Electric Dipole Polarizability of $^{48}$Ca and Implications for the Neutron Skin

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The electric dipole strength distribution in $^{48}$Ca between 5 and 25 MeV has been determined at RCNP, Osaka, from proton inelastic scattering experiments at forward angles. Combined with photoabsorption data at higher excitation energy, this enables for the first time the extraction of the electric dipole polarizability $\alpha_D^{(48)}$Ca = 2.07(22) fm$^3$. Remarkably, the dipole response of $^{48}$Ca is found to be very similar to that of $^{40}$Ca, consistent with a small neutron skin in $^{48}$Ca. The experimental results are in good agreement with $ab$ initio calculations based on chiral effective field theory interactions and with state-of-the-art density-functional calculations, implying a neutron skin in $^{48}$Ca of 0.14 – 0.20 fm.

Introduction.— The equation of state (EOS) of neutron-rich matter governs the properties of neutron-rich nuclei, the structure of neutron stars, and the dynamics of core-collapse supernovae [1,2]. The largest uncertainty of the EOS at nuclear densities for neutron-rich conditions stems from the limited knowledge of the symmetry energy $J$, which describes the difference of neutron to symmetric nuclear matter at saturation density, and the slope of the symmetry energy $L$, which is related to the pressure of neutron matter.

The symmetry energy plays an important role also in nuclei, where it contributes to the formation of neutron skins in presence of a neutron excess. Calculations based on energy density functionals (EDFs) pointed out that $J$ and $L$ can be correlated to isovector collective excitations of the nucleus such as pygmy dipole resonances (PDRs) [3] and giant dipole resonances (GDRs) [4], thus suggesting that the neutron skin thickness, the difference of the neutron to proton root-mean-square radii, could be constrained by studying properties of collective isovector observables at low energy [5]. One of such observables is the nuclear electric dipole polarizability $\alpha_D$, which represents a viable tool to constrain the EOS of neutron matter and the physics of neutron stars [6–11].

While correlations among $\alpha_D$, the neutron skin and the symmetry energy parameters have been studied extensively with EDFs [12–16], only recently $ab$ initio calculations based on chiral effective field theory (χEFT) interactions successfully studied such correlations in medium-mass nuclei [17,18]. By using a set of chiral two- plus three-nucleon interactions [19,20] and exploiting correlations between $\alpha_D$ and the proton and neutron radii, Hagen et al. [17] predicted for the first time the electric dipole polarizability and a neutron skin thickness of 0.12 – 0.15 fm for $^{48}$Ca from an $ab$ initio calculation [17]. Since the electric dipole polarizability can be measured rather precisely, this offers novel insights to the properties of neutron-rich matter from a study of the dipole response of $^{48}$Ca. The properties of neutron-rich matter also connect this to the physics of the neutron-rich calcium isotopes, with recent pioneering measurements of the masses and $2^+$ excitation energies up to $^{54}$Ca [21,22] and of the charge radius up to $^{52}$Ca [23].

The neutron skin thickness can be obtained by comparison of matter radii deduced, e.g., from elastic proton scattering [24,25] or coherent photoproduction of neutrons [26] with well-known charge radii from elastic electron scattering. It can also be measured directly with antiproton annihilation [27,28]. A direct determination of the neutron radius is possible with parity-violating elastic electron scattering. Such an experiment (PREX) has been performed at JLAB for $^{208}$Pb but at present statistical uncertainties limit the precision [29]. An further measurement is approved and a similar experiment on $^{48}$Ca (CREX) is presently under discussion [30,31]. Here, we focus on the electric dipole polarizability,

$$\alpha_D = \frac{8\pi}{9} \int \frac{B(E1, E_X)}{E_X} dE_X = \frac{hc}{2\pi^2} \int \frac{\sigma_\gamma(E_X)}{E_X^2} dE_X,$$

(1)

where $B(E1)$ and $\sigma_\gamma$ denote the electric dipole (E1) strength distribution and the E1 photoabsorption cross
section, respectively, and \( E_X \) is the excitation energy. The evaluation of Eq. \( 1 \) requires a measurement of the complete E1 strength distribution which is dominated by the GDR [32].

