The Coulomb dissociation method

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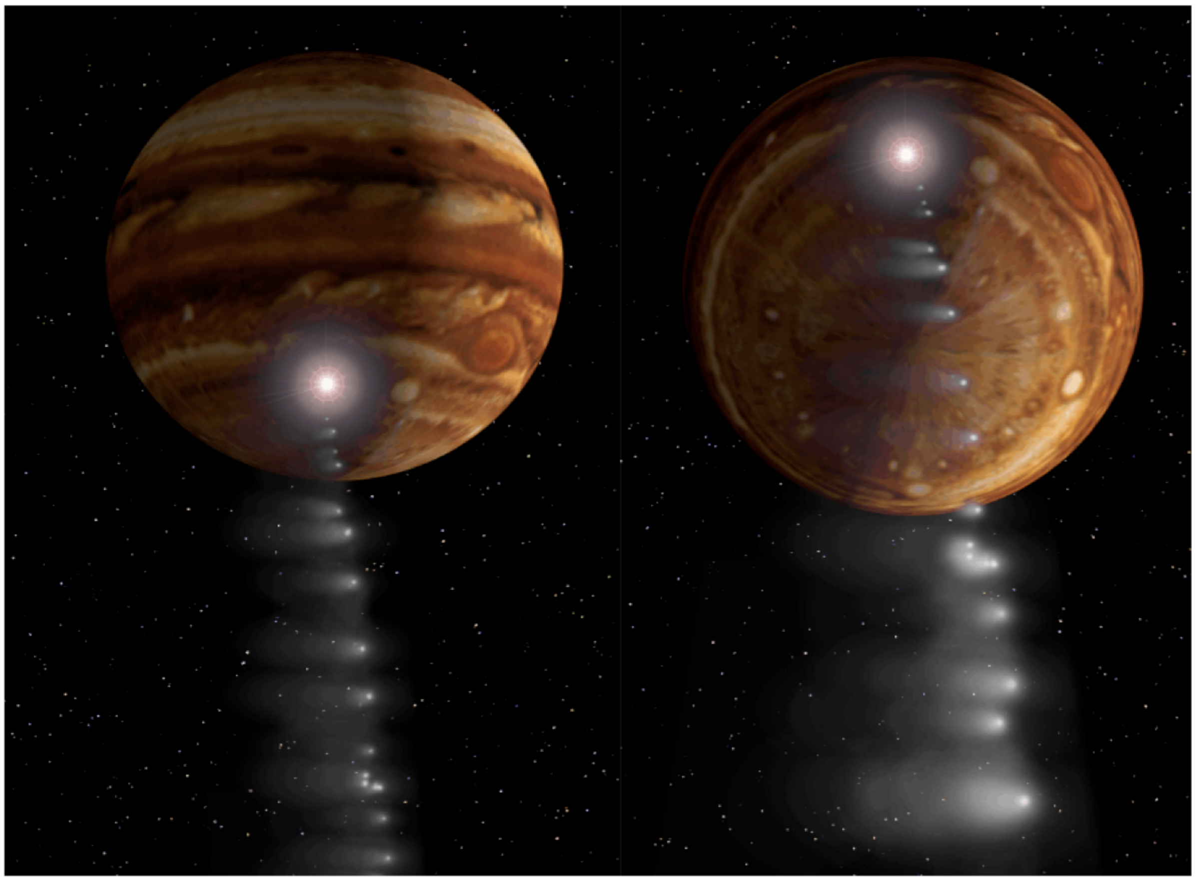
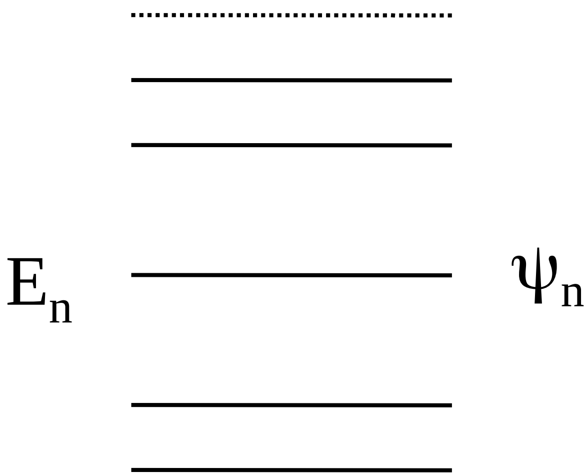
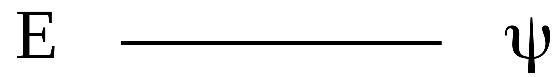


Review: **http://lanl.arxiv.org/pdf/0908.4307**

Coupled-channels equations





H_0 spectrum: $H_0\psi_n = E_n\psi_n$

$H = H_0 + V$ solution

$$\psi = \sum_n a_n(t)\psi_n e^{-iE_n t/\hbar}$$

$$H\psi = i\hbar\frac{\partial\psi}{\partial t}$$

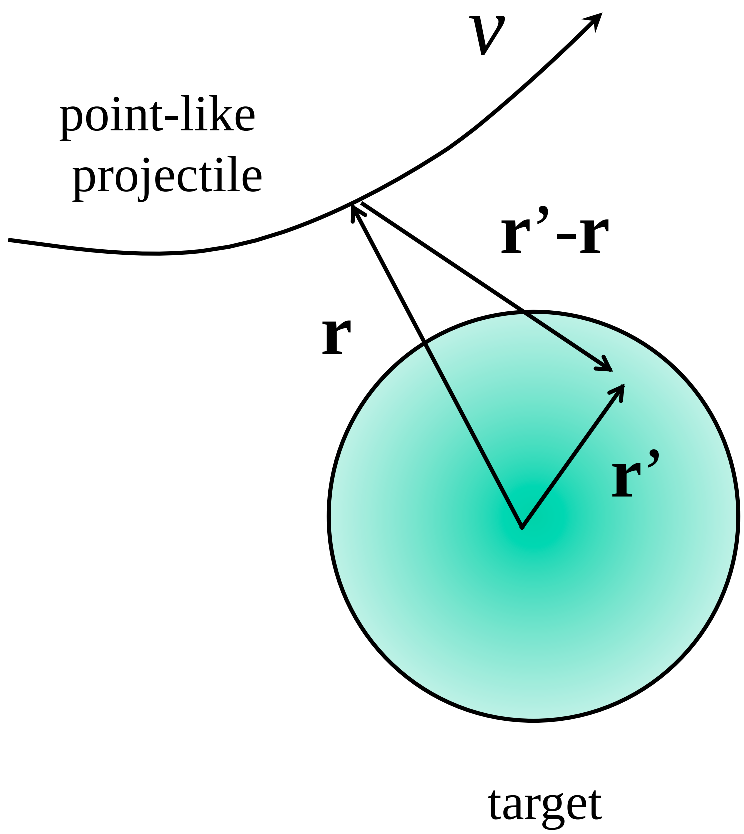
⇒

$$\frac{da_k}{dt} = -\frac{i}{\hbar}\sum_n a_n(t)V_{kn}(t)e^{i(E_k-E_n)t/\hbar}$$

$$V_{kn}(t) = \int \psi_k^* V(t)\,\psi_n\, d^3r$$

1st order: $a_n \sim \delta_{n0}$ ⇒

$$a_k = -\frac{i}{\hbar}\int dt\, V_{k0}(t)e^{i(E_k-E_0)t/\hbar}$$



Multipole expansion

$$V_C(r,r') = Z_p e \int \frac{\rho(\mathbf{r}')}{|r-r'|} d^3r' = \frac{Z_p e}{r} + \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^3} + \frac{1}{2}\frac{Q_{ij} r_i r_j}{r^5} + \cdots$$

$$\mathbf{p} = \int \mathbf{r}'\rho(\mathbf{r}') d^3r'$$ (dipole)

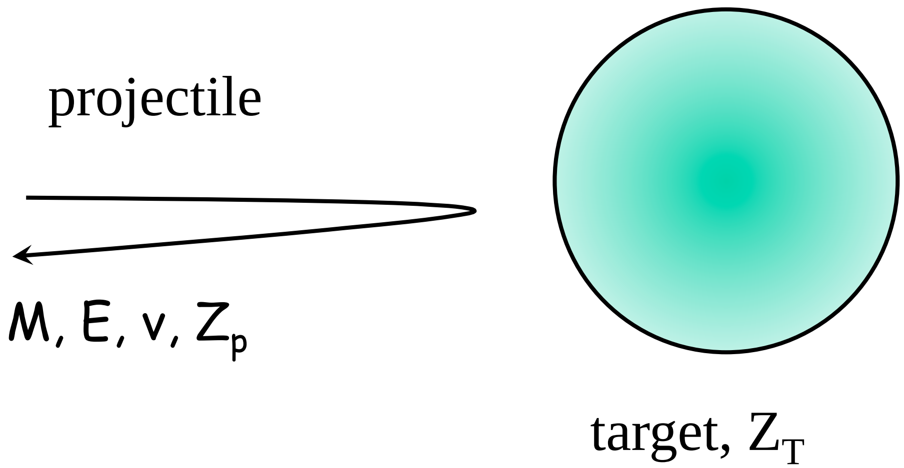
$$Q_{ij} = \int \left(3r'_i r'_j - r'^2 \delta_{ij}\right)\rho(\mathbf{r}') d^3r'$$

(Quadrupole)

Semiclassical method: r = r(t)

Validity: $\eta = \dfrac{\text{distance of closest approach}}{\text{wavelength}} = \dfrac{Z_1 Z_2 e^2}{\hbar v} >> 1$

Example: low-E central collision



$$V_C(t) = \frac{1}{2}\frac{Z_p e^2 Q_{fi}}{r^3(t)}$$

$$Q_{fi} = \int \psi_f^*(\mathbf{r}')\left(3z'^2 - r'^2\right)\psi_i(\mathbf{r}')\, d^3r'$$

excitation amplitude:

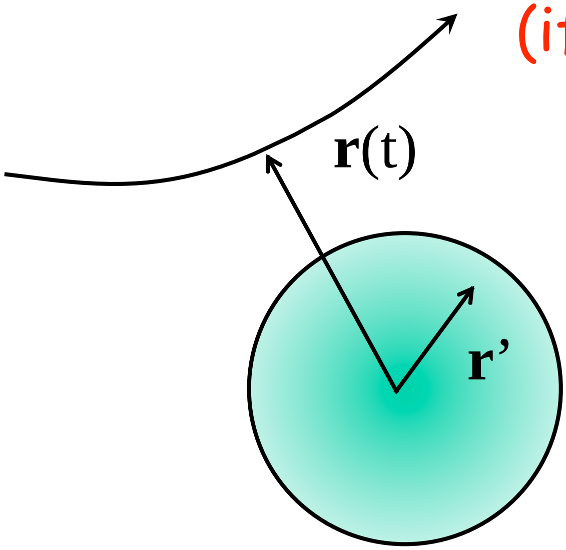
$$a_{fi} = \frac{Z_p e^2 Q_{fi}}{2i\hbar}\int_{-\infty}^{\infty}\frac{e^{i\omega t}}{r^3(t)}dt = \frac{4Q_{fi}E^2}{3Z_p Z_T^2 e^2 \hbar v}$$

Cross section:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=180^0} = \left.\frac{d\sigma_R}{d\Omega}\right|_{\theta=180^0} \times \left|a_{fi}\right|^2 = \frac{ME\left|Q_{fi}\right|^2}{18\hbar^2 Z_T^2}$$

HW1: X-section does not depend on Z_p! Why is it larger for heavier projectiles?

General multipole expansion



(if r > r')

$$\frac{1}{|\mathbf{r}(t)-\mathbf{r}'|} = \sum_{L,M} \frac{4\pi}{2L+1} \frac{r'}{r^{L+1}(t)} Y_{LM}\left(\hat{\mathbf{r}}(t)\right) Y_M^*\left(\hat{\mathbf{r}}'\right)$$

Calculate a_{fi} and average over spins:

$$w_{fi} = \frac{1}{2J_i+1} \sum_{M_i M_f} \left| a_{fi} \right|^2$$

Cross section:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_R}{d\Omega} \cdot w_{fi} = \sum_{L>0} \frac{d\sigma_L}{d\Omega}$$

orbital integral

$$\frac{d\sigma_L}{d\Omega} \sim Z_P^2 \, B(EL) \left| I_L\left(\omega_{fi}\right) \right|^2$$

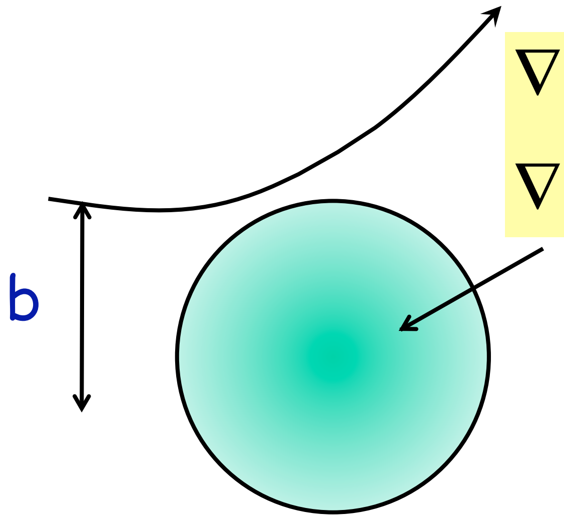
$$I_L(\omega) = \int_{-\infty}^{\infty} dt \, \frac{1}{r^{L+1}(t)} Y_{LM}\left(\hat{\mathbf{r}}(t)\right) e^{i\omega t}$$

$$\omega_{fi} = \frac{E_f - E_i}{\hbar}$$

$$B(EL) \sim \left| \int r^L \delta\rho_{fi} \, d^3 r \right|^2$$

reduced matrix element

Virtual photon numbers



$$\nabla \cdot \mathbf{E}(t) = 0$$

$$\nabla \cdot \mathbf{B}(t) = 0$$

E, B-field of projectile divergence free

$$\frac{d\sigma_L}{d\Omega} = \int \frac{dE_\gamma}{E_\gamma} \frac{dn_L}{d\Omega}\left(E_\gamma,\theta\right)\sigma_L^\gamma\left(E_\gamma\right)$$

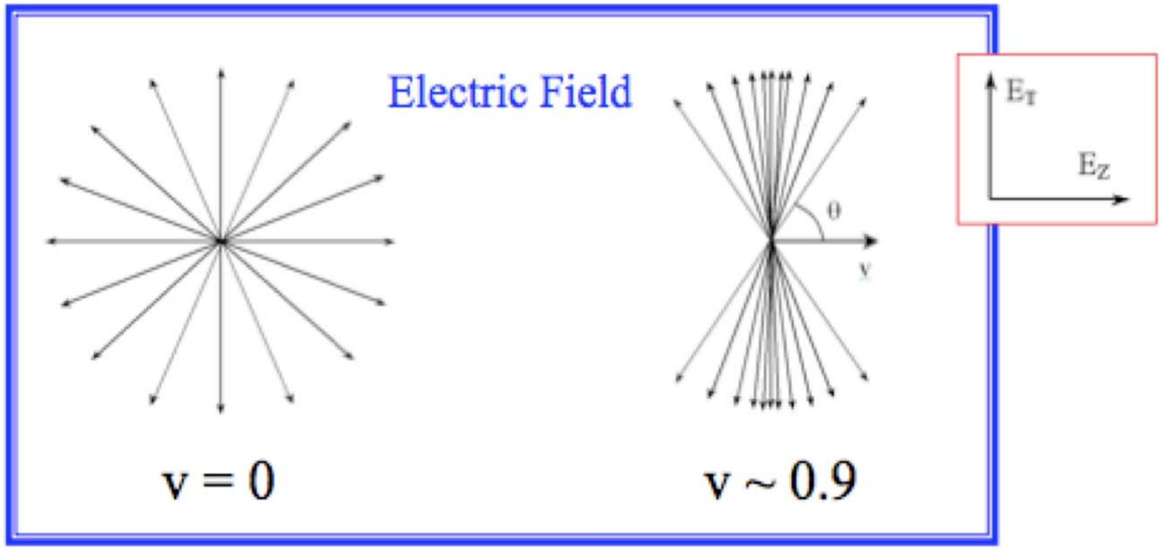
photonuclear X-section: $\sigma_L^\gamma \sim E_\gamma^{2L+1} B(EL)$ $\quad E_\gamma = E_f - E_i$

virtual photon numbers: $\frac{dn_L}{d\Omega} \sim Z_P^2 \left| I_L\left(\omega_{fi},\theta\right)\right|^2$

impact parameter dependence: $n_L\left(E_\gamma,b\right) \equiv \frac{dn_L}{2\pi b\,db} \sim \sin^4\left(\theta/2\right)\frac{dn_L}{d\Omega}$

Magnetic excitations:

more complicated (involves currents, spins), but straight-forward.

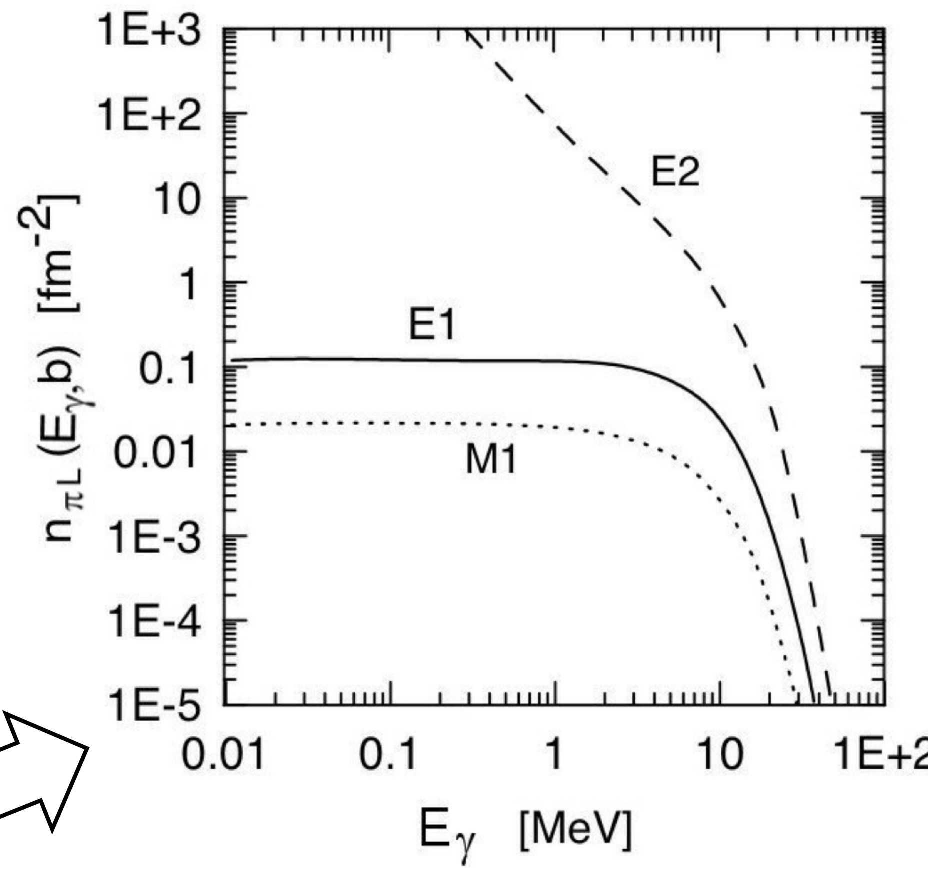


Virtual photon numbers

low energy scattering:

$$n_{E2} >> n_{E1} >> n_{M1} = \frac{v^2}{c^2} n_{E1}$$

Virtual photons "seen" by a Pb target due to the passage of an O projectile at 100 MeV/nucleon and b = 15 fm

