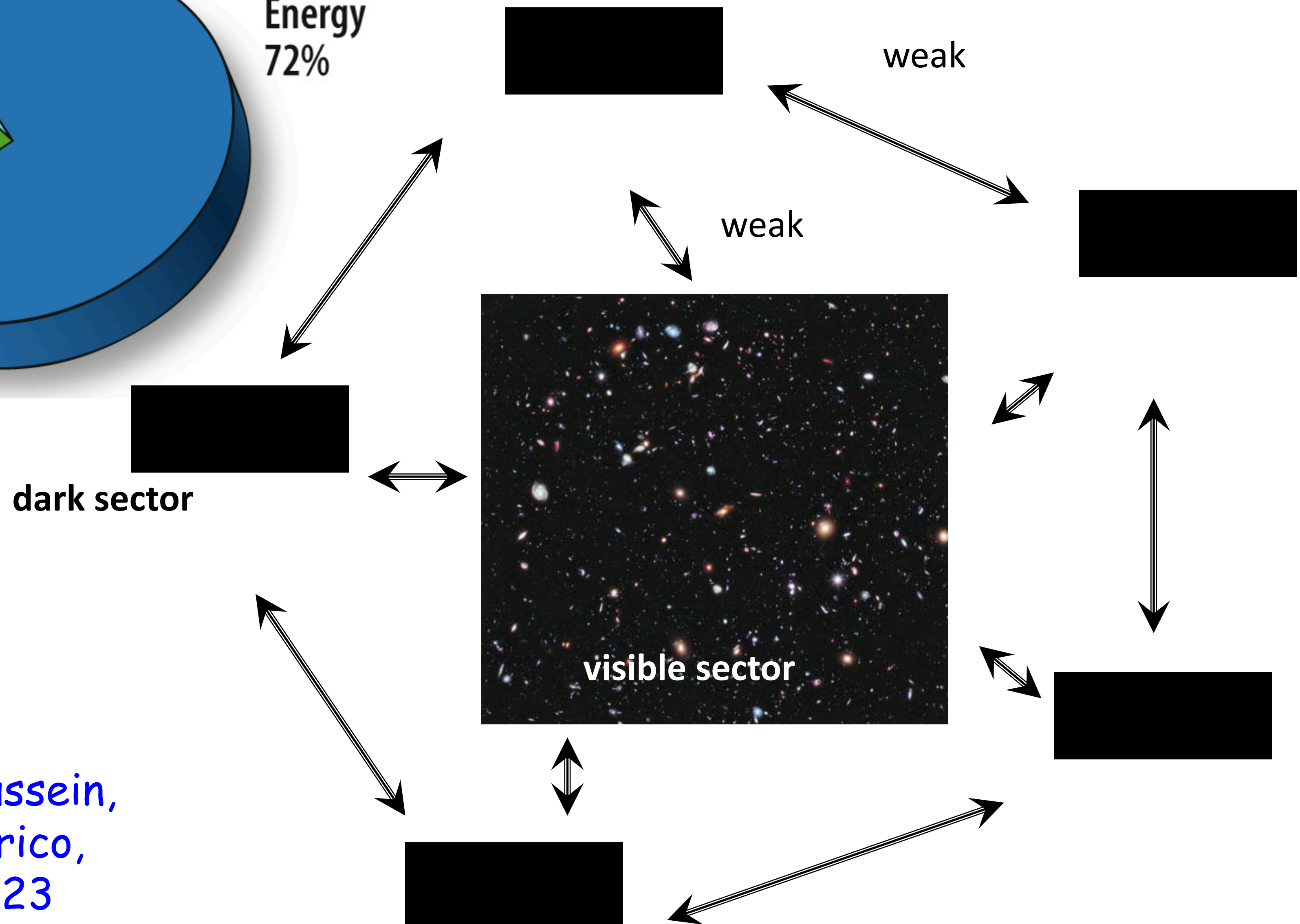
	BNN	Non-ext. q=0.5	Non-ext. q=2	Observation
^{4}He	0.249	0.243	0.141	0.256 ± 0.006
D/H	2.62	3.31	570	$(2.82 \pm 0.26) \times 10^{-5}$
$^{3}He/H$	0.98	0.091	69.1	$(0.9 - 1.3) \times 10^{-5}$
$^{7}LI/H$	4.39	6.89	356.	$(1.58 \pm 0.31) \times 10^{-10}$

non-extensive statistics

standard BBN

4.3.4 - Parallel universes of dark + visible matter





Oliveira, CB, Hussein, de Paula, Frederico, arXiv:1108.2723 AIP, 1498, 134 (2013)

Parallel Universes of Dark Matter

1. Leaves unchanged long distance properties of SM and Gravity

2. No Higgs Mechanism

3. Compatible with Cosmological constraints and BBN

$$L = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_f \bar{Q}_f \left[i\gamma^\mu D_\mu - m_f \right] Q_f + \frac{1}{2} \left(D^\mu \phi^a \right) \left(D_\mu \phi^a \right) - V_{oct} \left(\phi^a \phi^a \right)$$