A promising new method to measure the E1 strength distribution from low energies across the GDR is inelastic proton scattering under extreme forward angles including \( 0^\circ \) at energies of a few hundred MeV [33, 34]. In these kinematics the cross sections are dominated by relativistic Coulomb excitation, while the nuclear excitation of collective modes, except for the spinflip M1 resonance [35], is suppressed. Results for \( \alpha_D \) extracted for \(^{208}\text{Pb} \) [36] and \(^{120}\text{Sn} \) [37] have shown to provide important constraints [38] on the respective neutron skins of these nuclei and, together with data on the exotic nucleus \(^{68}\text{Ni} \) from experiments in inverse kinematics [39], on EDFs [14]. In this Letter, we report the first measurement for the electric dipole polarizability of \(^{48}\text{Ca} \) and compare this with results from \textit{ab initio} calculations based on \( \chi \)EFT interactions and with state-of-the-art EDF calculations.

\textbf{Experiments.}– The \(^{48}\text{Ca}(p,p') \) reaction has been measured at RCNP, Osaka, with an incident proton energy of 295 MeV. Data were taken with the Grand Raiden spectrometer [40] in the laboratory scattering angle range \( 0^\circ - 5.5^\circ \) for excitation energies 5 – 25 MeV. A \(^{48}\text{Ca} \) foil with an isotopic enrichment of 95.2 \% and an areal density of 1.87 mg/cm\(^2 \) was bombarded with proton beams of 4 to 10 nA. Dispersion matching techniques were applied to achieve an energy resolution of about 25 keV (full width at half maximum). The experimental techniques and the raw data analysis are described in Ref. [41] while details for the present experiment can be found in Ref. [41]. Cross sections due to relativistic Coulomb excitation can be separated from the spinflip M1 resonance dominating the nuclear response at small momentum transfers using spin transfer observables [36, 37] or a multipole decomposition analysis (MDA) of angular distributions [42, 43]. Comparison of the two independent methods shows good agreement. No polarization measurements were performed for \(^{48}\text{Ca} \) since the spinflip M1 strength in \(^{48}\text{Ca} \) is concentrated in a single transition [41].

Figure 1 shows representative spectra taken at laboratory scattering angles \( \Theta_{lab} = 0.4^\circ \) (blue), 1.7\(^\circ \) (red), and 3.2\(^\circ \) (green). At lower excitation energies, a few discrete transitions are observed, mostly of E1 character [41]. The prominent transition at 10.23 MeV carries the spinflip M1 strength [41]. The cross sections above 10 MeV show a broad resonance structure identified with excitation of the GDR. The steep decrease of cross sections with increasing scattering angle is consistent with relativistic Coulomb excitation.

\textbf{E1 strength and photoabsorption cross sections.}– Cross sections due to Coulomb excitation were extracted by a MDA following the methods described in Ref. [42]. In order to reduce the degrees of freedom in the \( \chi^2 \) minimization procedure, the cross sections from excitation of the isoscalar giant monopole and quadrupole resonance were determined from the experimental strength functions in \(^{48}\text{Ca} \) [45] with the method described in Ref. [43] and subtracted from the spectra. The contributions to the cross sections are small at the most forward angle (below 4\% in any given energy bin).

A complication of the analysis in \(^{48}\text{Ca} \) compared to that in heavier nuclei is the absence of detailed information on a phenomenological angular distribution of the nuclear background due to quasi-free scattering. This was determined, e.g., in \(^{208}\text{Pb} \) in the excitation region above the GDR [12]; however, the centroid of the GDR in \(^{48}\text{Ca} \) lies at much higher \( E_X \) while the momentum acceptance of the magnetic spectrometer is fixed at \( 0^\circ \). Motivated by the approximate constancy of the angular distributions at the high-energy end of the spectra at \( E_X \approx 25 \text{ MeV} \) (cf. Fig. 1), this contribution was modeled as a constant plus additional contributions from different multipoles.