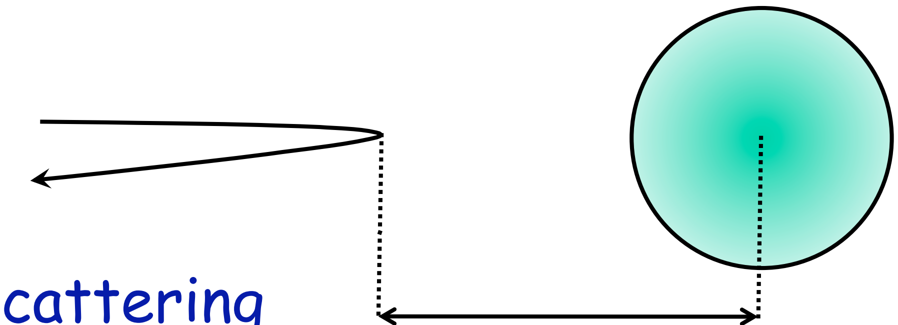


Adiabacity

Maximum effective excitation energy
Maximum effective impact parameter.

$$I_L(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{r^{L+1}(t)} Y_{LM}(\hat{\mathbf{r}}(t)) e^{i\omega t}$$

orbital integral



low energy scattering

(1/2) distance of closest approach

$$a = \frac{Z_P Z_T e^2}{2E_{c.m.}}$$

if $|t| > t_{exc} \sim \frac{1}{\omega}$ then $e^{i\omega t}$ oscillates too fast: I_L small

if $|t| > t_{coll} \sim \frac{a}{\mathrm{v}}$ then $\frac{1}{r^{L+1}}$ too large: I_L small

excitation possible if $\frac{t_{coll}}{t_{exc}} \lesssim 1$ ⇒

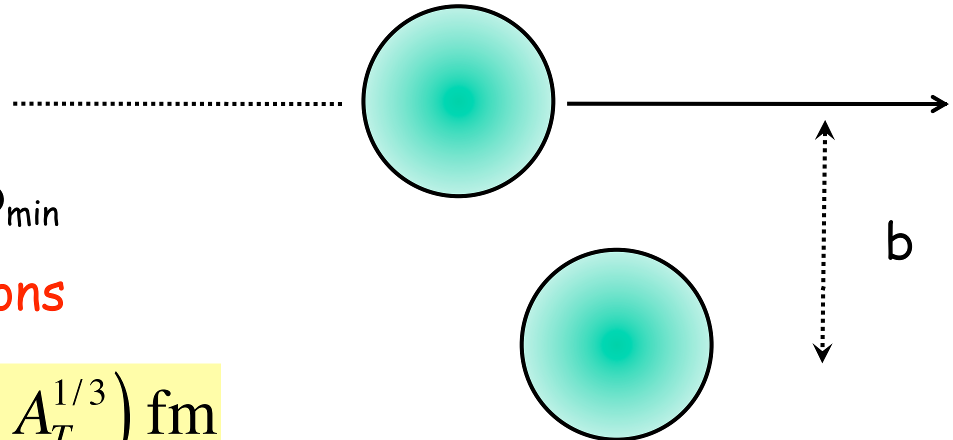
$$\zeta = \frac{a\omega}{\mathrm{v}} \lesssim 1$$

adiabacity parameter

$$I_L(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{r^{L+1}(t)} Y_{LM}(\hat{\mathbf{r}}(t)) e^{i\omega t}$$

orbital integral

high energy collisions



Closest approach distance = b_{min}

$b < b_{min}$ → nuclear interactions

$$b_{mim} \sim R_P + R_T \sim 1.2\left(A_P^{1/3} + A_T^{1/3}\right) \text{ fm}$$

$$t_{coll} \sim \frac{R}{\gamma \mathrm{v}}$$

(γ due to contraction)

$$\gamma = \frac{1}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}$$

Lorentz γ-factor

Excitation possible if

$$\xi = \frac{\omega R}{\gamma \mathrm{v}} \lesssim 1$$

Estimates

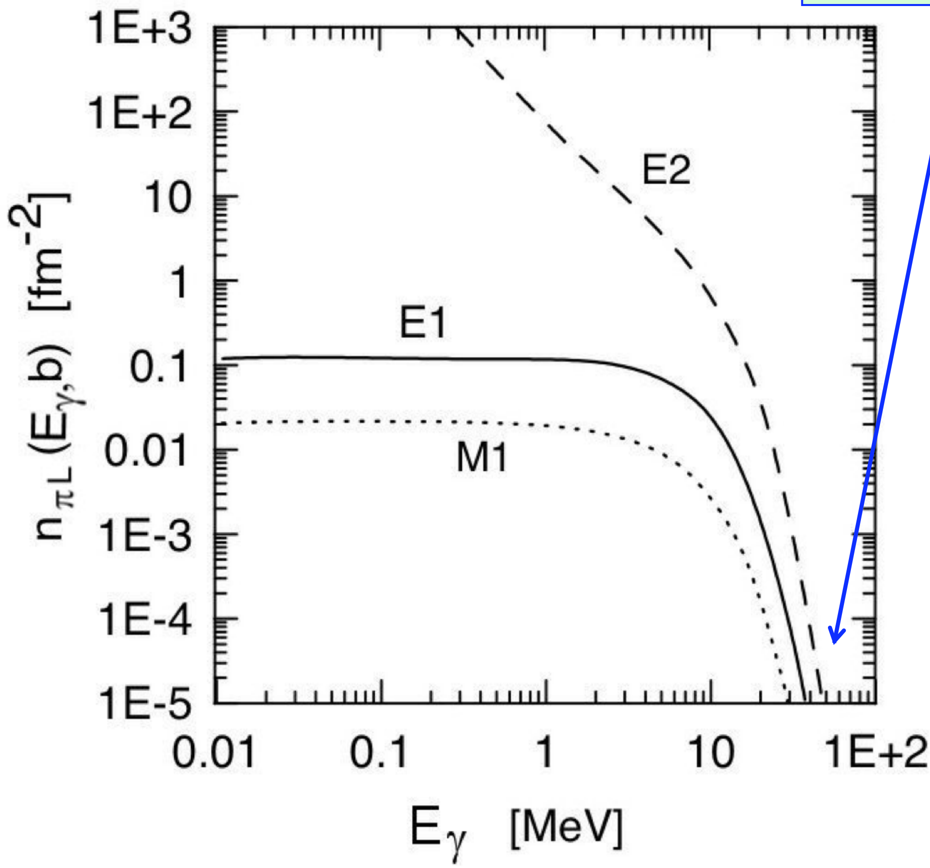
$$\zeta = \frac{E_\gamma a}{\hbar v}$$

low energy collisions

$$\xi = \frac{E_\gamma R}{\gamma \hbar v}$$

high energy collisions

$$a,\ b_{min} \sim 20\ \text{fm} \quad \Rightarrow \quad E_\gamma \lesssim \frac{200\,\text{MeV.}\ \text{fm}\ \gamma}{20\ \text{fm}} = 10\,\text{MeV.}\ \gamma$$



- small γ's: giant resonances

- large γ's: giant resonances,
  quasi-deuteron,
  deltas,
  mesons (ex: J/ψ)

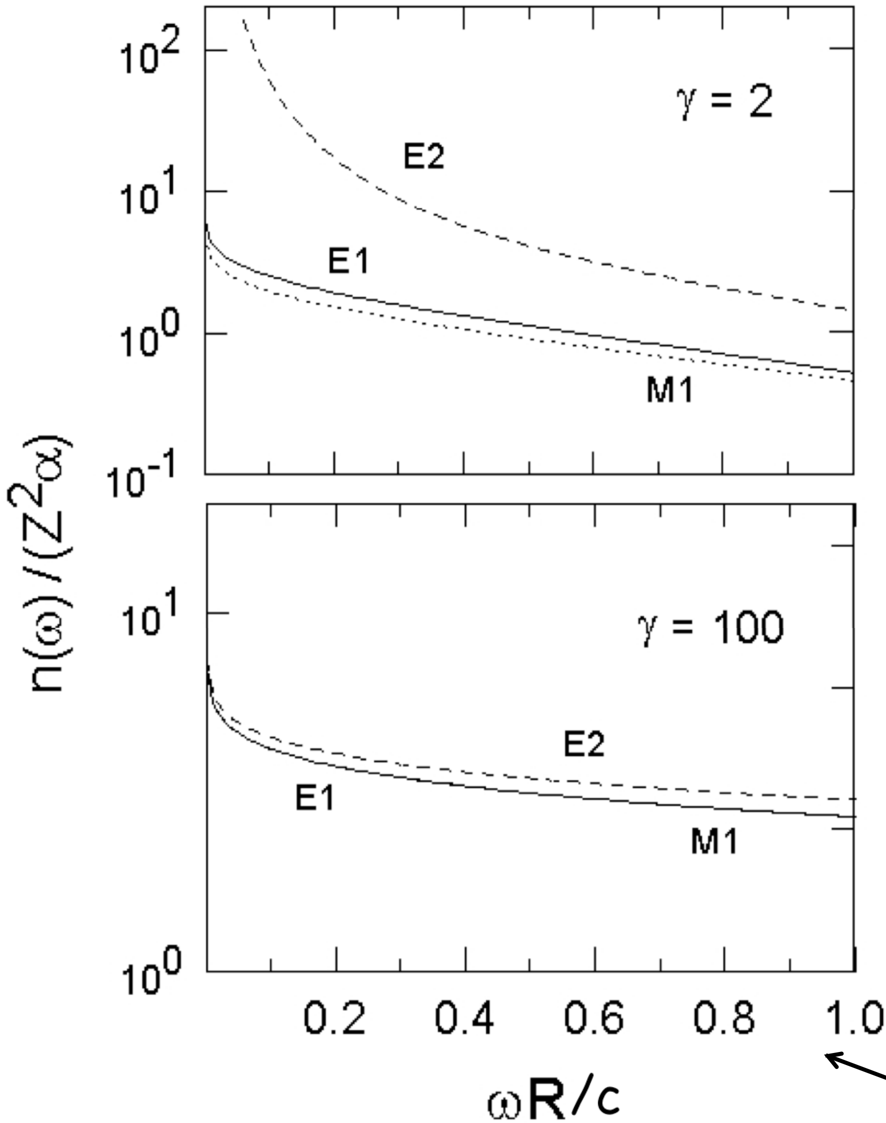
$$I_L(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{r^{L+1}(t)} Y_{LM}(\hat{\mathbf{r}}(t)) e^{i\omega t}$$

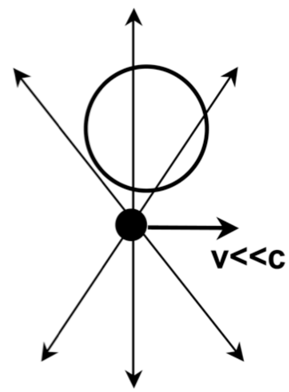
orbital integral

ω large, $e^{i\omega t}$ oscillates fast: I_L small

$$n_L(E_\gamma, b) \sim \left| I_L(\omega_{fi}, \theta) \right|^2$$

also smal

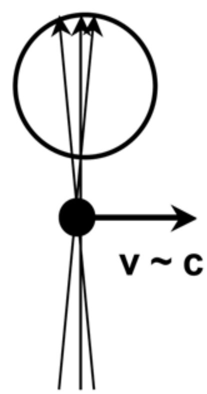




low-energy:

$$n_{E2} >> n_{E1} >> n_{M1} = \frac{v^2}{c^2} n_{E1}$$

high-energy:



$$n_{E2} \sim n_{E1} \sim n_{M1}$$

Low-energies: multipolarities of virtual photons have different weights

High energies: multipolarities have same weight

$$n_L(\omega) = \int db\, n_L(\omega, b)$$

Virtual x real photons

$$\frac{d\sigma}{db} = \int \sum_L \frac{dE_\gamma}{E_\gamma} n_L(E_\gamma, b)\sigma_L^\gamma(E_\gamma)$$

Coulomb excitation: virtual photons

Each part (multipolarity) of a real photon has a different weight n_L

$$\sigma^\gamma(E_\gamma) = \sum_L \sigma_L^\gamma(E_\gamma)$$

Real photons

All parts (multipolarities) have the same weight

High-energy:

$$n_{E2} \sim n_{E1} \sim n_{M1}$$

$$\frac{d\sigma}{db} = \int \frac{dE_\gamma}{E_\gamma} n(E_\gamma, b)\sigma^\gamma(E_\gamma)$$

Coulomb excitation for a fixed energy E_γ probes the same physics as a real photon.

But each E_γ has a different weigth.

Z_P^2 makes number of photons large.

Nuclear response to multipolarities

$$\sigma_L^\gamma \sim E_\gamma^{2L+1} B(EL)$$

$$B(EL) \sim \left| \int r^L \delta\rho_{fi} \, d^3 r \right|^2$$

Estimate

$$\delta\rho_{fi} = \psi_f^* \psi_i$$

$$\psi_f \sim \psi_i \sim \frac{1}{\sqrt{R^3}}, \quad \text{if r < R}, \quad \text{0 otherwise}$$

$$B(EL) \sim R^{2L} \quad \Rightarrow \quad \sigma_L^\gamma \sim (kR)^{2L} \qquad k = \frac{E_\gamma}{\hbar c}$$

$$\frac{\sigma_{L+1}}{\sigma_L} \sim (kR)^2 < 1 \text{ for low lying states}$$

Sum-rules

$$S = \sum_f \left(E_f - E_i\right) \left| \langle f | \hat{O} | i \rangle \right|^2 \qquad \hat{O} \equiv \hat{O}(z) \qquad \hat{H} = \frac{\hat{p}_z^2}{2m} + V(z)$$

$$\Rightarrow \quad S = \frac{1}{2} \langle i | \left[[H, \hat{O}], \hat{O} \right] | i \rangle$$

$$\hat{O} = f(z) \quad \Rightarrow \quad S = \frac{\hbar^2}{2m} \left\langle i \left| \left| \frac{df}{dz} \right|^2 \right| i \right\rangle$$

Example: electric dipole operator

$$\hat{O} = e\,z \quad \Rightarrow \quad S = \frac{\hbar^2 e^2}{2m}$$

independent of V(z) !

A-particles:

$$\hat{H} = \sum_b \left[\frac{\hat{p}_b^2}{2m_b} + V(z_b) \right] \qquad \hat{O} = \sum_a e_a z_a \quad \Rightarrow \quad S = \sum_a \frac{\hbar^2 e_a^2}{2m_a}$$

Effective charges, c.m. motion

$$d_z = \sum_a e_a z_a = \sum_a e_a r_a Y_{10}(\hat{\mathbf{r}}_a)$$

$$z_a \to z_a - R_z$$

$$R_z = \sum_a \frac{z_a}{A} \quad \text{(center of mass)}$$

$$d_z = \sum_a e_a (z_a - R_z) = e\sum_p z_p - \frac{Ze}{A}\left(\sum_p z_p + \sum_n z_n\right)$$

$$= e_p \sum_p z_p + e_n \sum_n z_n$$

$$e_p = \frac{N}{A}e, \quad e_n = -\frac{Z}{A}e$$

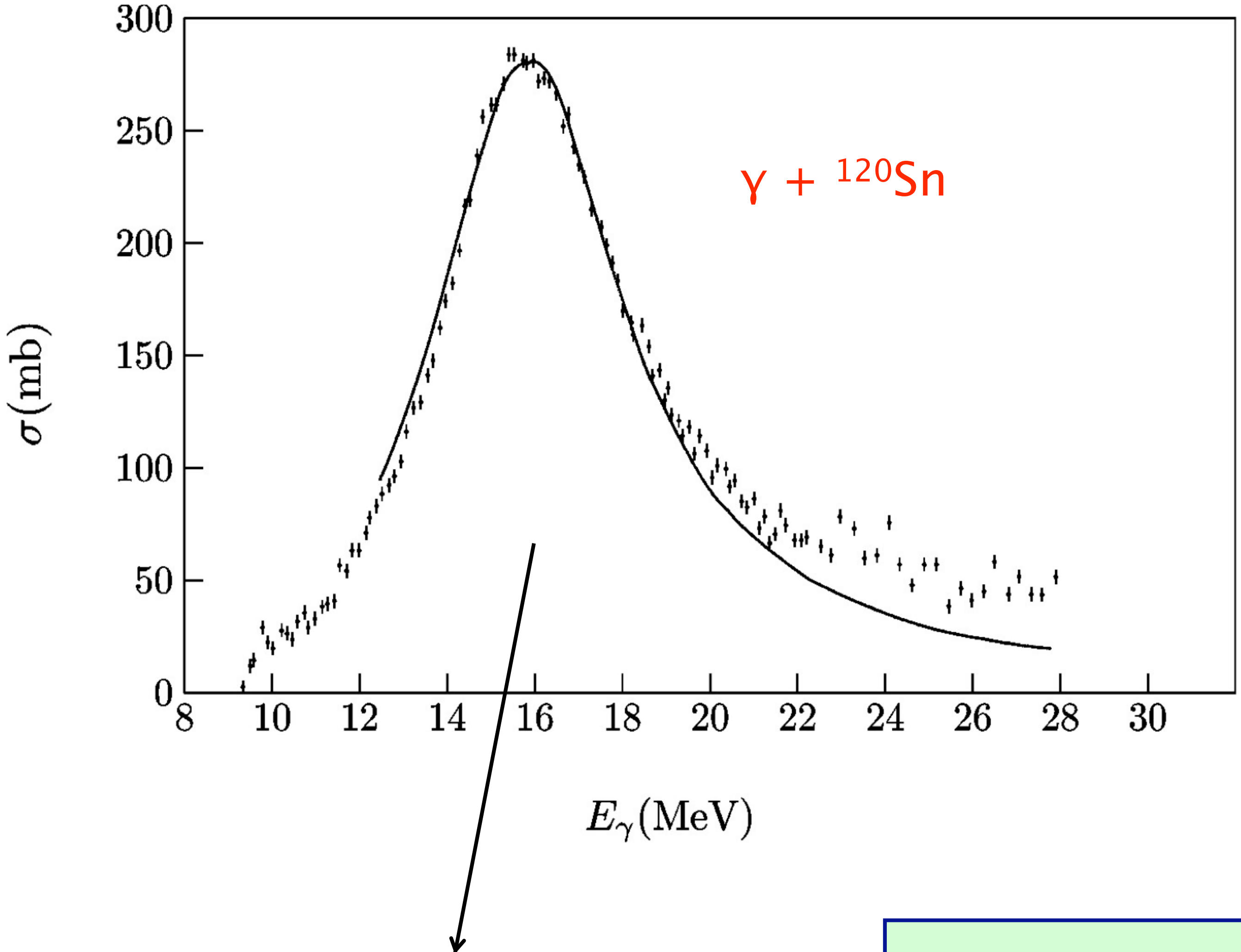
effective charges

$$S = \sum_f E_{fi} \left| d_{fi}^z \right|^2 = \frac{\hbar^2 e^2}{2m_N}\left[Z\left(\frac{N}{A}\right)^2 + N\left(-\frac{Z}{A}\right)^2\right]$$

$$= \frac{\hbar^2 e^2}{2m_N}\frac{NZ}{A}$$

Thomas-Reiche-Kuhn sum-rule

Nuclear response to photon energies

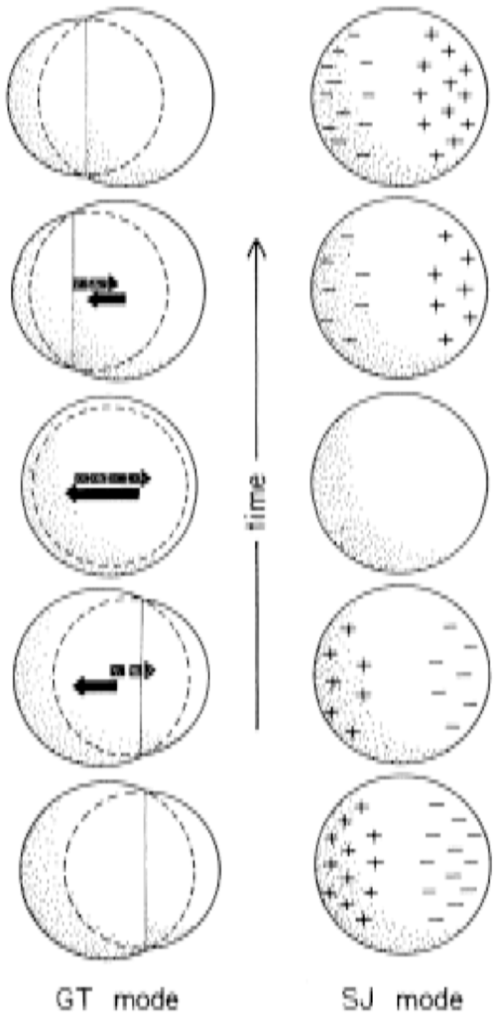


Area ~ 100% TRK sum-rule:

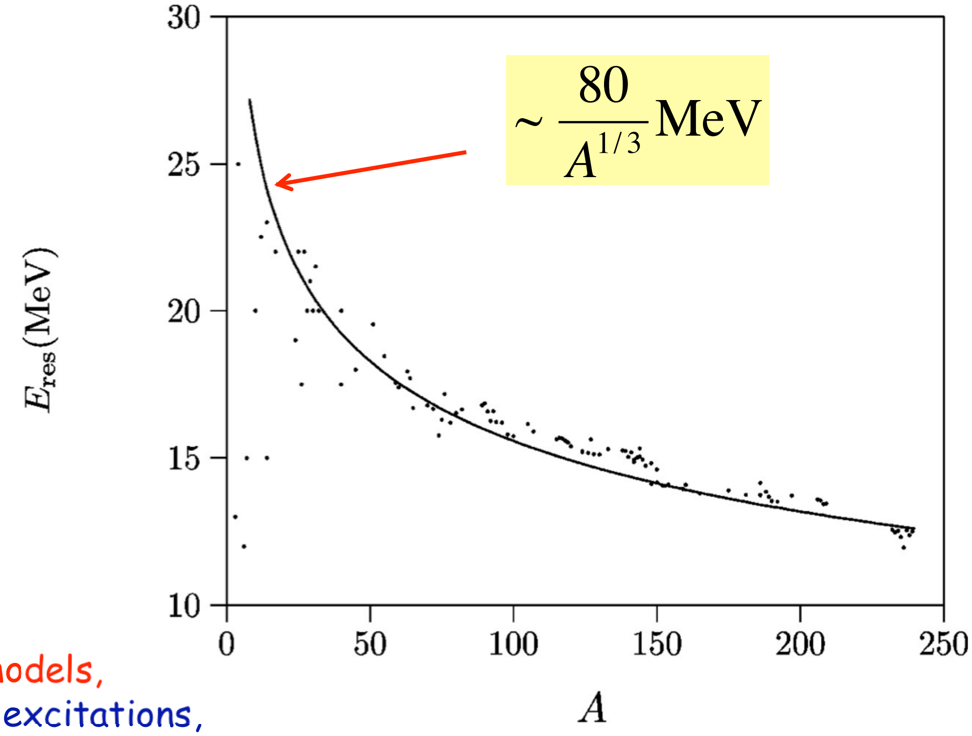
$$\int dE_\gamma \, \sigma^\gamma\left(E_\gamma\right) = 2\pi^2 \frac{\hbar e^2}{m_N c} \frac{NZ}{A}$$

Giant resonances

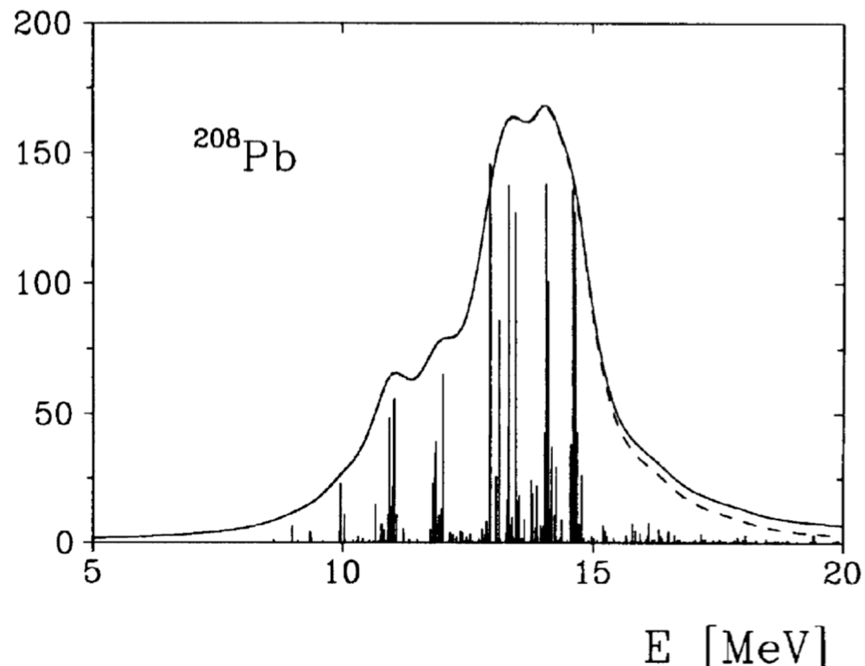
Macroscopic models, liquid drop



GT: Goldhaber-Teller
SJ: Steinwedel-Jensen



Microscopic models,
particle-hole excitations,
RPA



Coulomb excitation of GRs

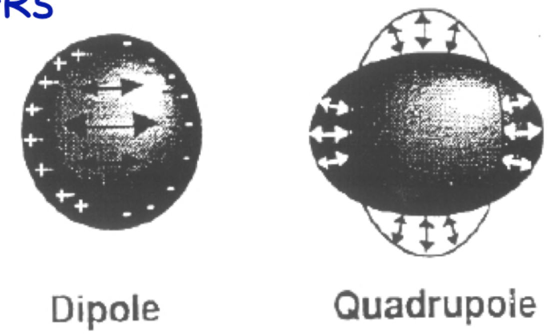
Estimate

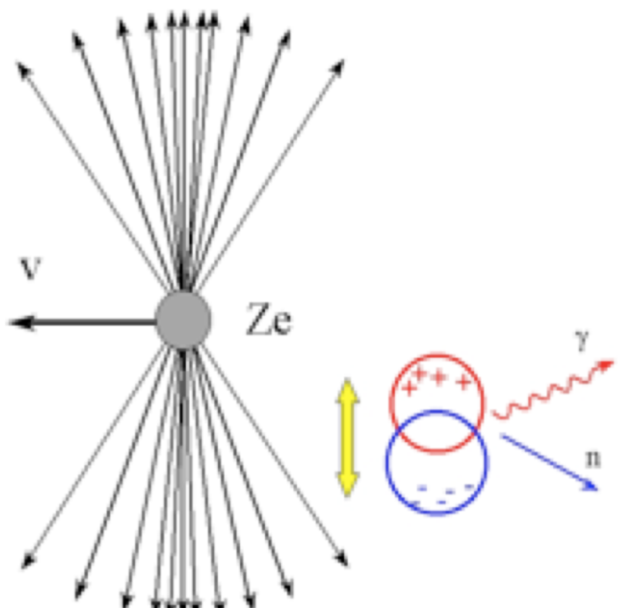
sum-rules

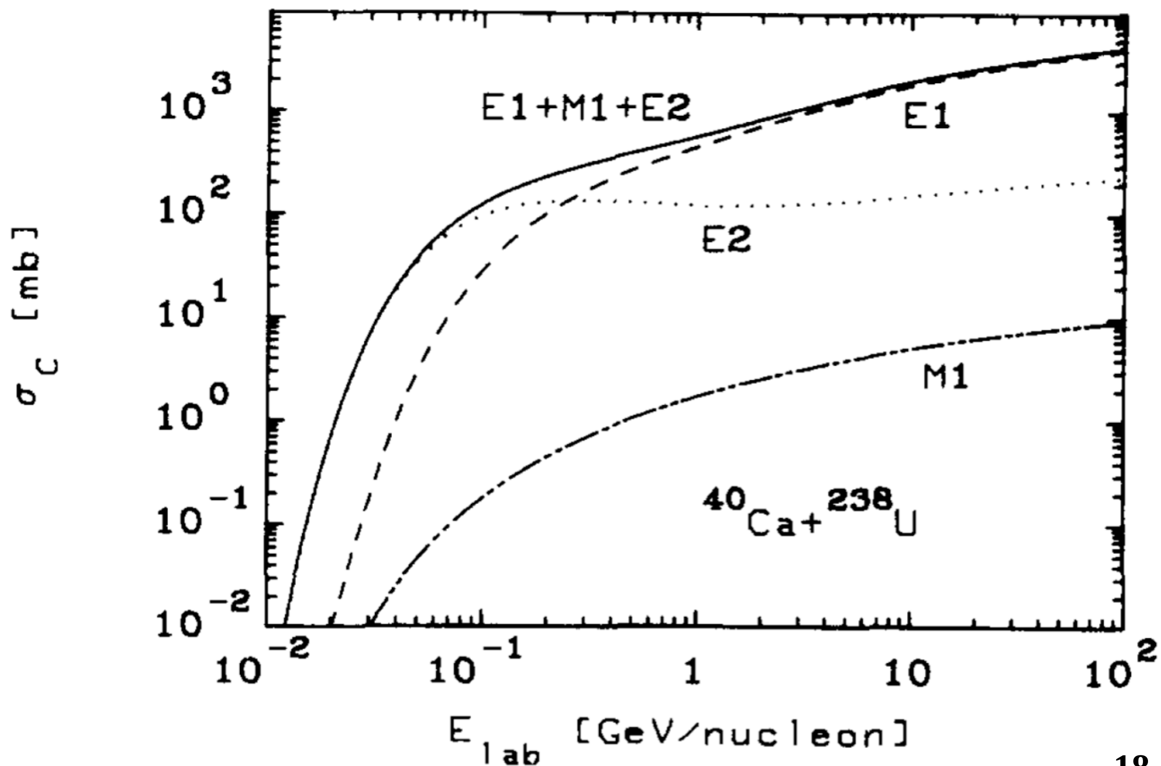
$$\sigma_C \approx \frac{n_{E1}(E_{GDR})}{E_{GDR}} \int dE_\gamma \sigma^\gamma_{GDR}(E_\gamma) + n_{E2}(E_{GQR}) E_{GQR} \int \frac{dE_\gamma}{E_\gamma^2} \sigma^\gamma_{GQR}(E_\gamma)$$

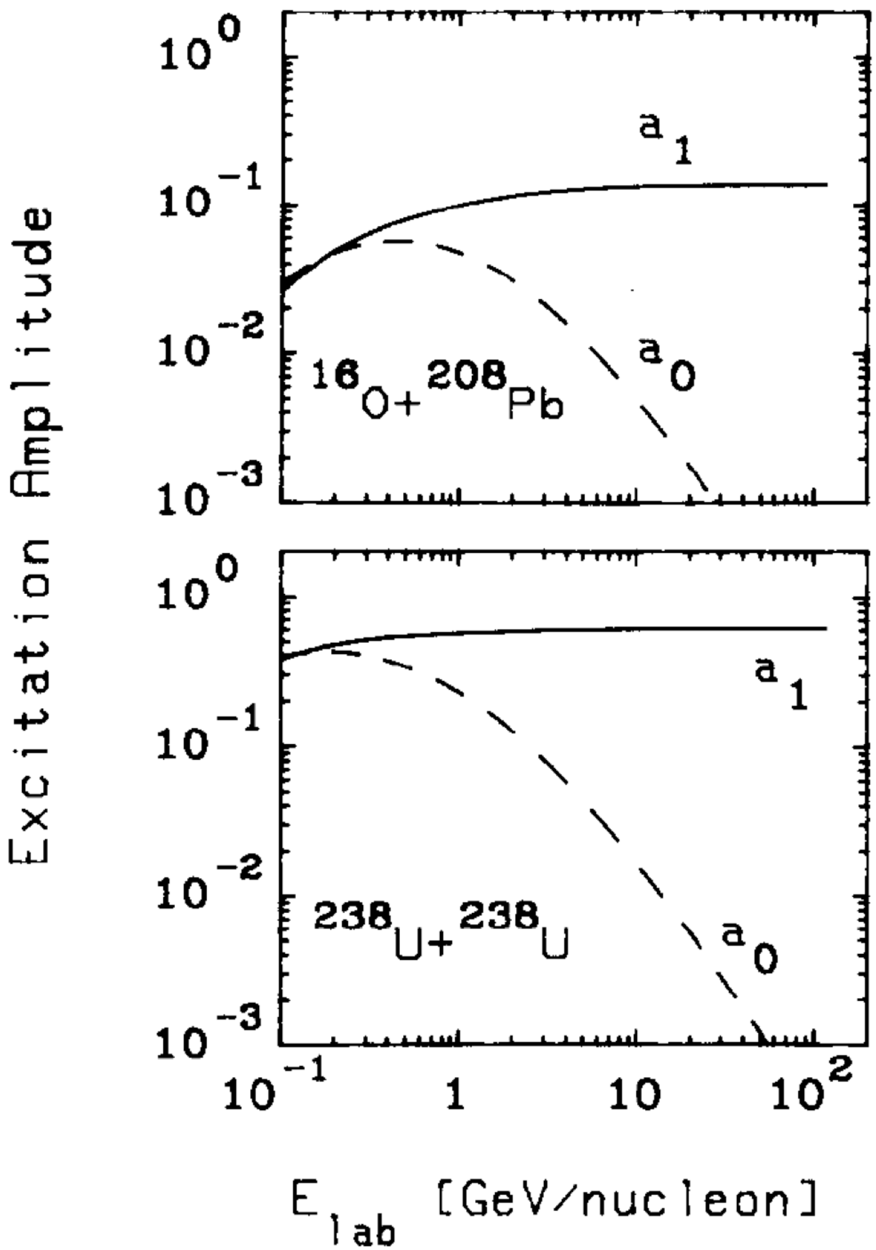
$$\int \frac{dE_\gamma}{E_\gamma^2} \sigma^\gamma_{GQR}(E_\gamma) \approx 0.22 A^{2/3} \text{ MeV}$$

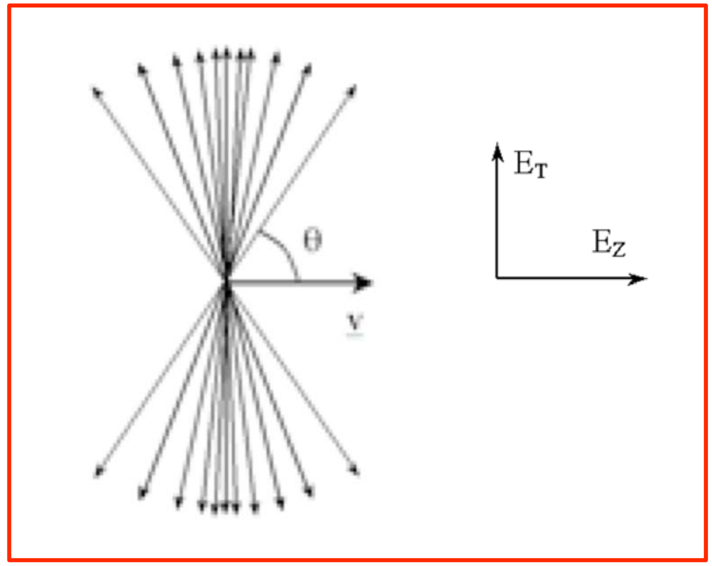
Other GRs

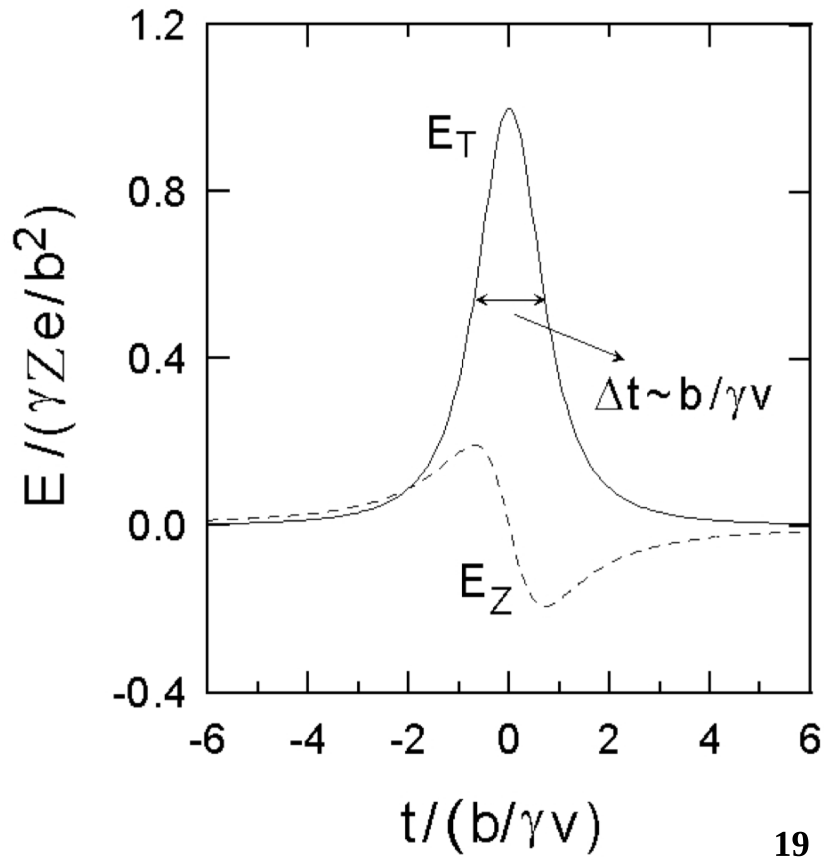












Non-negligible possibility to excite a giant resonance excited on top of another giant resonance

Multiphonon GRs

Estimate

Harmonic oscillator:

1- $\Phi = |a_{fi}|^2$ from 1st order

2- All orders:

$$P_{N\ \mathrm{phonons}} = \frac{\Phi^N}{N!} e^{-\Phi}$$

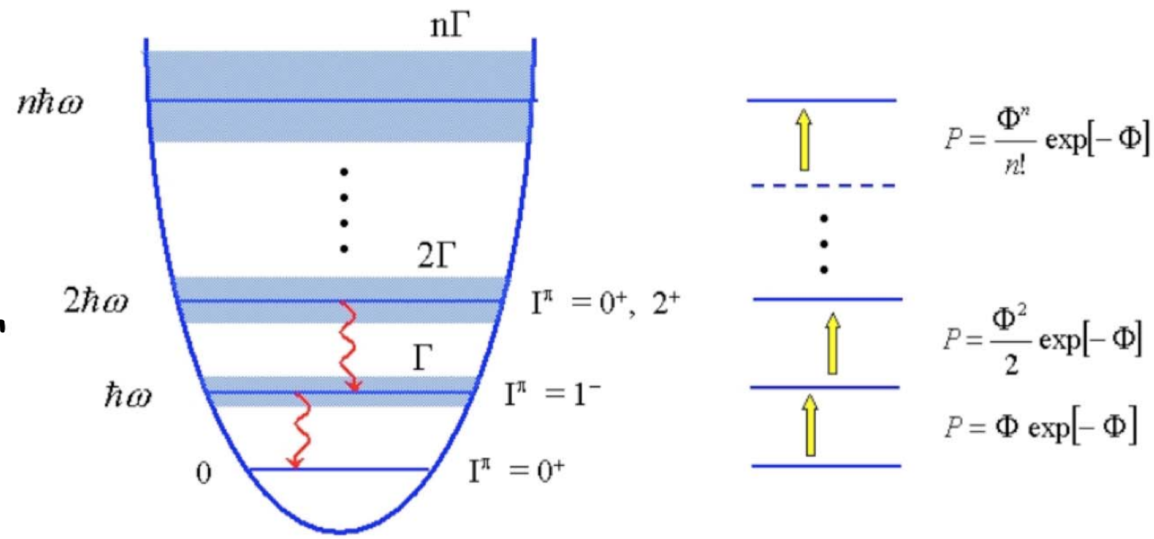
$$\sigma_{DGDR} \approx Z_P^4 \left(\frac{NZ}{A^{2/3}} \right)^2$$