(4.44)

Grey Boson + Matter Field + Scalar

$$D_\mu = \partial_\mu + i g_M T^a M^a_\mu \qquad (4.45)$$

Grey-boson

ϕ^a Mass generation

$$\frac{1}{2}\left(D^{\mu}\phi^{a}\right)\left(D_{\mu}\phi^{a}\right) \quad (4.46)$$

$$\frac{1}{2}g_M^2\phi^c\left(T^aT^b\right)_{cd}\phi^d M_\mu^a M^{b\mu} \quad (4.47)$$

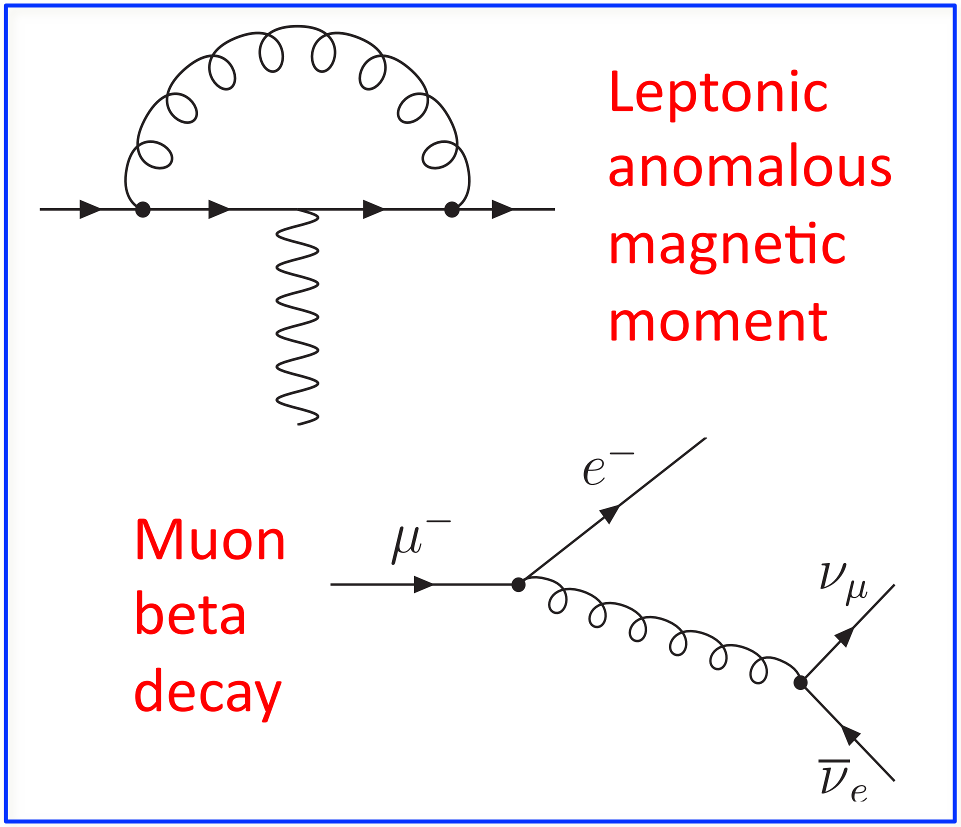
$$\left\langle \phi^a \right\rangle = 0 \quad (4.48)$$

$$\left\langle \phi^a \phi^b \right\rangle = v^2\delta^{ab} \quad (4.49)$$

gauge invariant

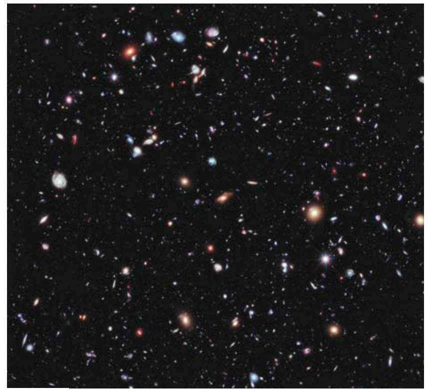
$$M^2 = 3g_M^2 v^2 \quad (4.50)$$

$$M \geq 9\ \text{TeV} \quad (4.51)$$

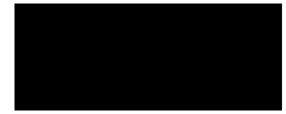


BBN → dark sectors are colder

T ordinary matter



T' dark sectors



$$\rho = \frac{\pi^2}{30} g_*(T) T^4 \quad (4.52)$$

$$g_*(T) = \sum_B g_B \left(\frac{T_B}{T}\right)^4 + \frac{7}{8} \sum_F g_F \left(\frac{T_F}{T}\right)^4 \quad (4.54)$$

$$s = \frac{2\pi^2}{45} g_s(T) T^3 \quad (4.53)$$

$$g_s(T) = \sum_B g_B \left(\frac{T_B}{T}\right)^3 + \frac{7}{8} \sum_F g_F \left(\frac{T_F}{T}\right)^3 \quad (4.55)$$