The Coulomb excitation cross sections resulting from the MDA were converted into equivalent photoabsorption cross sections, respectively a \( B(E1) \) strength distribution, using the virtual photon method [46]. The virtual photon spectrum was calculated in an eikonal approach [47]. The resulting \( B(E1) \) strength distribution is displayed as full circles in Fig. 1. The error bars include systematic uncertainties of the absolute cross sections due to charge collection, dead time of the data acquisition, target thickness, as well as a variation of the minimum impact parameter in the calculation of the virtual photon spectrum. The model dependence of the MDA was considered by including the variance of \( \chi^2 \) values obtained for fits with all possible combinations of theoretical input curves. The latter contribution dominates the overall uncertainty.
There exist two other measurements of E1 strength in $^{48}\text{Ca}$ in the energy region of the GDR. A form factor decomposition of an $^{48}\text{Ca}(e,e'\pi)$ experiment at the S-DALINAC [48] resulted in the strength distribution shown as open triangles in Fig. 2. Considering that the error bars shown do not include an additional 10% uncertainty from the model dependence of the form factor decomposition [48] the two data sets are in good agreement. However, the proton emission contributes to the cross sections above threshold ($S_\alpha = 15.8$ MeV) although it is expected to be weak in a neutron-rich nucleus. Another result [49] (open squares) shows rather large deviations at the high-energy flank of the GDR. It was obtained from excitation functions of the activity of residual isotopes after particle emission. The photoabsorption cross sections were deduced in an unfolding procedure with the bremsstrahlung spectrum as input [50] leading to sizable systematic uncertainties not reflected in the quoted error bars. Furthermore, the contribution from the $(\gamma,2n)$ channel contributing at higher $E_\gamma$ was estimated from statistical model calculations assuming a large fraction of direct decay inconsistent with the results of Ref. [48]. Thus, these results are discarded in the further discussion.

From the present work, photoabsorption cross sections in the range $E_\gamma = 10 - 25$ MeV could be extracted and are displayed in Fig. 3(a) as solid dots. They are well described by a Lorentzian with a centroid energy $E_C = 18.9(2)$ MeV and a width $\Gamma = 3.9(4)$ MeV. The centroid energy is consistent with systematics of the mass dependence [51]

$$E_C = 31.2 A^{-1/3} + 20.6 A^{-1/6}. \tag{2}$$

The integrated strength in the measured energy range corresponds to an exhaustion of the E1 energy-weighted sum rule of 85%. It is instructive to compare to photoabsorption data for $^{40}\text{Ca}$ (open squares) [52] which again are well described by a Lorentzian. Figure 3(b) compares the two data sets after shifting the $^{40}\text{Ca}$ centroid by the amount predicted by Eq. (2). It is evident that the GDR in $^{40}\text{Ca}$ and $^{48}\text{Ca}$ exhibit nearly identical widths. The contributions to the electric dipole polarizability from the energy region 10 – 25 MeV are $\alpha_D^{(40}\text{Ca}) = 1.50(2)$ fm$^3$ and $\alpha_D^{(48}\text{Ca}) = 1.73(18)$ fm$^3$.

Although the GDR strength dominates, contributions to $\alpha_D^{(48}\text{Ca})$ at lower and higher excitation energies must be considered. Electric dipole strength below the neutron threshold ($S_n = 9.9$ MeV) was measured with the $(\gamma,\gamma')$ reaction [54]. Unlike in heavy nuclei, where the low-energy strength is a sizable correction [42, 43], the contribution $0.0101(6)$ fm$^3$ is negligibly small in $^{48}\text{Ca}$. For the energy region above 25 MeV, in analogy to the procedure described in Ref. [37] we adopt the $^{40}\text{Ca}$ photoabsorption data of Ref. [53], but shifted by the difference of centroid energies for mass-48 and 40 predicted by Eq. (2). Figure 3(a) summarizes the combined data used for the determination of $\alpha_D^{(48}\text{Ca})$.