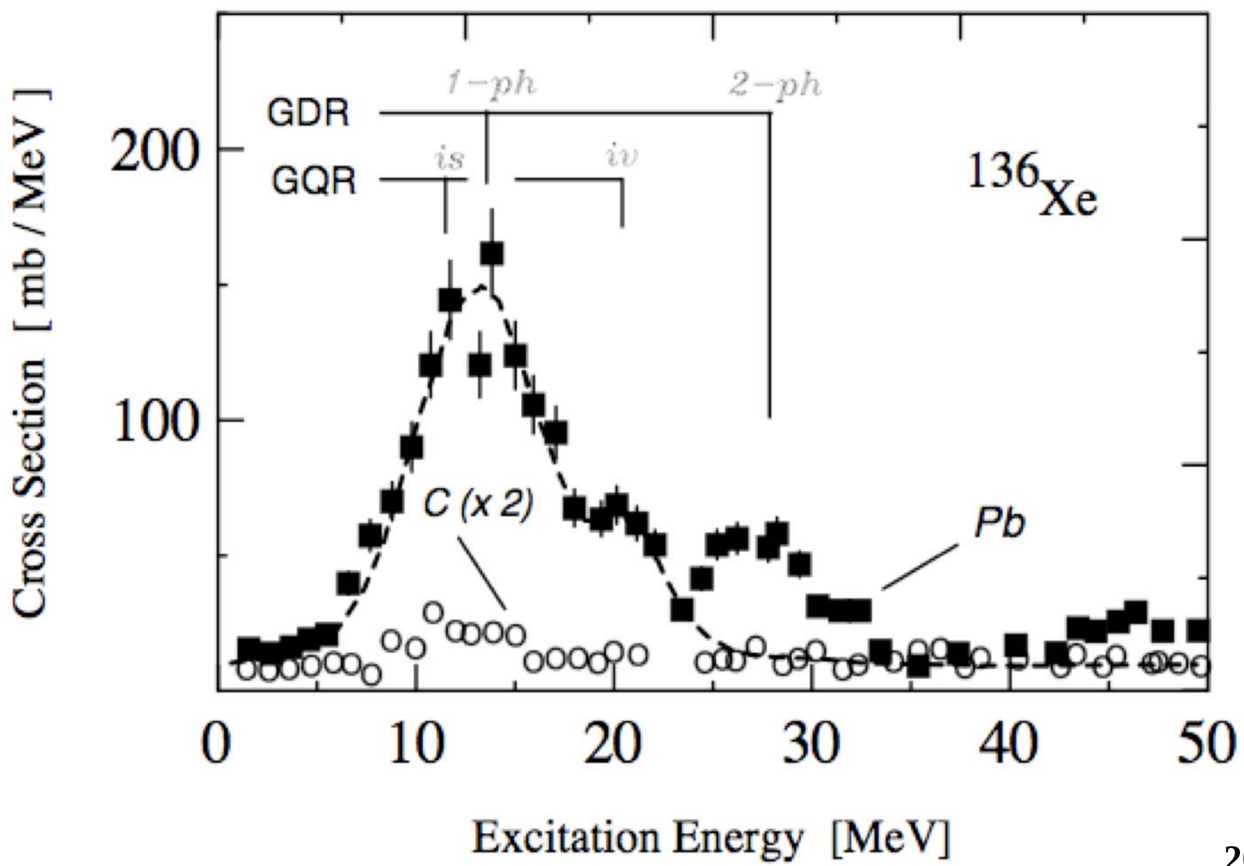
^{136}Xe + C

^{136}Xe + Pb

~ 1 GeV/nucleon

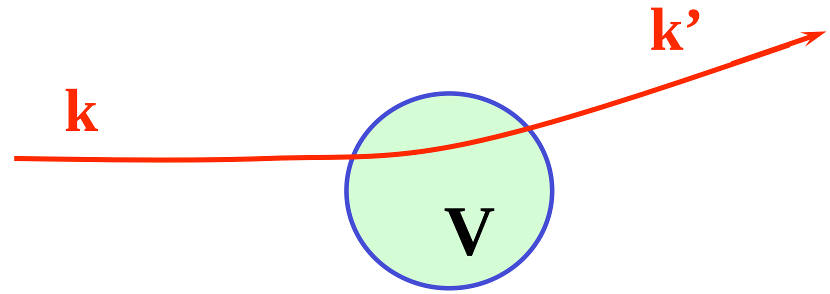
GSI/Darmstadt, 1993





Quantum scattering effects

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V\right]\Psi = E\Psi$$



Partial wave expansion:

$$u_l(r) \xrightarrow[r\to\infty]{} \frac{i}{2}\left\{H_l^{(-)}(kr) - S_l H_l^{(+)}(kr)\right\}$$

Incoming wave

"Survival" amplitude (S-matrix)

Outgoing wave

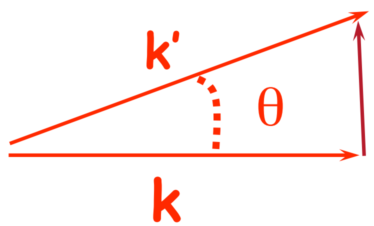
$$S_l = e^{2i\delta_l}$$ (δ_l = Phase shift)

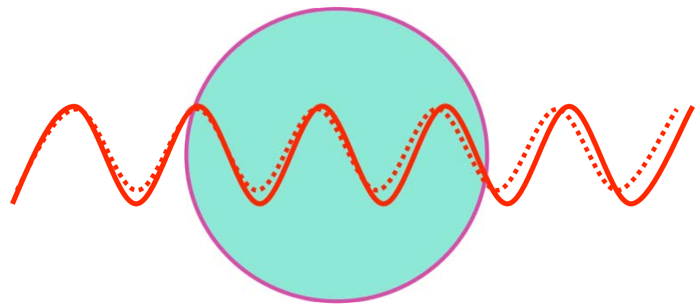
$|S_l|^2$ = "Survival" probability ≤ 1

High energy collisions (E_{lab} > 50 MeV/nucleon)

Eikonal Waves

$\Delta E << E, \quad \theta << 1$ radian , $\quad |\Delta\psi/\psi|_{\Delta r=\lambda} << 1$





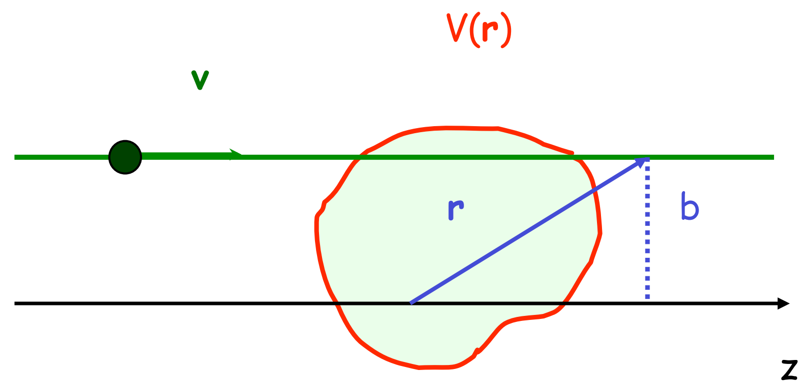
$$\Psi(\mathbf{r}) = S(\mathbf{b},z)\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$S(\mathbf{b},z) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{z} V(\mathbf{r}')\,dz'\right\}$$

$$\mathbf{r}' = (\mathbf{b},z')$$

$z \rightarrow \infty$ after the collision:

$$\Psi(\mathbf{r}) = S(\mathbf{b})\, e^{i\mathbf{k}\cdot\mathbf{r}}$$



$$S(\mathbf{b}) = e^{i\chi(\mathbf{b})}$$
$$= \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{\infty} V(\mathbf{r}')\,dz'\right\}$$

Eikonal waves (reactions)

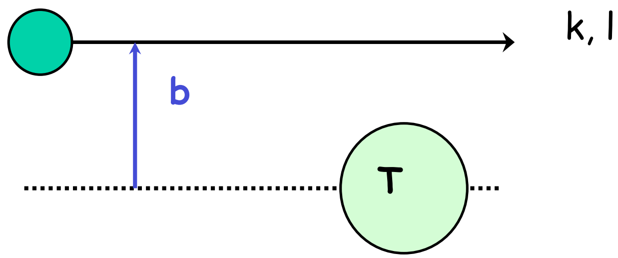
Harmonic oscillator (structure)

Pearls of quantum mechanics

Eikonal Waves: Applications
(sometimes called "Glauber theory")

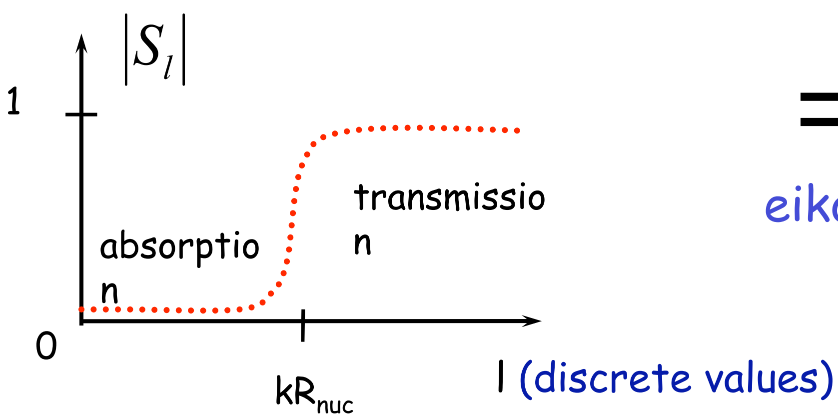
Roy Glauber in Tucson, University of Arizona
Few days after winning 2005 Nobel prize
(another "Glauber theory")

S-matrices ("Survival" Amplitudes)



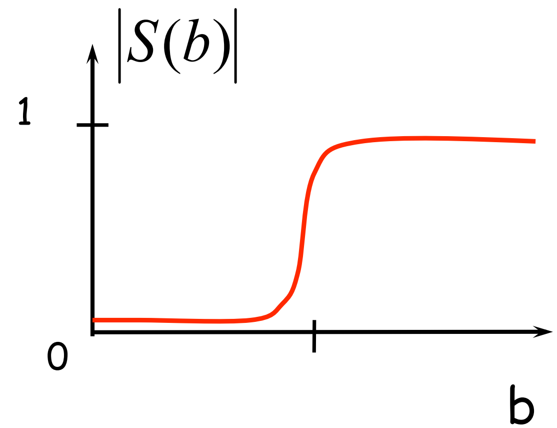
b = impact parameter

$l = kb$ (actually $l + 1/2 = kb$)



$\Longrightarrow$

eikonal



Ex: Elastic Scattering

$$f(\theta) = \frac{i}{k} \sum_l (l + \tfrac{1}{2})(1 - S_l) P_l(\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

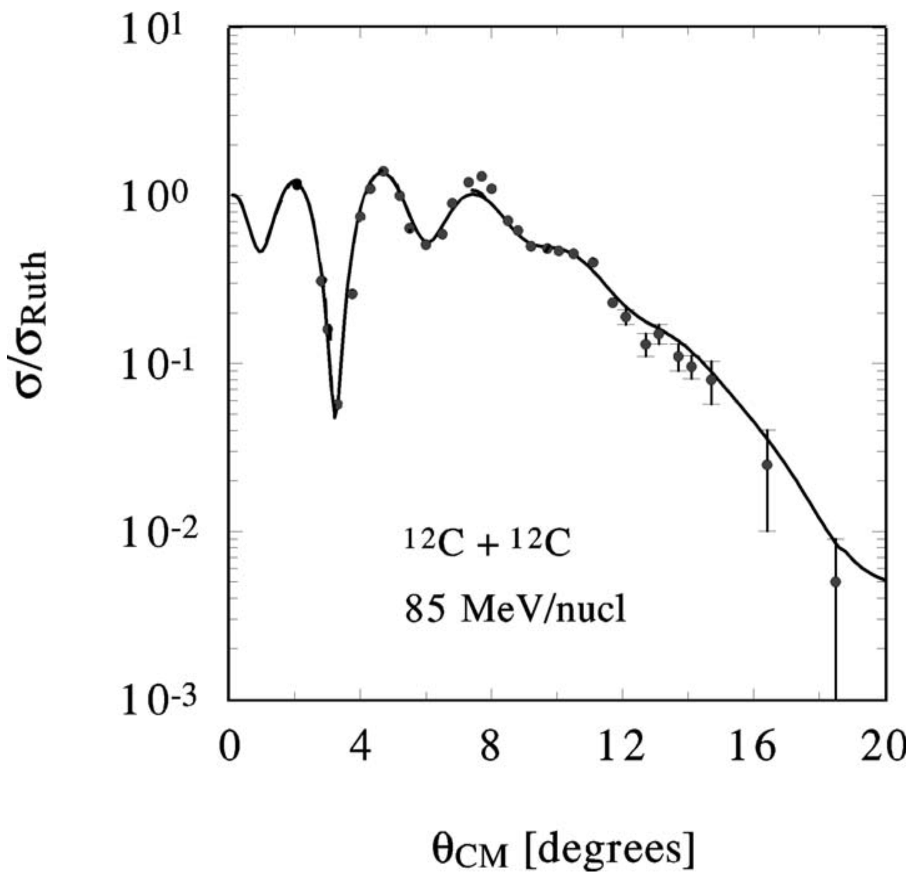
$$f(\theta) = ik \int db\, b\, J_0(kb) \{1 - S(b)\}$$

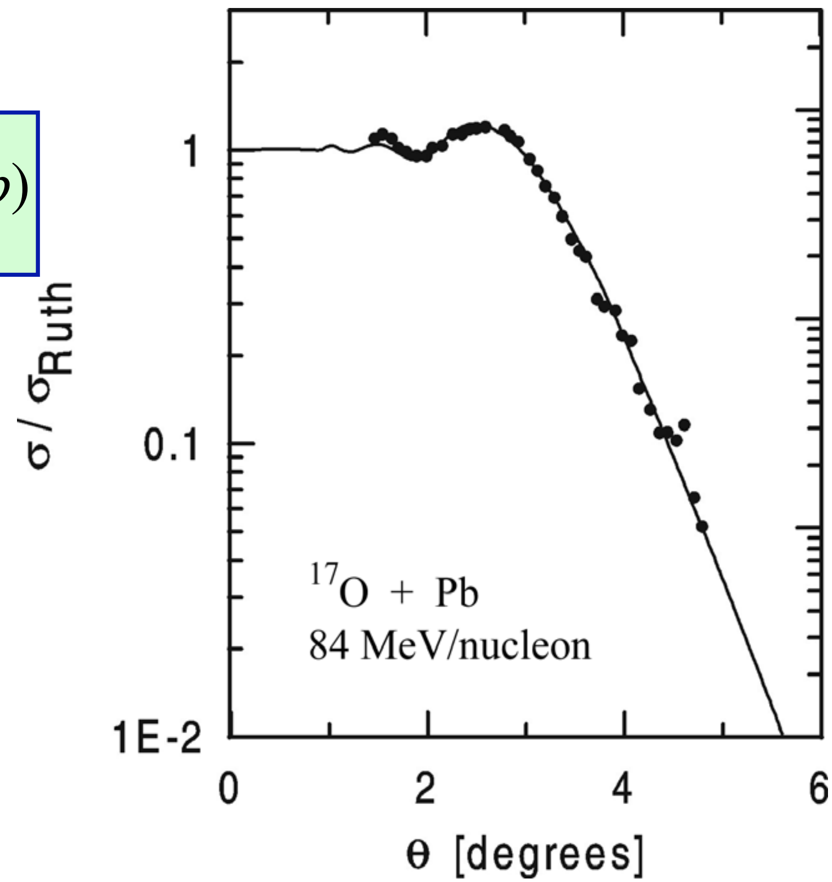
Ex: Probing nuclear densities

$$\chi_{AB}^{(N)}(b) = \frac{1}{k_{nn}} \int_0^\infty dq\, q\, \tilde{\rho}_A(q)\tilde{\rho}_B(q)\, f_{nn}(q) J_0(qb)$$

$$f_{nn}(q) = \frac{k_{nn}}{4\pi}\sigma_{nn}\left(i + \alpha_{nn}\right)e^{-\beta_{nn} q^2}$$

(from nn scattering)





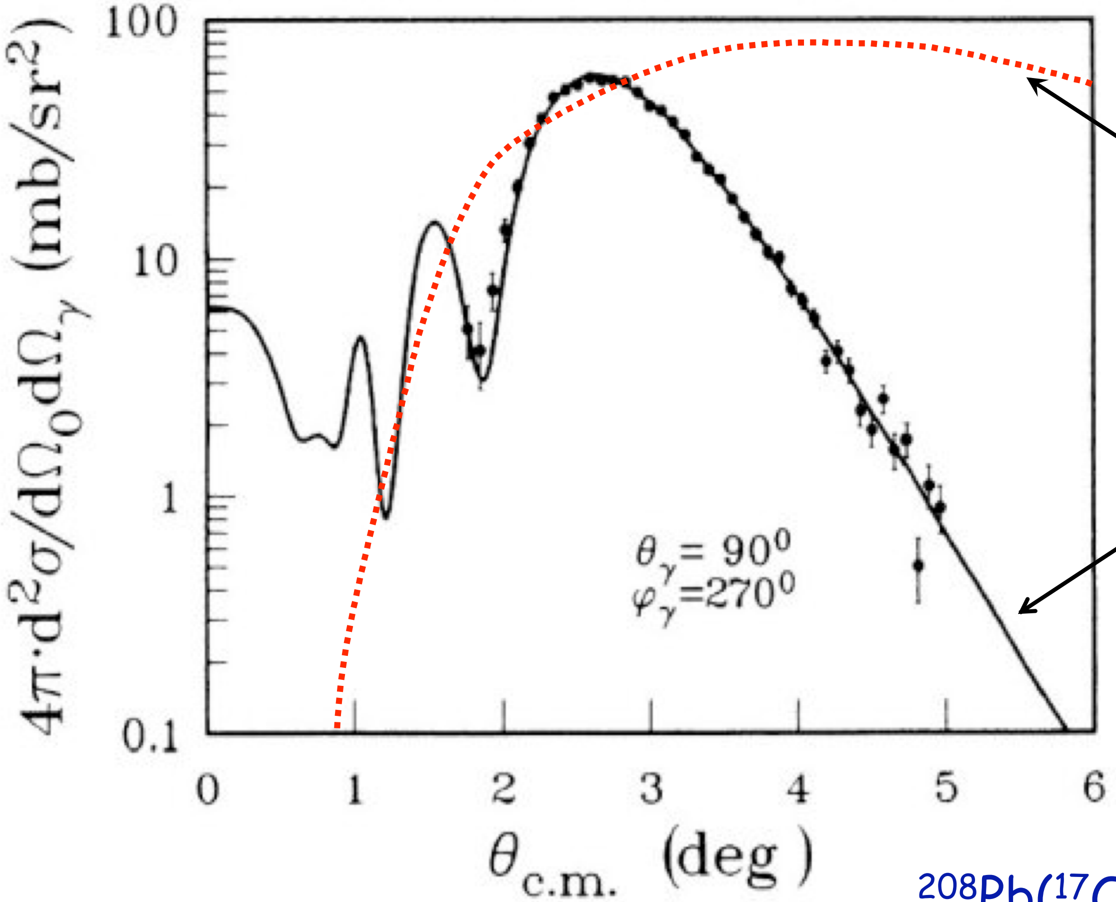
solid curves: Glauber

Quantum treatment of Coulomb excitation

$$f_{inel}^{C}(\theta) \approx \int d^3r\, d^3r'\, \Psi_{\mathbf{k}'}^{(-)*}(\mathbf{r})\, \varphi_f(\mathbf{r}')\, V_C(\mathbf{r},\mathbf{r}')\, \Psi_{\mathbf{k}}^{(+)}(\mathbf{r})\, \varphi_i(\mathbf{r}')$$