T' = dark T

Do it for T and T' + use Friedmann equation + evolve to BBN time

$$\bar{g}_s(T) = g_* \left[1 + N_D \left(\frac{T'}{T}\right)^4\right] \quad (4.56)$$

$$g_s(T)\Big|_{T=1\,\mathrm{MeV}} = 10.75 \quad (4.57)$$

BBN + ^{4}He, ^{3}He, D and ^{7}Li constraints →

$$\frac{T'}{T} < \frac{0.78}{N_D^{1/4}} = 0.52 \quad (4.58)$$

Baryon asymmetry and DM halo dynamics

$$\eta = \frac{\text{density of baryons}}{\text{density of photons}}$$

$$\frac{\text{dark baryons}}{\text{ordinary baryons}} = \frac{\eta'}{\eta}\left(\frac{T'}{T}\right)^3 \sim 1 \qquad (4.59)$$

$$\rightarrow \quad \eta' = 7\eta \qquad (4.60)$$

Acoustic oscillations → to change CMB:

$$\frac{T'}{T} \geq 0.6 \qquad (4.61)$$

$$\rightarrow \quad N_D \geq 0.35$$

lower bound (4.62)

DM-DM interactions

$$\sigma = \left(g^2 \frac{T}{\Lambda^2}\right)^2 \qquad (4.63)$$

$$\rightarrow \quad \frac{\sigma'}{\sigma} \sim \left(\frac{T'}{T}\right)^2 \qquad (4.64)$$

BBN →

$$\frac{\sigma'}{\sigma} \sim \frac{0.61}{\sqrt{N_D}} \qquad (4.65)$$

→ **Dark sectors are essentially collisionless**

Model is compatible with cosmological, BBN, and CMB constraints
But does not do anything to solve the Lithium problem

BBN incredibly successful, except for Lithium problem

This has led to a large number of speculations, such as if the MB distribution is valid for the BBN scenario, or if another statistics should be adopted.
See, e.g.,

- " Big bang nucleosynthesis with a non-Maxwellian distribution", CB, J. Fuqua and M.S. Hussein, The Astrop. J. 767, 67 (2013).

Other possibilities even include electron screening, the effect of dark matter, or parallel universes.
See, e.g.,
- "Electron screening and its effects on Big-Bang nucleosynthesis", Biao Wang, CB and A.B. Balantekin, Phys. Rev. C 83, 018801 (2011).
- "Dark/Visible Parallel Universes and Big Bang Nucleosynthesis", CB, T. Frederico, J. Fuqua, M.S. Hussein, O. Oliveira, W. de Paula, AIP Conf. Proc., 1498, 134 (2013).

For more details on current status of the lithium problem, see:
- The Cosmological Lithium Puzzle Revisited, CB, A. Mukhamedzhanov, and Shubhchintak, AIP Conf. Proc. 1753, 040001 (2016)

Practice

1 – At the end of BBN ^4He and ^1H made up 24% and 76% of the total mass respectively. Assuming that all the free neutrons became bound in the ^4He isotope and that the process was fast compared with the neutron lifetime, estimate using the Boltzmann distribution the temperature at which the n/p ratio was frozen.

2 – From the atomic masses below, calculate the energy released per kilogram of matter in the production of helium assuming once again that 24% by mass is converted (1 a.m.u. = 1.6605 10^{-27} kg = 931.494 MeV/c^2).

Element/Nucleon	Atomic Mass
^4He	4.002602 u
^1H	1.00794 u
n	1.008664 u

3 – The mean lifetime for free neutrons is 880 s. The n/p ratio was frozen at 0.2 but the formed deuterons were destroyed by photodisintegration until the mean photon energy was reduced by the decreasing temperature. By that time neutron decay had lowered the n/p ratio to 0.134. Estimate the duration of this time period.

Practice

4 – Estimate the height of the Coulomb barrier for the reaction

$$^4He + {}^3H \rightarrow {}^7Li + \gamma$$

and the temperature at which the nuclei will have kinetic energies of approximately this value (assume that the nuclear radius is given by $1.2 \times A^{1/3}$ fm).

Computing practice

1 – Get together in groups of maximum 4 persons.

2- Use 5 different values of the neutron lifetime within a window of 200 seconds around the accepted experimental value and compute the final BBN 4He abundance as a function of the baryon to photon ratio, η.

3 - Use 5 different values of the number of neutrino families within a window of Δn_ν = 1 around the accepted experimental value and compute the final BBN 4He abundance as a function of the baryon to photon ratio, η.

4 – Compute the abundances of 4He, D, 3He, 6Li and 7Li as a function of baryon to photon ratio, η.

5 - Compute the abundances of 4He, D, 3He, 6Li and 7Li as a function of time and temperature for the accepted value of the baryon to photon ratio, η.

6 – Find the lines within the code where the rates of the BBN reactions are entered. Modify some of the reaction rates (keep track of it, so that you can undo the changes) by a factor 10 or 1/10 and make a table with the results in the final prediction for all elements. Explain why the results changed in terms of the reaction chain.