DWBA

eikonal waves

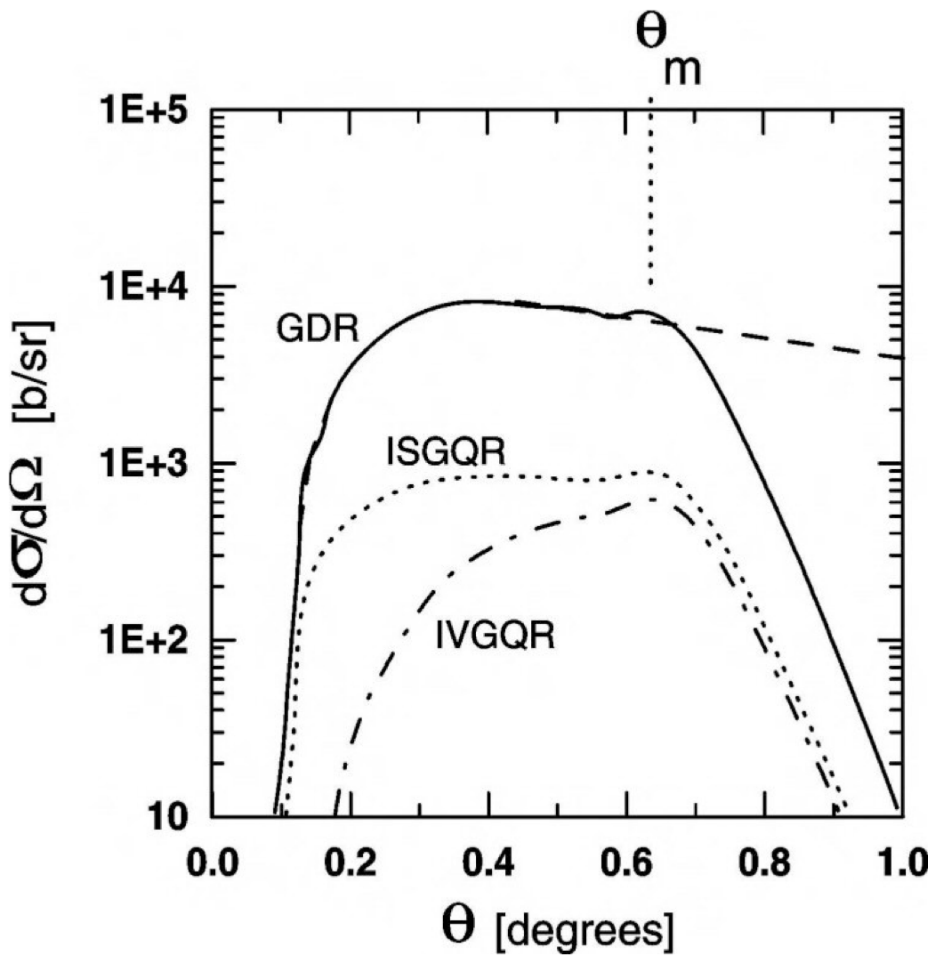


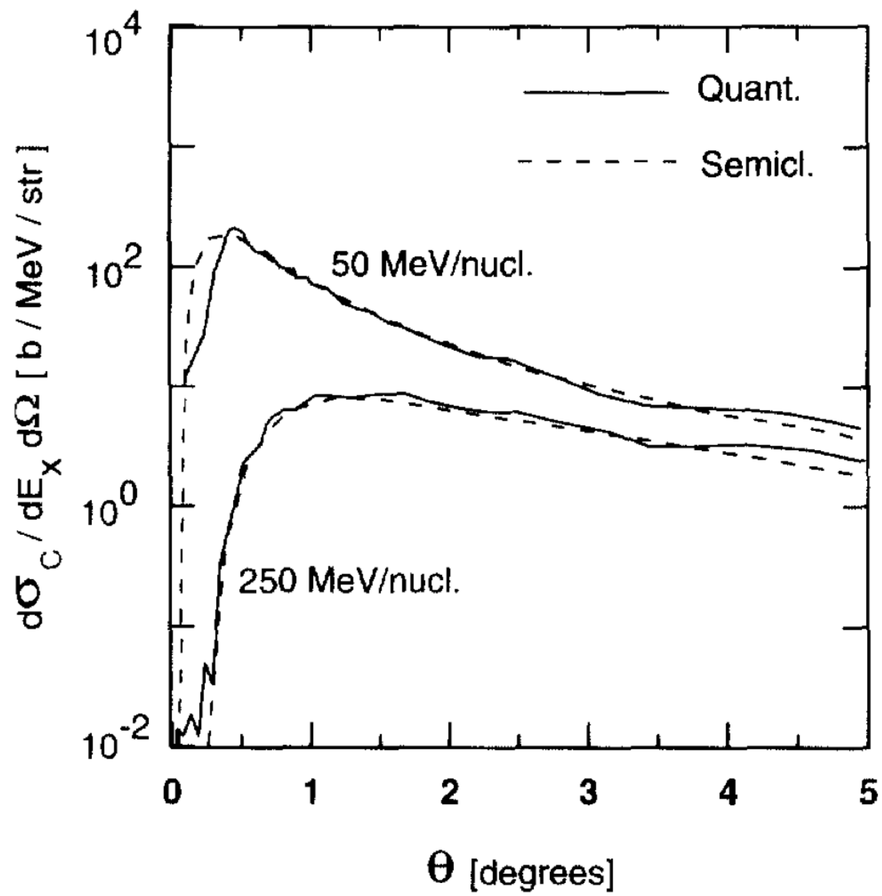
semiclassical

eikonal

Excitation of GDR
followed by γ-decay

$^{208}Pb(^{17}O,^{17}O'\gamma)$ at 84 MeV/nucleon
for fixed angle $\theta_\gamma = 90^o$ and $\varphi_\gamma = 270^o$





Excitation of GRs in Pb+Pb collisions at 640 MeV/nucleon

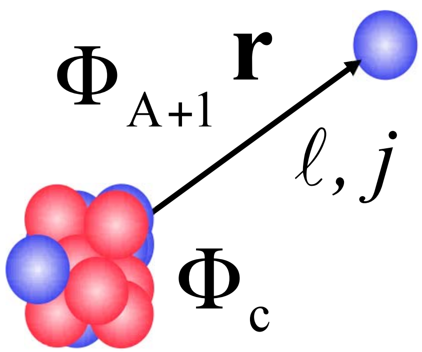
Excitation of ^{8}B projectiles on Pb at 50 and 250 MeV/nucleon.

E_γ = 1.2 MeV

Coulomb excitation of loosely-bound nuclei

Spectroscopic factors

Nucleon removal from Φ_{A+1} will leave mass A residue in the ground or an excited state - amplitude for finding nucleon with sp quantum numbers ℓ, j, about core state Φ_c in Φ_{A+1} is



$$O^{c}_{\ell j}(\mathbf{r}) = \langle \mathbf{r}, \Phi_c \,|\, \Phi_{A+1} \rangle, \quad S_N = E_{A+1} - E_c$$

overlap integral

$$\int d^3 r \, |O^{c}_{\ell j}(\mathbf{r})|^2 = C^2 S(\ell j)$$

Spectroscopic factor – occupancy of the state

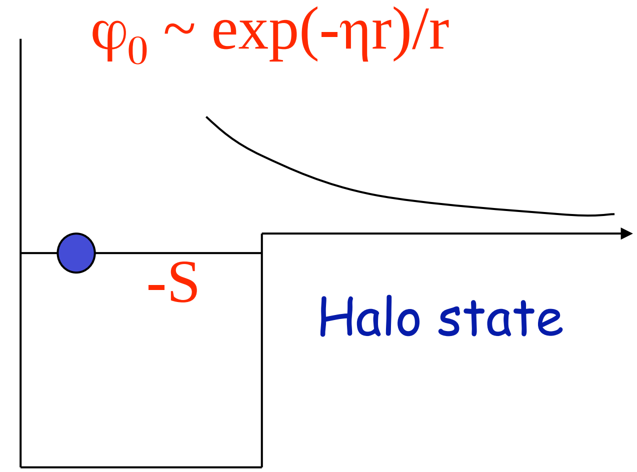
Usual to write

$$O^{c}_{\ell j}(\mathbf{r}) = \sqrt{C^2 S(\ell j)} \; \phi_0(\mathbf{r}) \; ; \qquad \int d^3 r \, |\phi_0(\mathbf{r})|^2 = 1$$

Coulomb excitation of loosely-bound nuclei

$$\frac{d\sigma_C}{dE_x} \approx n_{EL}(E_\gamma) \times \mathrm{C}^2\mathrm{S} \left| \left\langle \psi_{\mathbf{k}} \left\| r^L Y_L \right\| \phi_0 \right\rangle \right|^2 \frac{d^3k}{(2\pi)^3}$$

$$\frac{dB(EL)}{dE_\gamma}$$



Estimate

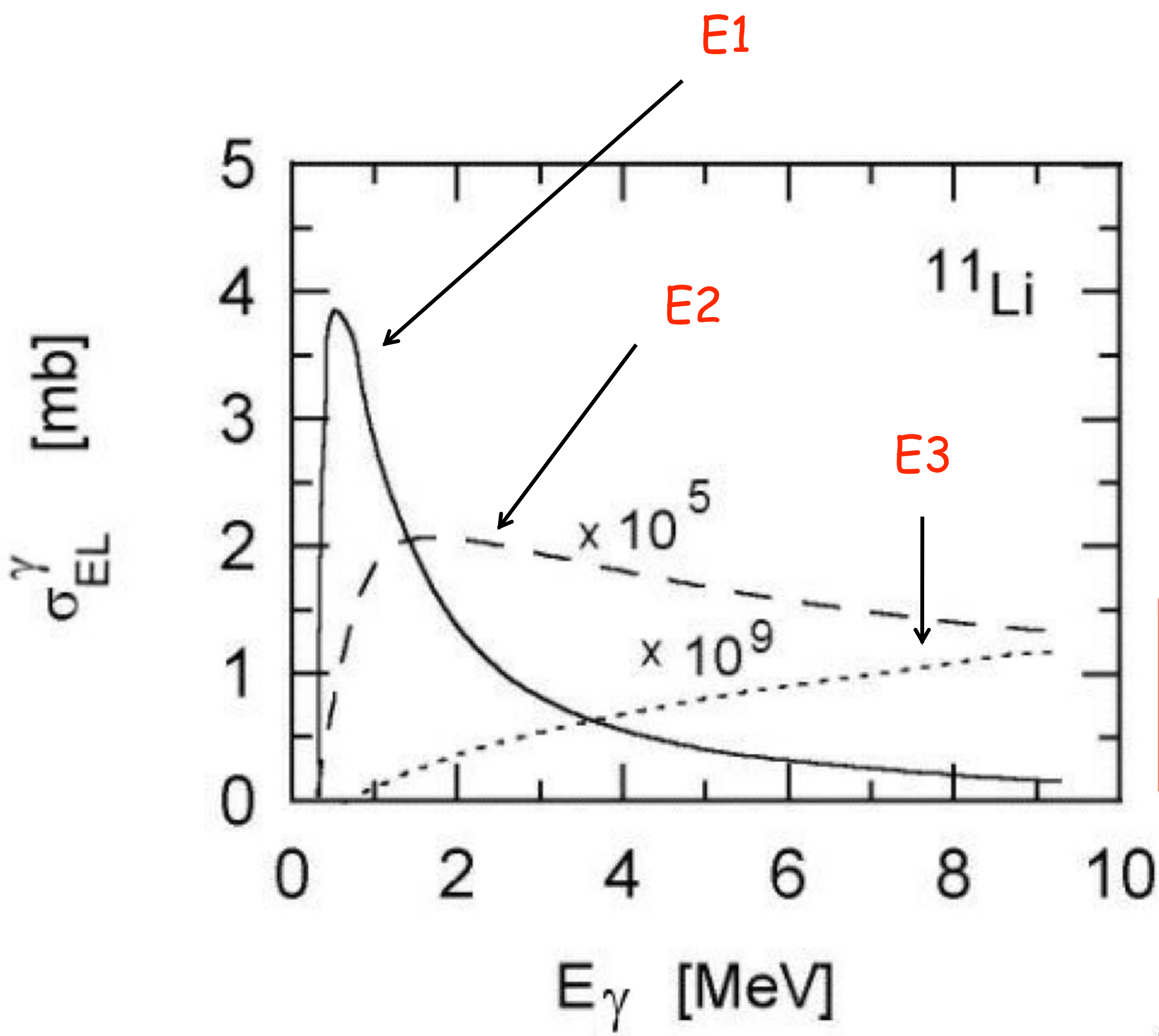
$$\psi_{\mathbf{k}} \approx e^{i\mathbf{k}\cdot\mathbf{r}} \qquad \phi_0 \approx \frac{1}{r} e^{-\eta r}$$

$$\Rightarrow \quad \frac{dB(EL)}{dE_\gamma} \approx \frac{\sqrt{S}\, E_{rel}^{L+1/2}}{(E_{rel} + S)^{2L+2}}$$

$$E_{rel} = E_\gamma - S \qquad S = \frac{\hbar^2 \eta^2}{2\mu}$$

Separation energy of fragments with reduced mass μ

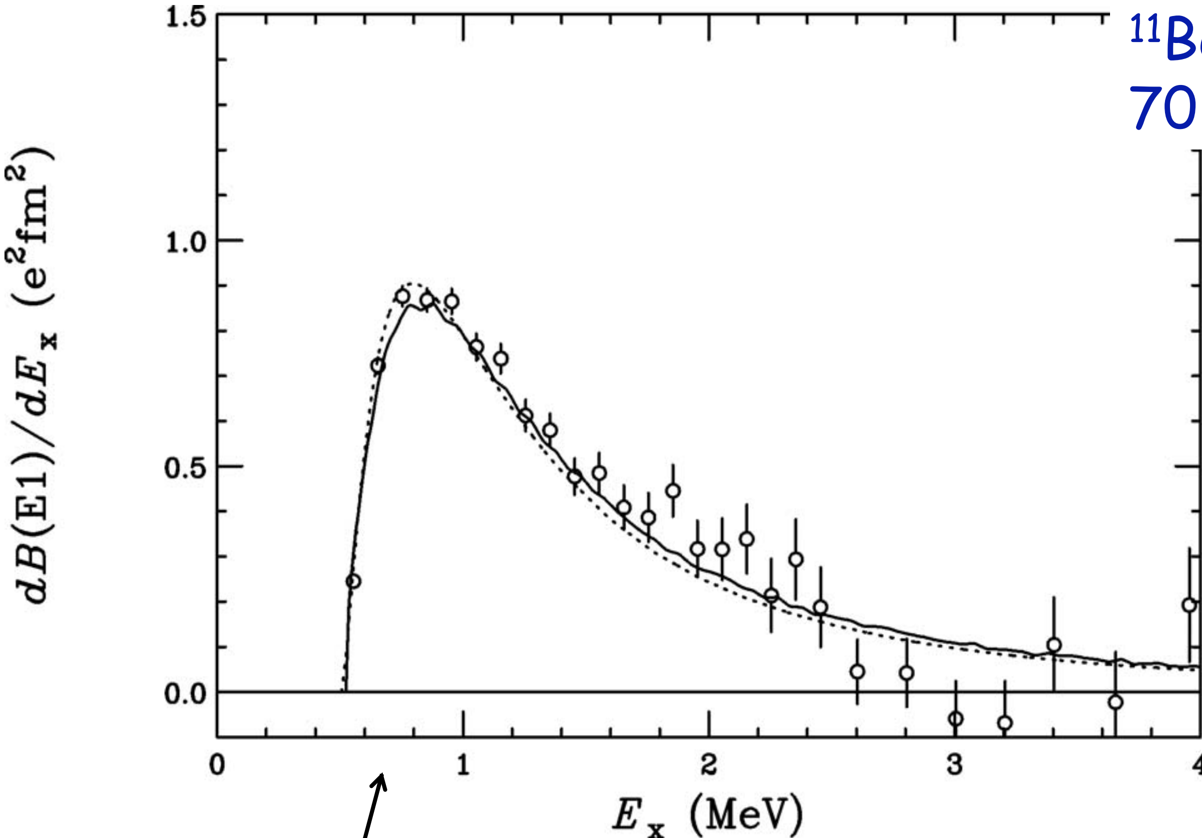
Electric response of loosely-bound nuclei



$$\sigma_L^{\gamma} \sim E_{\gamma}^{2L+1} B(EL)$$

$$\frac{dB(EL)}{dE_{\gamma}} \approx \frac{\sqrt{S}\, E_{rel}^{L+1/2}}{\left(E_{rel}+S\right)^{2L+2}}$$

Ex: Coulomb breakup of ^{11}Be



^{11}Be+Pb → ^{10}Be + n + Pb
70 MeV/nucleon

$$B(E1) = 1.05 \pm 0.06\ e^2\text{fm}^2$$
$$(3.29 \pm 0.06\ \text{W.u})$$

$$S = 0.54\ \text{MeV}$$

peak at $E_x = \frac{8}{5} S = 0.76\ \text{MeV}$

$$\frac{dB(EL)}{dE_\gamma} \approx \frac{\sqrt{S}\left(E_x - S\right)^{3/2}}{E_x^{\ 4}}$$

$$\mathcal{I}_{sp} = \langle \psi_{\mathbf{k}} \| r^1 Y_1 \| \phi_0 \rangle \approx \frac{k^2}{(k^2+\eta^2)^2}\left[\cos\delta + \sin\delta \frac{\eta(\eta^2+3k^2)}{2k^3}\right]$$

$$\approx \frac{E_{rel}}{(S+E_{rel})^2}\left[1+\left(\frac{\mu}{2\hbar^2}\right)\frac{\sqrt{S}(S+3E_{rel})}{-1/a+(\mu E_{rel}/\hbar^2)r_{eff}}\right]$$

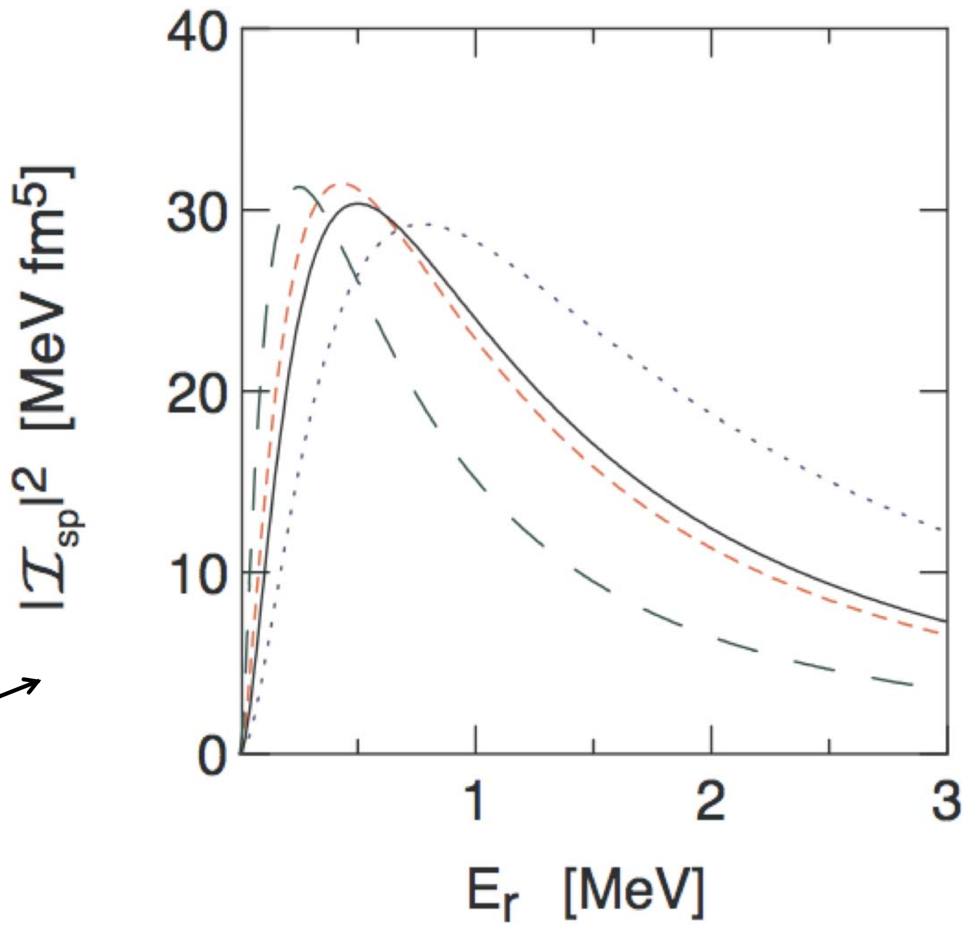
Final state interactions

δ = scattering phase shift

a = scattering length

r_{eff} = effective range

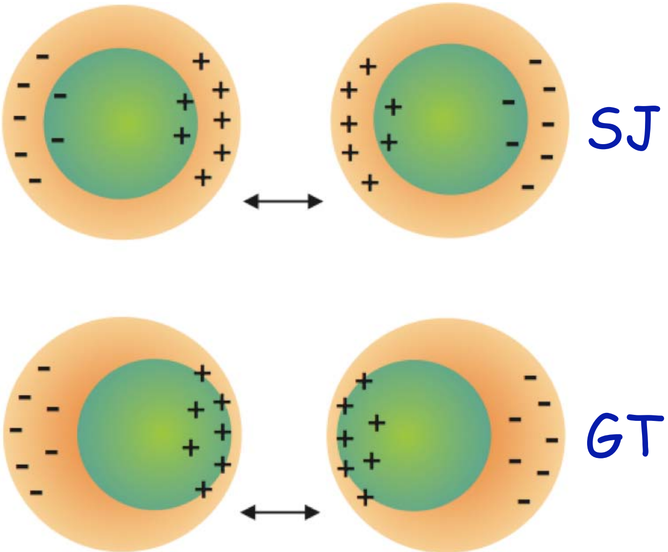
Strongly ddependent on final state interactions (phase-shifts)

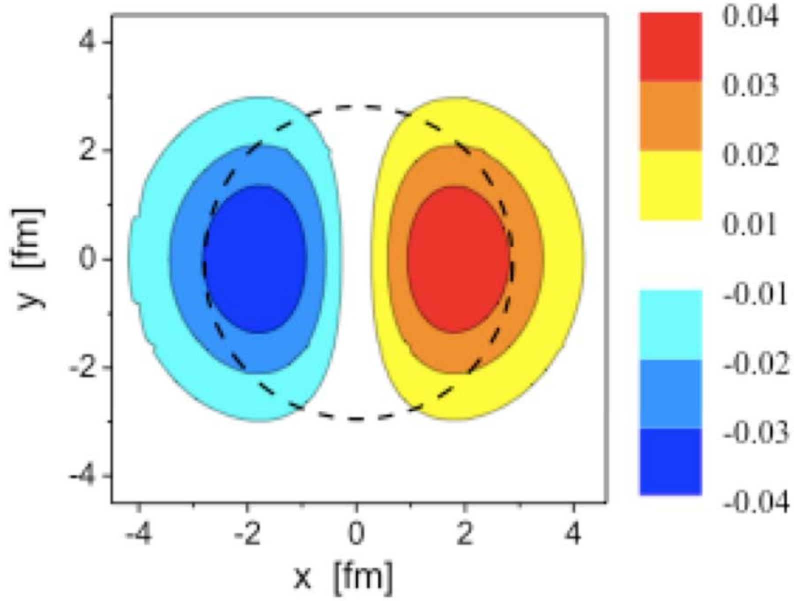


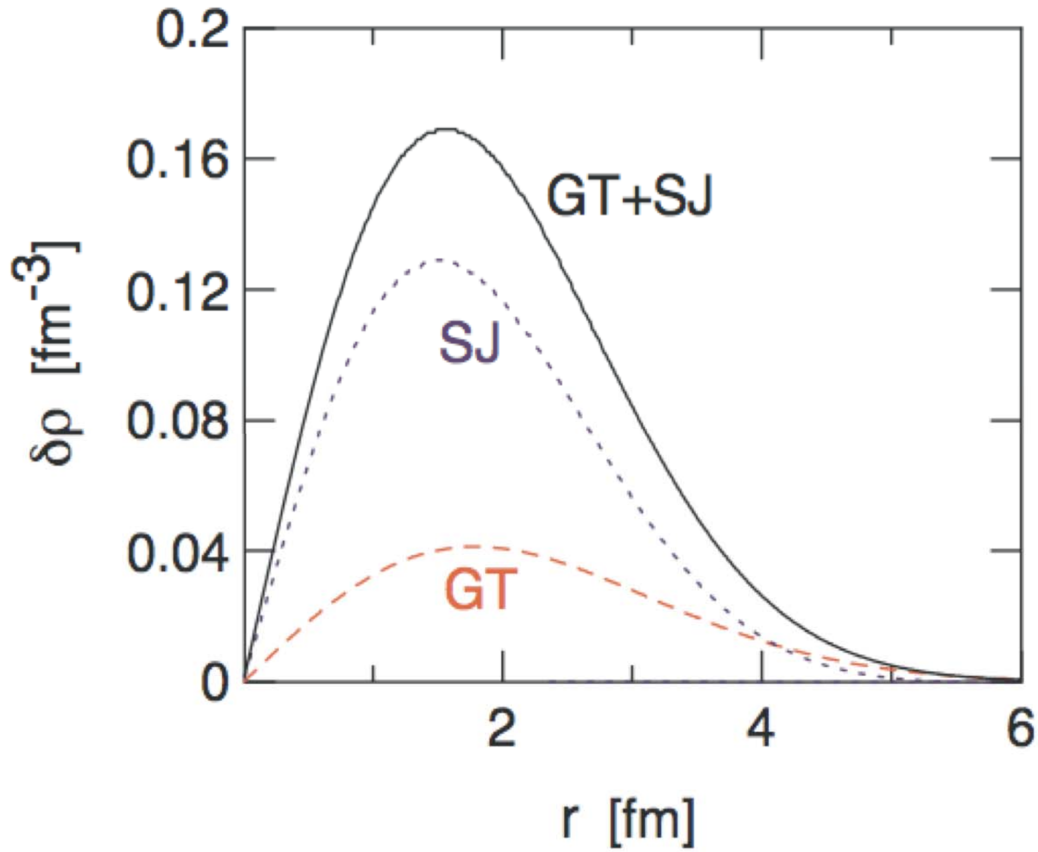
Pigmy resonances

$$B(EL) \sim \left| \int r^L \delta\rho_{fi}\, d^3 r \right|^2$$

$$\delta\rho_P(r) \approx Z_{eff}^{GT} \alpha_{GT} \frac{d\rho_0}{dr} + Z_{eff}^{SJ} \alpha_{SJ} j_1(kr) \rho_0(r)$$







Estimate

hydrodynamical model

$$E_{PR} = \left(\frac{3\hbar^2}{2aRm_N A_r} \right)$$

$$A_r = A_c(A - A_c)/A$$

R = nuclear size

a = difuseness

$$\Rightarrow \quad E_{PR} \approx 1 \text{ MeV}$$

$$\Gamma \sim \frac{\hbar \bar{v}_N}{R}$$

$$\bar{v}_N = \frac{3}{4} v_F = \frac{3}{4}\sqrt{\frac{2E_F}{m_N}}$$

$E_F \sim 35$ MeV

$R \sim 5$ fm

$$\Rightarrow \quad \Gamma \sim 6 \text{ MeV}$$

usual GR

pygmy GR

$\bar{v} \sim$ relative velocity of core and halo

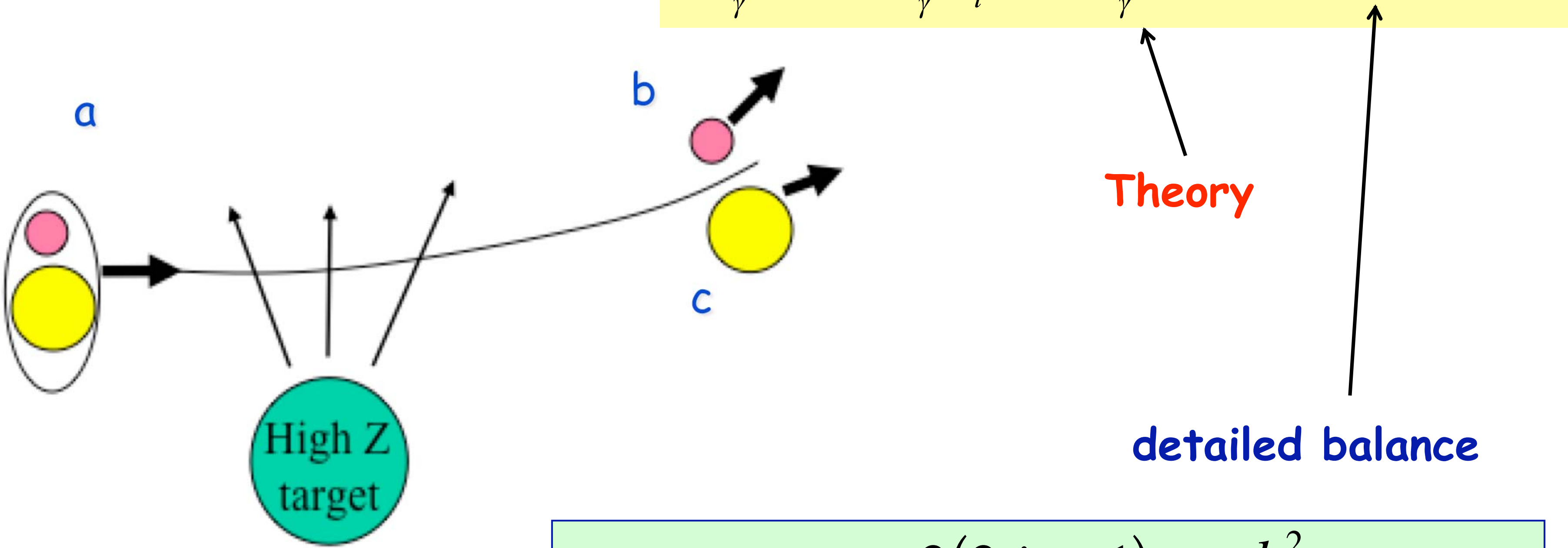
$E_F \rightarrow E_P \sim 1$ MeV

$$\Rightarrow \quad \Gamma \sim 1 \text{ MeV}$$

only accurate microscopic models
can resolve pygmy from direct breakup

The Coulomb dissociation method

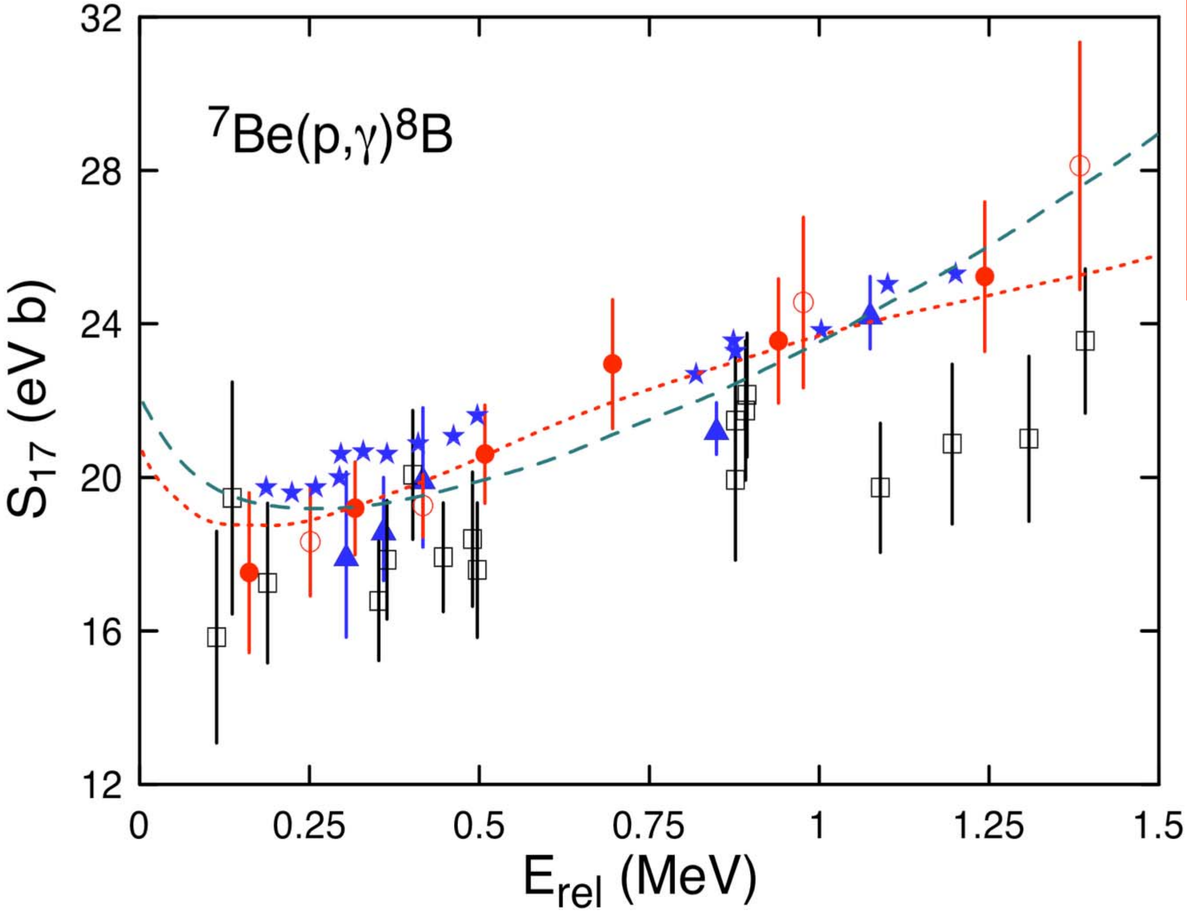
$$\frac{d\sigma}{dE_\gamma d\Omega} = \frac{1}{E_\gamma} \sum_l \frac{dn_l(E_\gamma, \Omega)}{dE_\gamma d\Omega} \sigma_{\gamma + \mathrm{a} \to \mathrm{b} + \mathrm{c}}(E_\gamma)$$



$$\sigma_{b+c \to a+\gamma} = \frac{2(2j_a+1)}{(2j_b+1)(2j_c+1)} \frac{k_{bc}^2}{k_\gamma^2} \sigma_{\gamma+a \to b+c}$$

Applications to radiative capture (n,γ) and (p,γ) reactions in nuclear astrophysics.

Ex: Coulomb breakup of ^8B



Solar neutrino problem is due to ν-oscillations

But this reaction needs to be known more accurately

- J. Bahcall

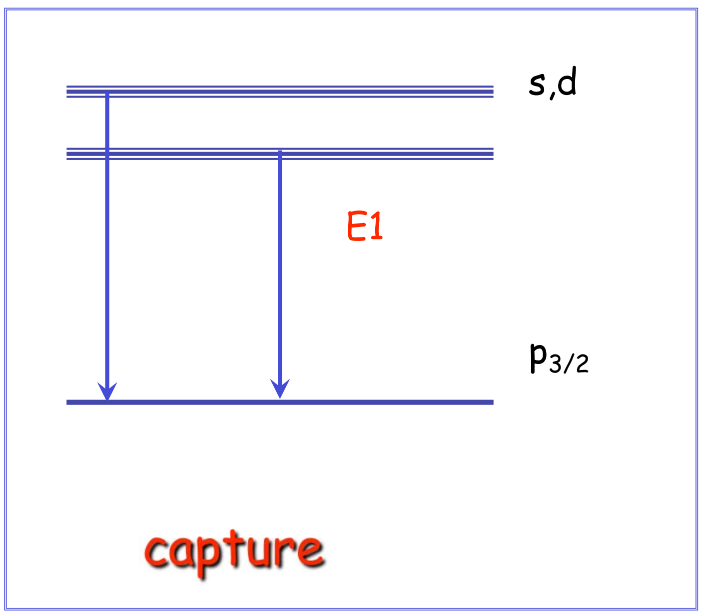
Needs consideration

1- Usual theoretical inputs: interactions, many-body problem.

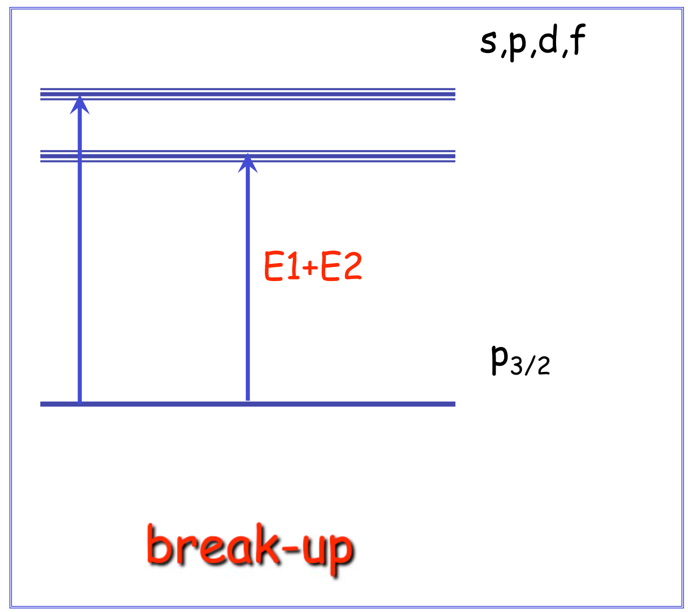
2- Nuclear breakup contribution

3- Additional reaction problems: inverse methods only studies ground state transtions, etc.

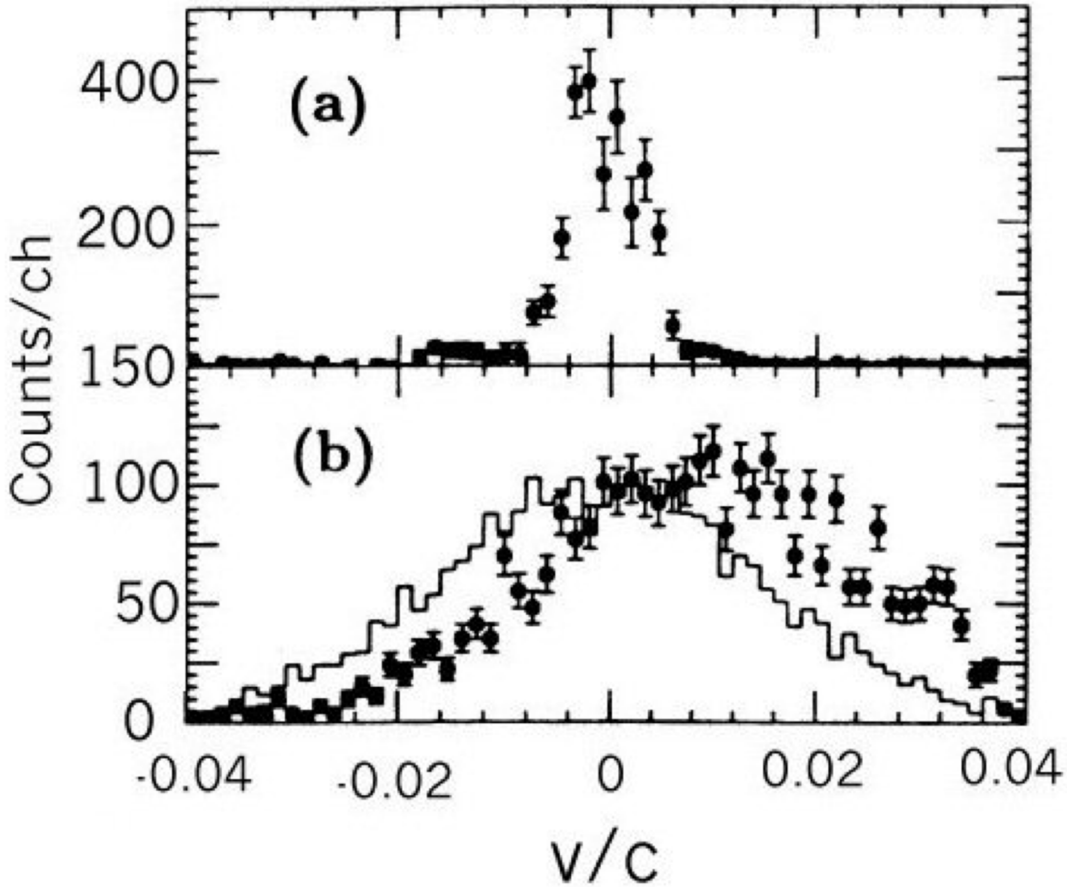
THIS occurs in stars



THIS is obtained in lab



Higher-order effects



Coulomb breakup of ^{11}Li on Pb at ~ 50 MeV/nucleon

velocity of neutrons

velocity of ^{9}Li

MSU, 1993

→ post-acceleration effect

Schroedinger equation on a space-time lattice

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$$

$$\psi(t+\Delta t) = \exp\left[-\frac{i}{\hbar}\hat{H}\Delta t\right]\psi(t)$$

$$H = \frac{\hat{p}^2}{2\mu} + V_N(x) + V_C(x,t)$$

discrete space lattice: x_j, $j=1, 2, \ldots, N$

$$\hat{S}\psi_j(t) = \sum_k V_{Ck}\psi_k(t)$$

$$\psi_j(t+\Delta t) = \frac{\left[\frac{1}{i\tau} + \Delta^{(2)} - \frac{\Delta t}{2\hbar\tau}V_{Nj} + \frac{\Delta t}{\hbar\tau}\hat{S}\right]}{\left[\frac{1}{i\tau} - \Delta^{(2)} + \frac{\Delta t}{2\hbar\tau}V_{Nj}\right]}\psi_j(t)$$

$$\tau = \frac{\hbar\Delta t}{4\mu(\Delta x)^2}$$

Good to order $(\Delta t)^3$
preserves unitarity

→ three-dimensions straightforward

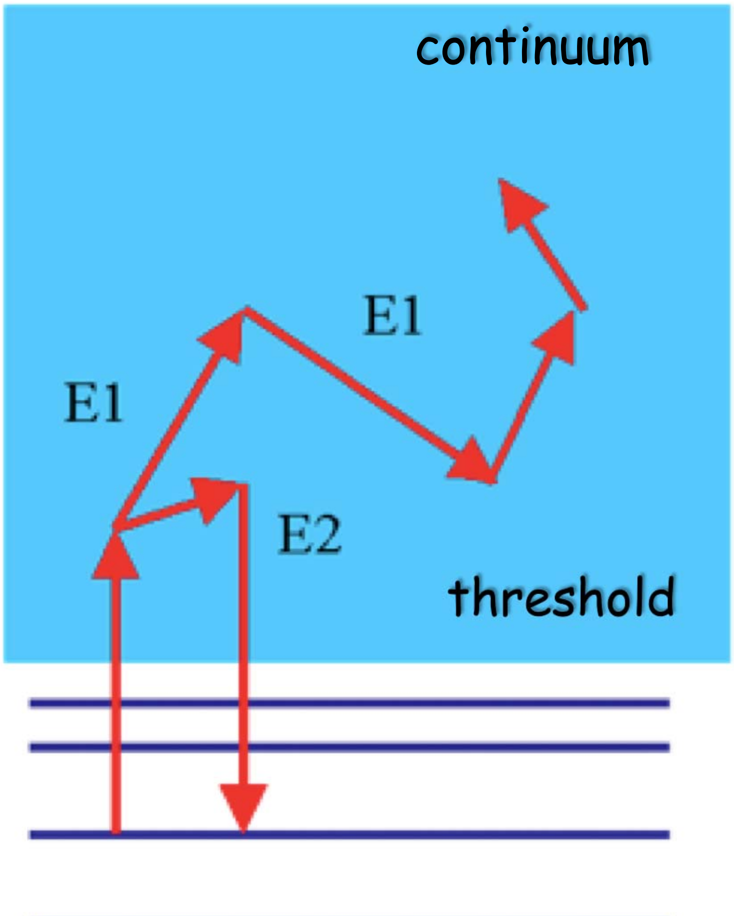
From ψ any observable can be calculated

Continuum discretized coupled-channels (CDCC)

$$\left|\varphi_0\right\rangle = e^{-iE_0 t/\hbar}$$

$$\left|\varphi_{jJM}\right\rangle = e^{-iE_j t/\hbar}\int \Gamma_j(E)\left|E,JM\right\rangle dE$$

$$\int \Gamma_i(E)\Gamma_j(E)\,dE = \delta_{ij}$$



semiclassical coupled-channels

$$\frac{da_k}{dt} = -\frac{i}{\hbar}\sum_j a_j(t)V_{kj}(t)e^{i(E_k-E_j)t/\hbar}$$

$$V_{kj}(t) = \int \psi_k^* V(t)\,\psi_j\, d^3r$$

From a_k calculate ψ and observables of interest

Eikonal CDCC

$$\psi = \sum_j S_j(z,b)\exp(ik_j z)\,\phi_{k_j}(\xi)$$

$$S_0 = \frac{1}{i\hbar}\int_{-\infty}^{z} V(r')dz'$$ (ground state)

$$V = V_C + V_N$$

$$i\hbar \mathrm{v}\frac{\partial S_j(z,b)}{\partial z} = \sum_m \langle j|V|m\rangle S_m(z,b)\exp\left[-\left(k_m - k_j\right)z\right]$$ (excited states)

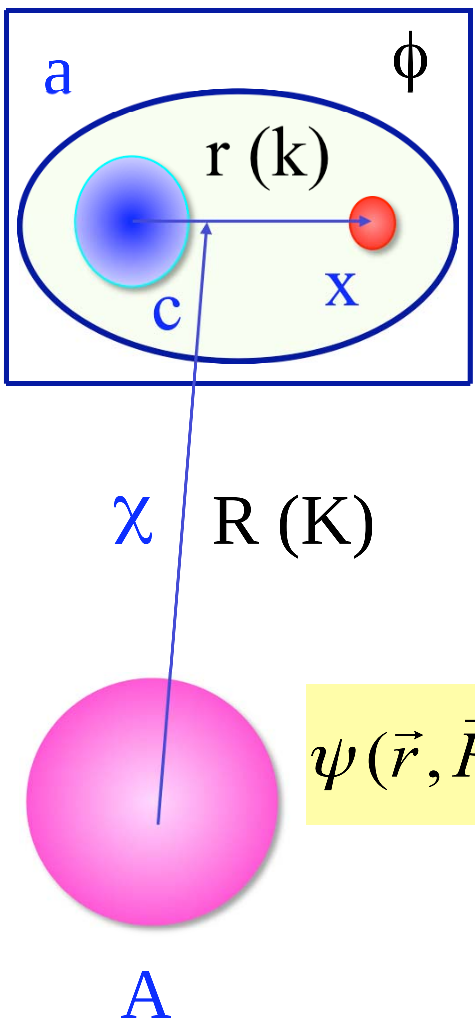
Note similarity with semiclassical t.d. equation with z = vt

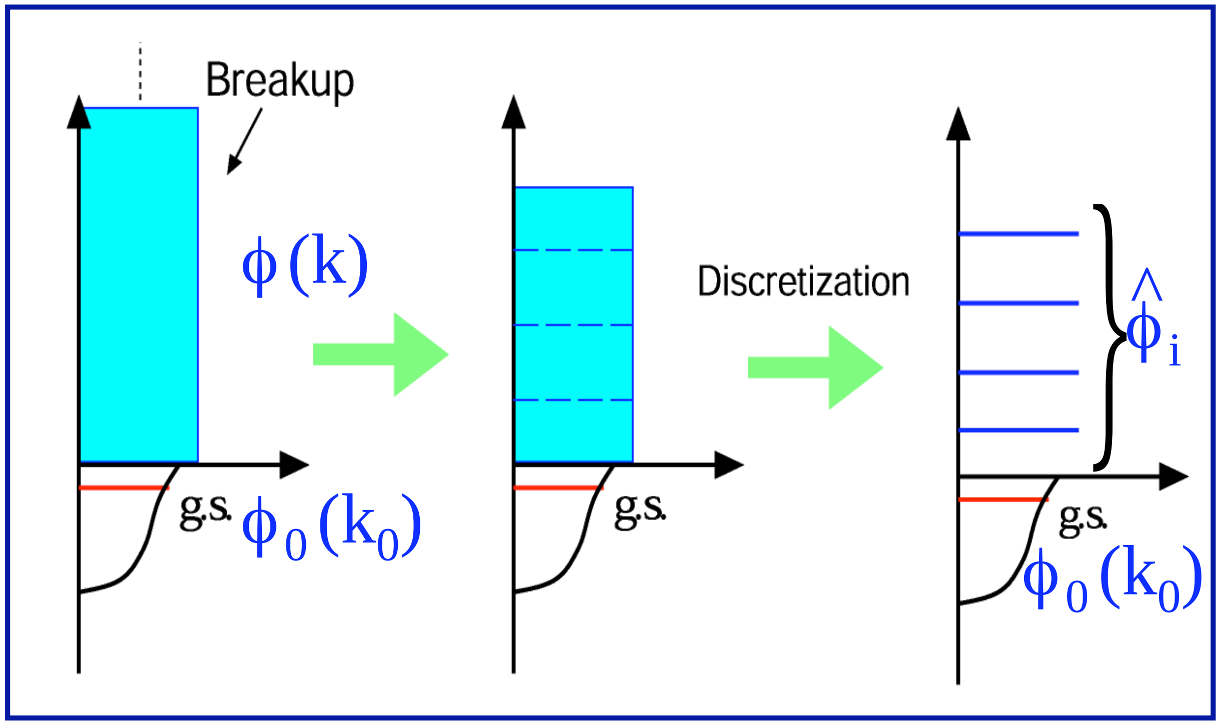
$$\frac{da_k}{dt} = -\frac{i}{\hbar}\sum_j a_j(t)V_{kj}(t)e^{i(E_k - E_j)t/\hbar}$$

Corrections due to energy conservation (v ≠ constant) straightforward

From S_j calculate ψ and observables of interest

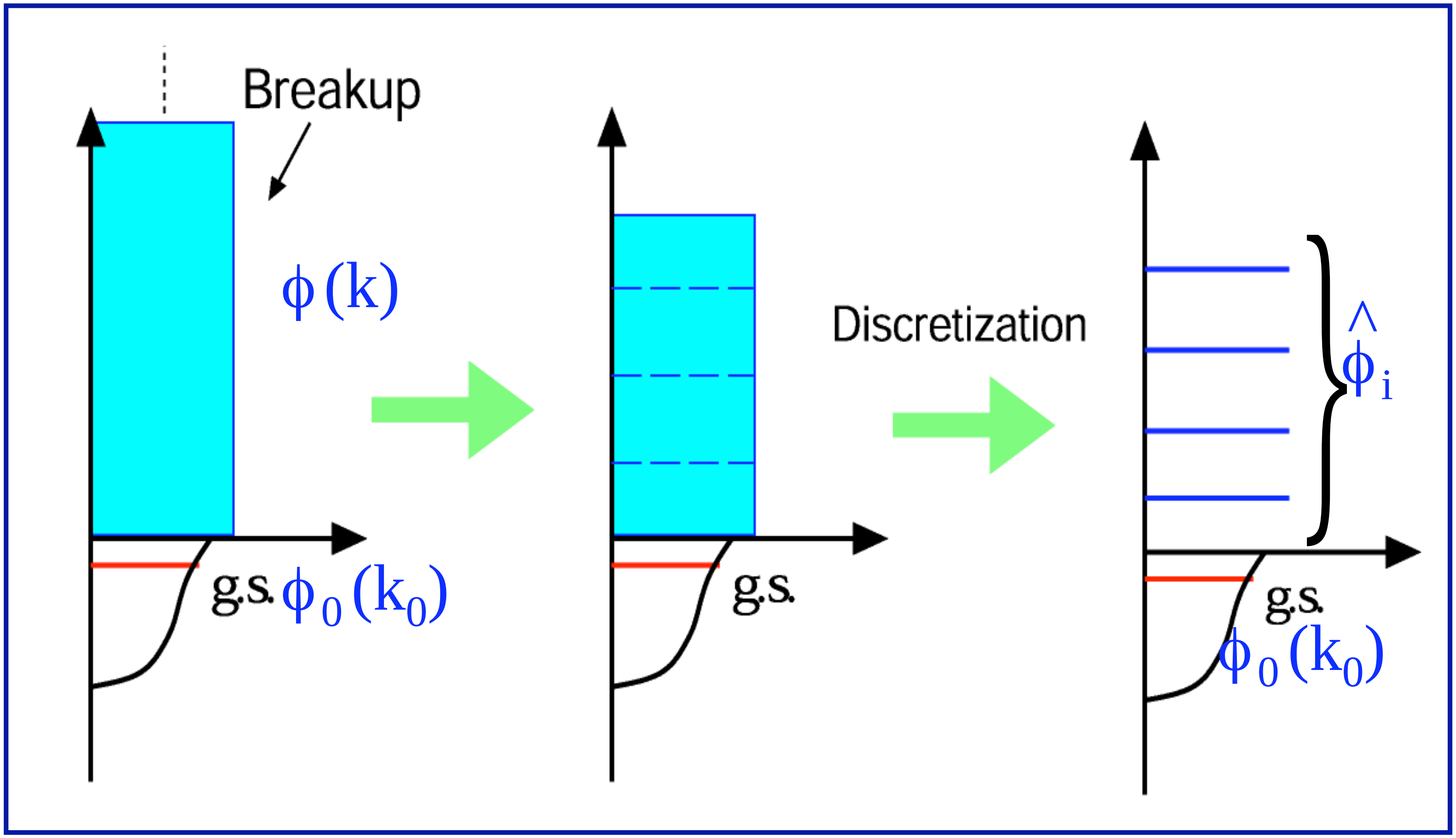
Theory movie in next 5 tranparencies (enjoy!)*



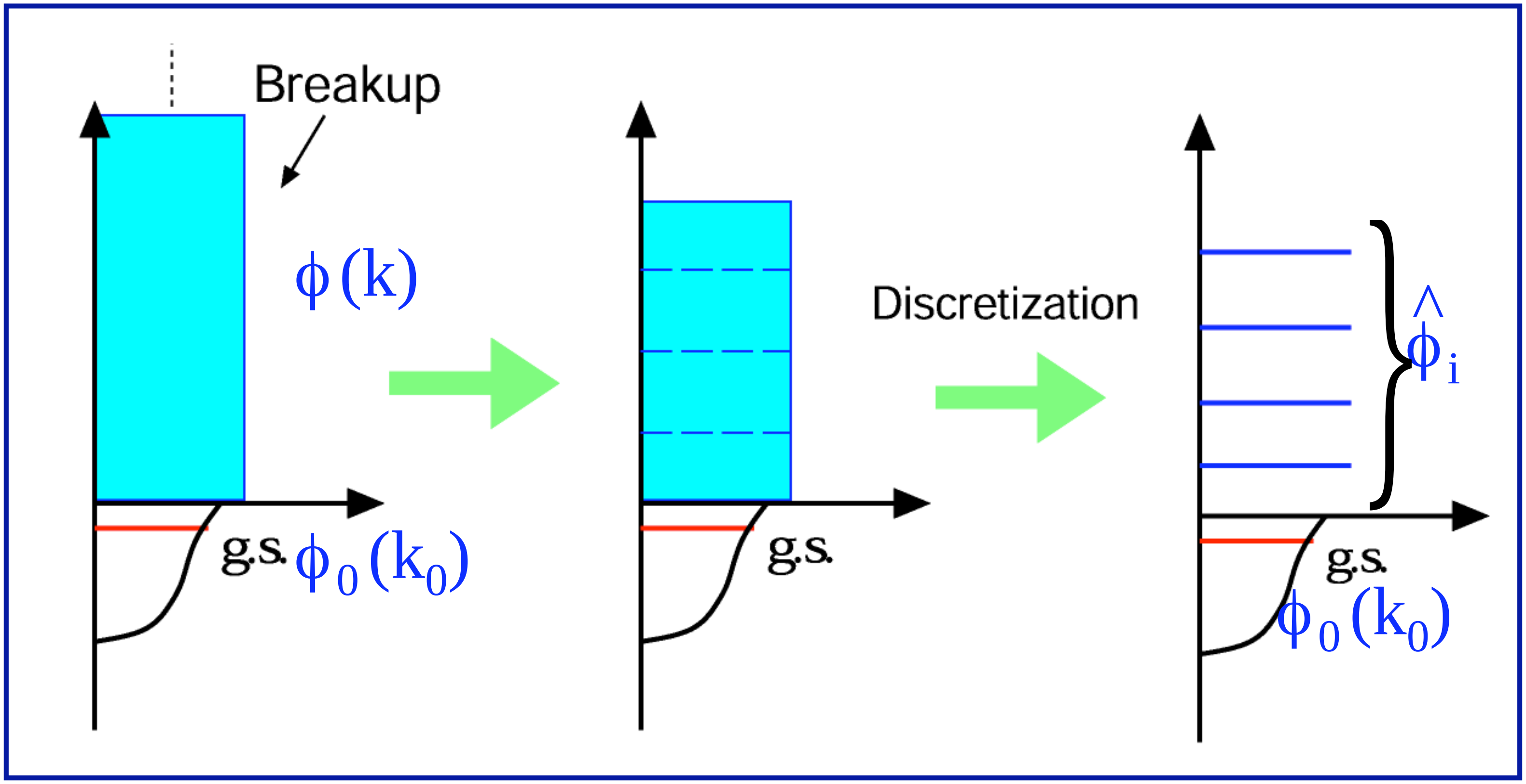


$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$

* Copyright: PIXAR

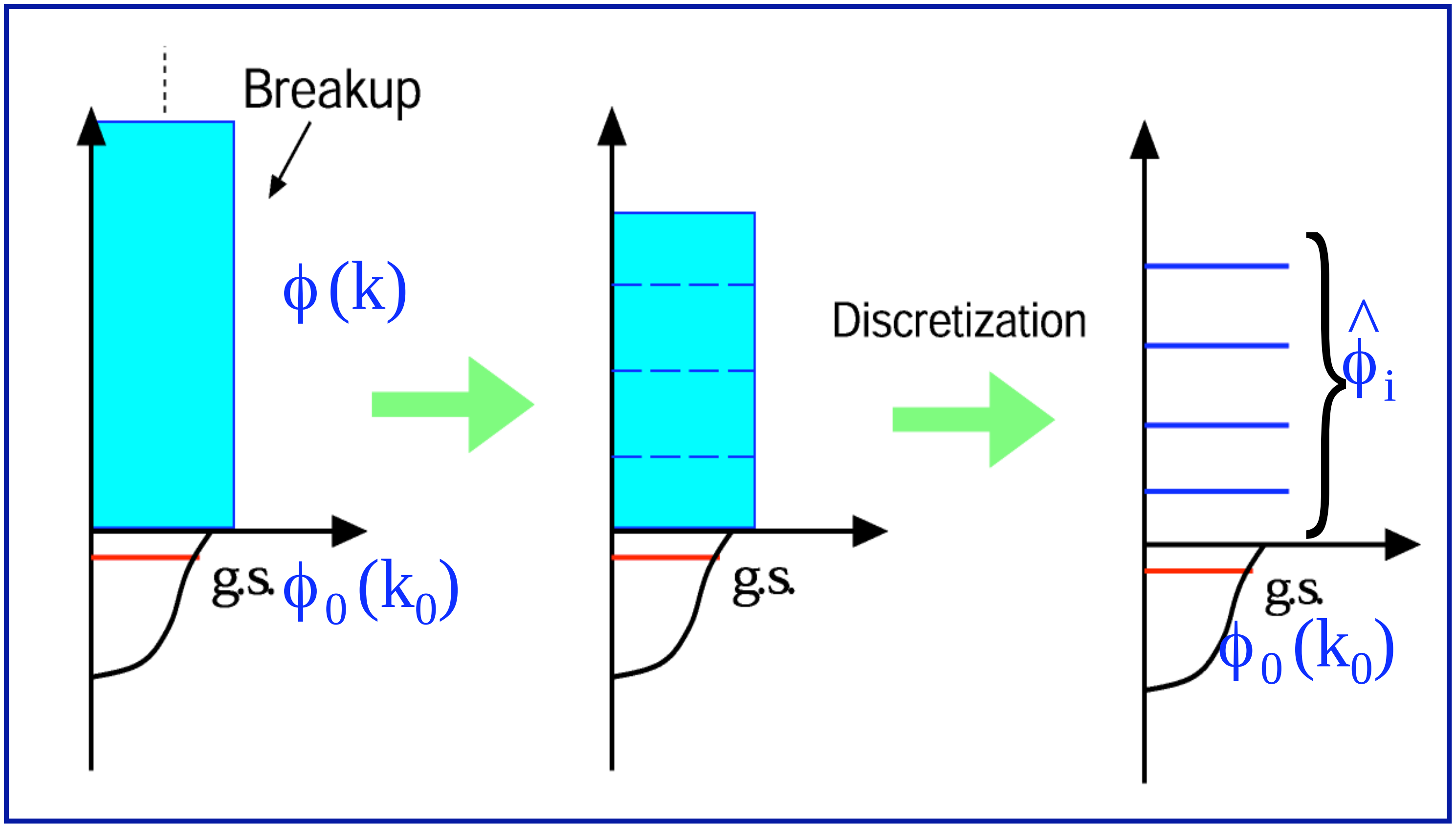


$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$



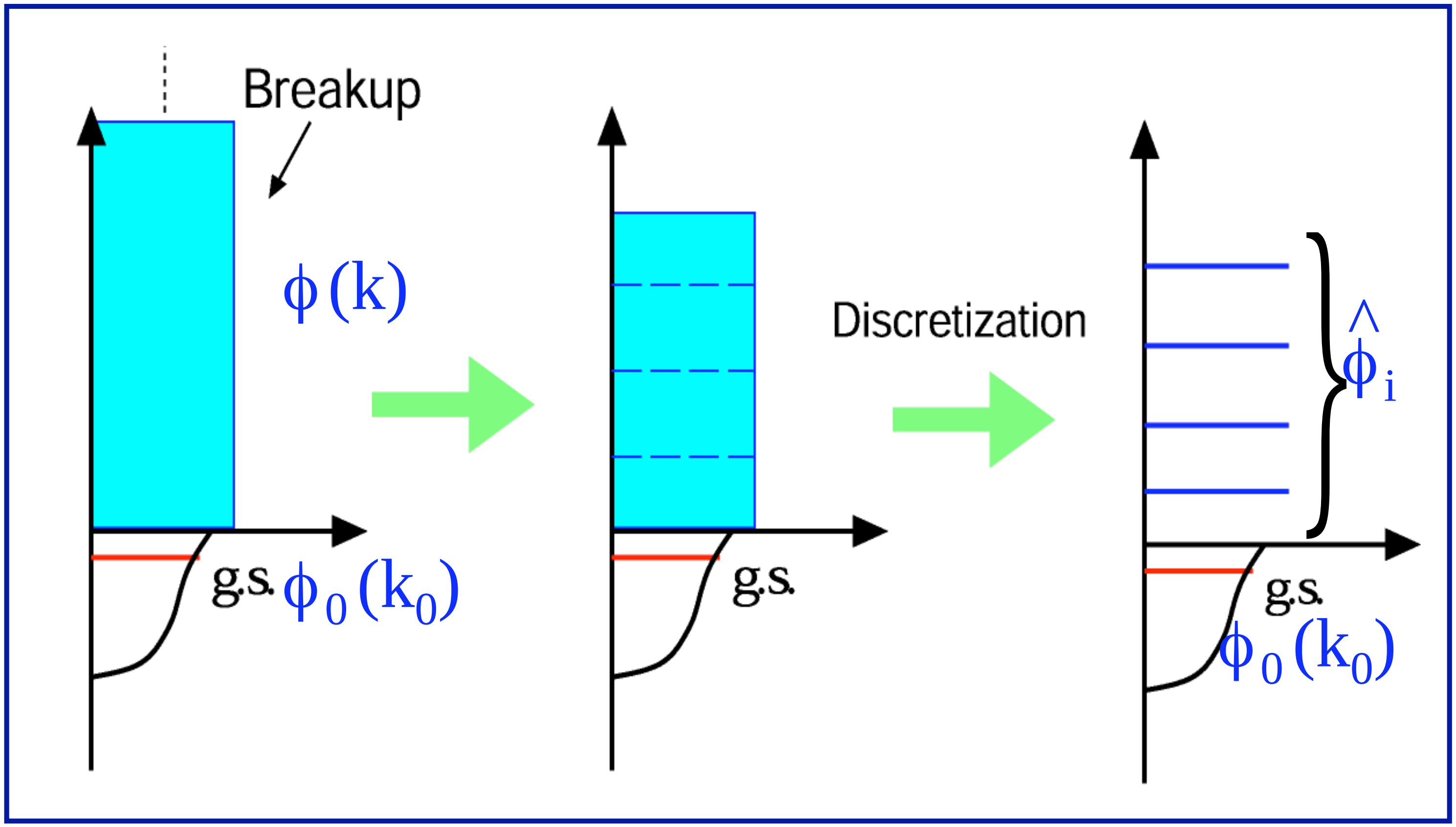
$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$

Truncation and Discretization



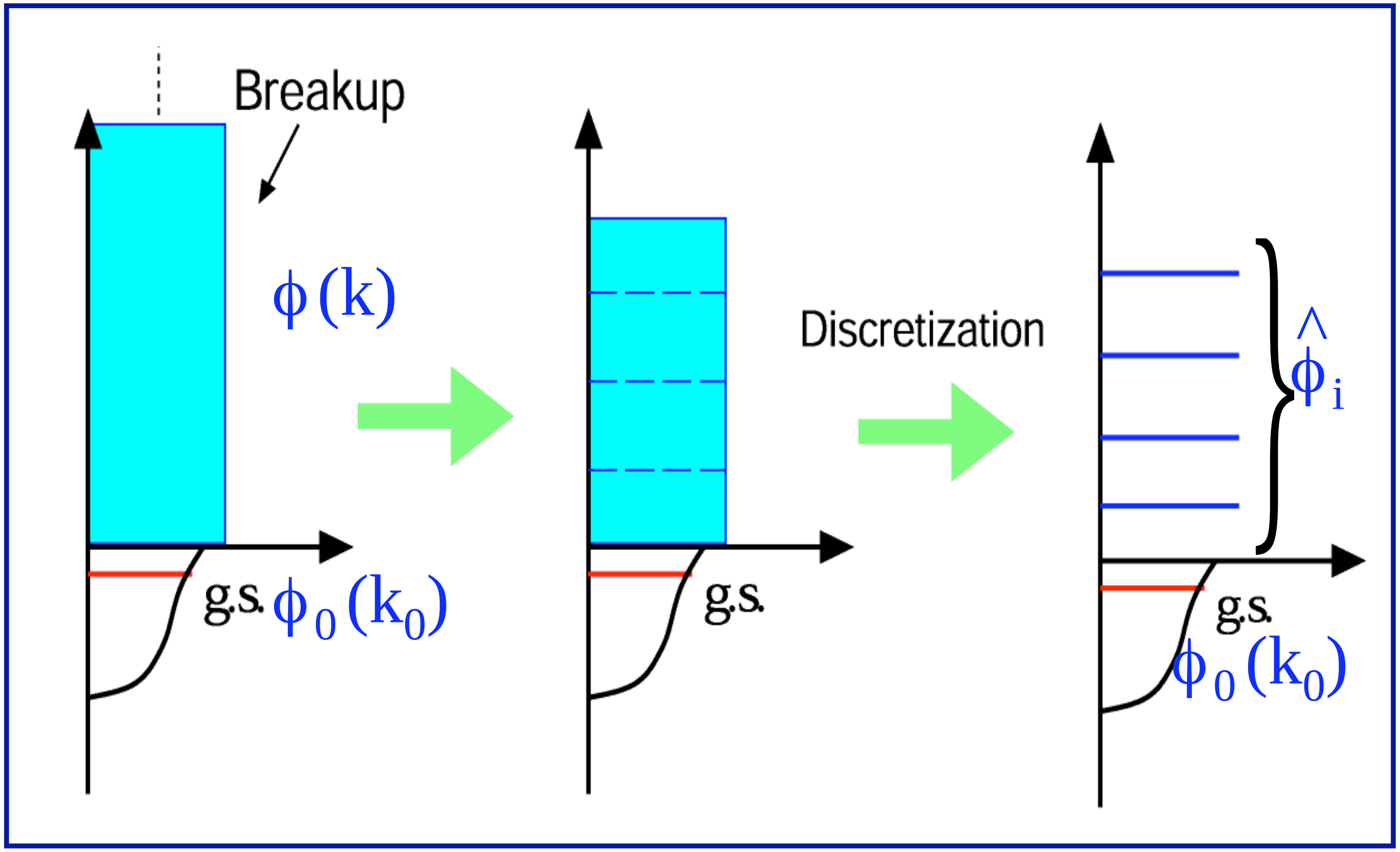
$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \sum_{i=1}^{i_{\max}} \int_{k_{i-1}}^{k_i} \phi(k,\vec{r})\chi(K,\vec{R})dk$$

Truncation and Discretization



$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \sum_{i=1}^{i_{\max}} \chi(K_i,\vec{R}) \int_{k_{i-1}}^{k_i} \phi(k,\vec{r})dk$$

Truncation and Discretization



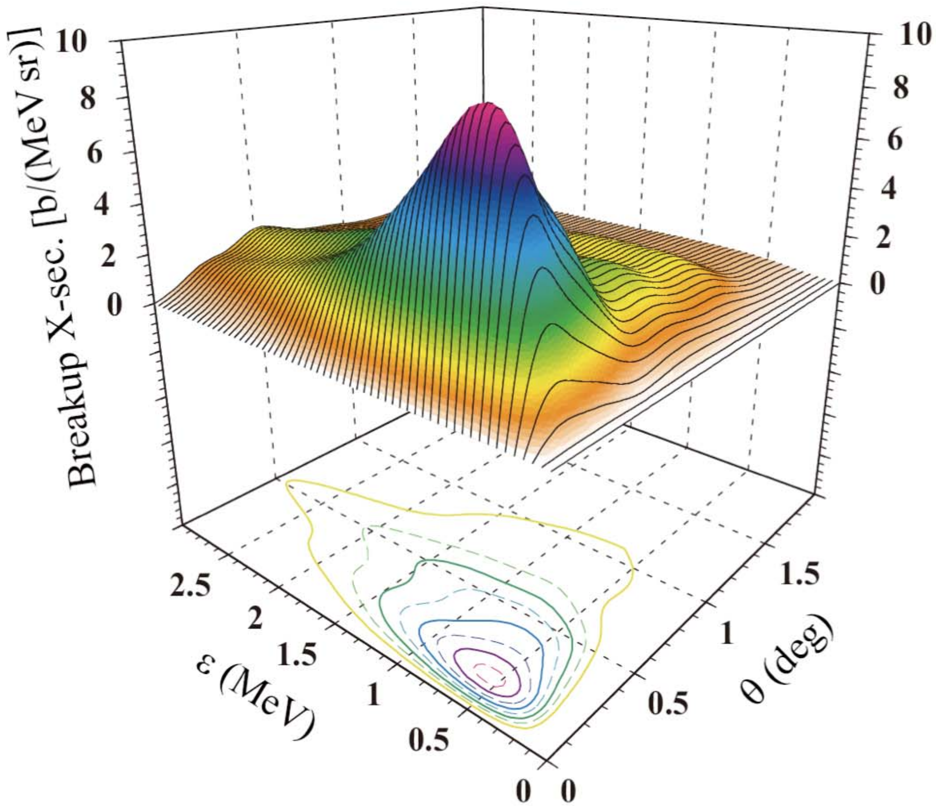
$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \sum_{i=1}^{i_{\max}} \chi(K_i,\vec{R})\int_{k_{i-1}}^{k_i} \phi(k,\vec{r})dk$$

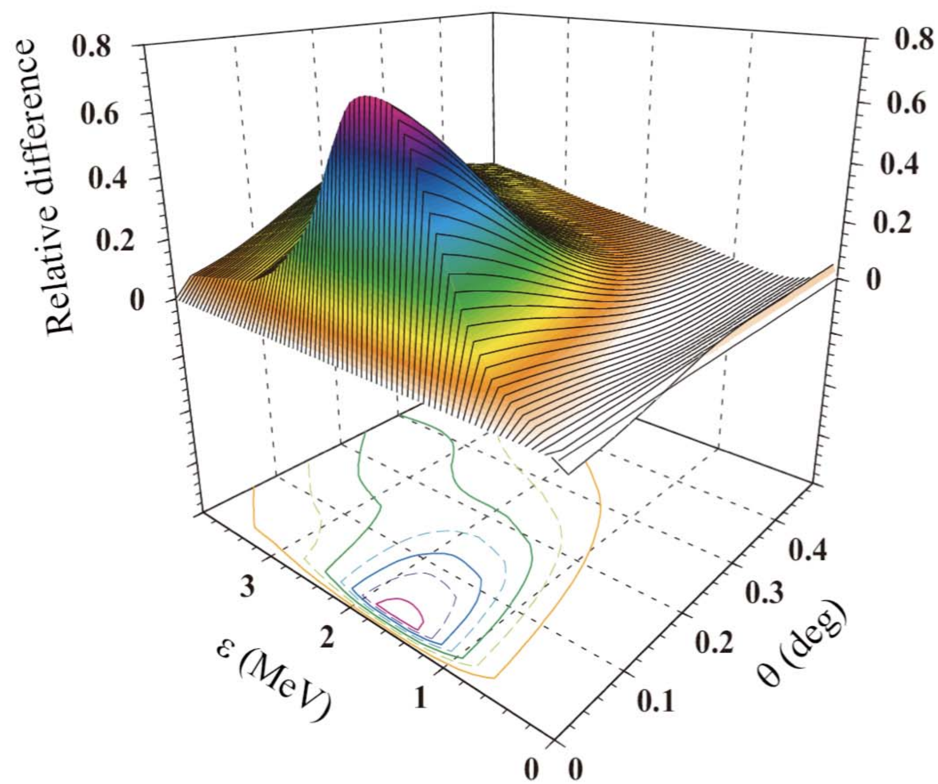
Truncation and Discretization

Ready to go.

$$\psi^{\mathrm{CDCC}}(\vec{r},\vec{R}) = \sum_{i=0}^{i_{\max}} \hat{\phi}_i(\vec{r})\hat{\chi}_i(K_i,\vec{R})$$

Ex: ^{8}B breakup by ^{208}Pb at 250 A MeV

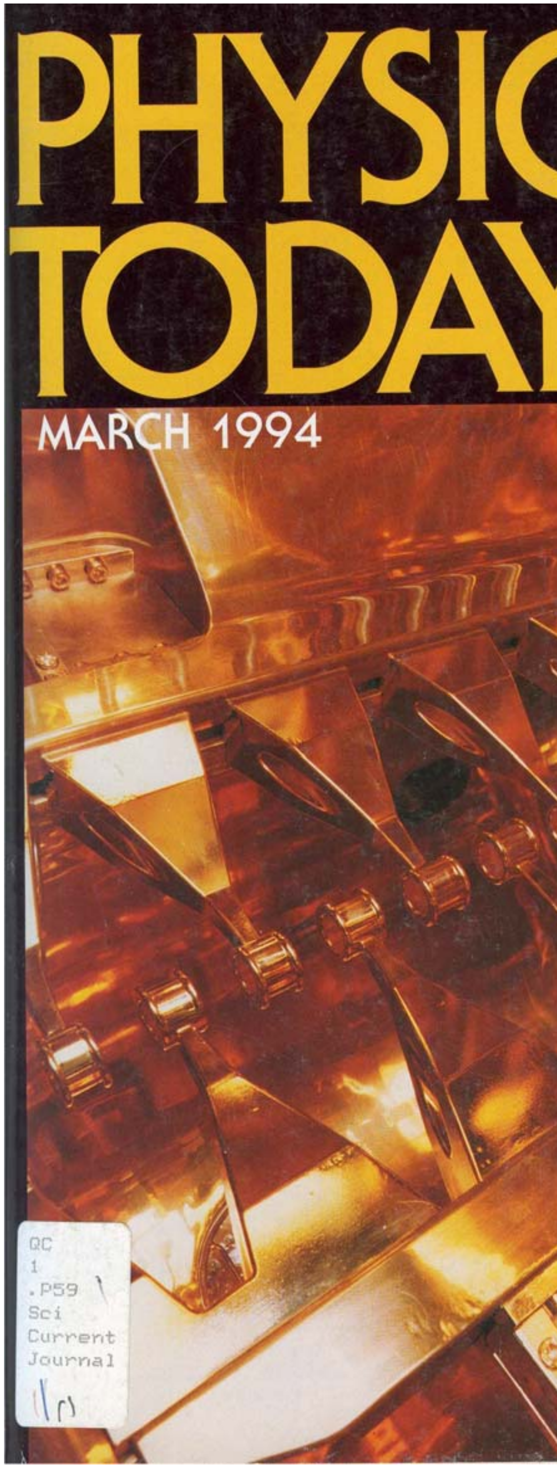




CDCC

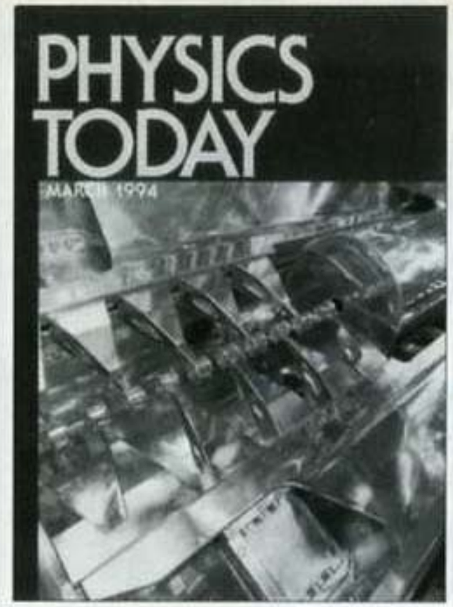
Correction to the CDCC due to retardation effects

(not discussed here)



15 years old summary

COVER: Inside of a compact high-frequency linear accelerator for heavy ions developed at the Technical University of Munich and at GSI in Darmstadt, Germany. The polished copper structure uses a quadrupole field to focus highly charged ions. Accelerators of this design at GSI and CERN bring ions up to high enough energies that the main accelerators can take them to relativistic energies. In their article on page 22, Carlos Bertulani and Gerhard Baur discuss the physics one can probe by colliding relativistic heavy ions without nuclear contact.



RELATIVISTIC HEAVY-ION PHYSICS WITHOUT NUCLEAR CONTACT

The large electromagnetic field generated by a fast heavy nucleus allows investigation of new electromagnetic processes not accessible with real photons.

Carlos Bertulani and Gerhard Baur

An increasing number of physicists are investigating nuclear collisions at relativistic energies. (See figure 1.) Accelerators completely devoted to the study of these collisions (such as the Relativistic Heavy Ion Collider at Brookhaven National Laboratory) are under construction. So are hadron colliders (such as the Large Hadron Col-

mately by $b/\gamma v$ and that the electric (or magnetic) field during this time interval is very intense: $E \simeq \gamma Ze/b^2$. The factor γ, which is $(1 - v^2/c^2)^{-1/2}$, is very large (on the order of 10^4–10^7) in relativistic heavy-ion colliders.

Theory