The data in Ref. [53] extend up to the pion threshold energy. However, here we evaluate $\alpha_D$ integrating the strength up to 60 MeV since, as will be shown in the following paragraphs, the sum rule is already well converged at these energies. With these assumptions we
deduce $\alpha_D^{48\text{Ca}} = 2.07(22) \, \text{fm}^3$.

For the comparison with theory it is instructive to also extract a corresponding value for $^{40}\text{Ca}$, which one would expect to be smaller than the one for $^{48}\text{Ca}$. As shown in Ref. [55], integrating the data for $^{40}\text{Ca}$ from Ref. [53] one obtains $\alpha_D^{40\text{Ca}} = 1.95(26) \, \text{fm}^3$. Here, we combine the data of Ref. [53] with a refined set of data in the giant resonance region measured by the same group [52] and find $\alpha_D^{40\text{Ca}} = 1.87(3) \, \text{fm}^3$. We note that a much higher value was quoted in Ref. [53] which would actually exceed our result for $^{48}\text{Ca}$. The preference of the data set from Ref. [52] is motivated by a preliminary comparison with $^{40}\text{Ca}(p,p')$ results taken at Osaka. Although no $E1$ strength has been extracted yet, a comparison of spectra at the most forward angles [Fig. 3(c)], again shifted by the centroid energy difference, demonstrates good correspondence of the Coulomb excitation cross sections and an absolute ratio similar to the one observed in Fig. 3(b).

**Comparison with theory.**—First principles calculations of $\sigma_\gamma(E_x)$ require the solution of the many-body scattering problem at all energies $E_x$, including those in the continuum, which is extremely challenging beyond few-nucleon systems. While an ab initio calculation of the full continuum is still out of reach for medium-mass nuclei, methods based on integral transforms that avoid its explicit computation [50,58] have been successfully applied to light nuclei (see Ref. [59] for a review) and recently extended to medium-mass nuclei [55,60] using coupled-cluster theory. Furthermore, it has been shown that energy-dependent sum rules, such as the polarizability, can be evaluated without the explicit knowledge of the continuum states or a cross-section calculation itself [62] and recent developments [18] have also allowed to calculate $\alpha_D$ as a function of the upper integration limit of Eq. (1).

We performed ab initio calculations of $\alpha_D$ using the Lorentz integral transform coupled-cluster-method described in Refs. [18,55]. The theoretical results are compared to experiment in Fig. 4(b), where the smooth band (blue and red) shows the running sum of the experimental dipole polarizability with error bars. The laddered (gray) band is based on different chiral Hamiltonians, using the same two- and three-nucleon interactions employed in Ref. [17], which well reproduce saturation properties of nuclear matter [19,20]. For each interaction, the estimated model-space dependence and truncation uncertainty is about 4% of $\alpha_D$, which is also included in the gray band. We find that the agreement between the experimental and theoretical results in Fig. 4(b) is better for higher excitation energies. However, we also observed that the position of the GDR is more affected by truncations, which could lead to a shift of $\approx 2 \, \text{MeV}$. In addition, we estimated that the contributions from coupled-cluster triples corrections could be important at low energies. Both of these truncation errors are not included in the uncertainty shown in Fig. 4(b), because it is difficult to quantify them without explicit calculations. With these taken into account, the steep rise in the theoretical band around 20 MeV indicates the position of the GDR peak is consistent with the experimental centroid.

In Fig. 5 we present a detailed comparison of the experimental electric dipole polarizability in $^{48}\text{Ca}$ (blue band) and predictions from $\chi$EFT (green triangles) and EDFs (red squares, for details on the functionals see [17], error bars from Ref. [63]). The green and black bars indicate the $\chi$EFT prediction selected to reproduce the $^{48}\text{Ca}$ charge radius [17] and the range of $\alpha_D$ predictions [13] from EDFs providing a consistent description of polarizabilities in $^{68}\text{Ni}$, $^{120}\text{Sn}$, and $^{208}\text{Pb}$, respectively.

**FIG. 4.** (Color online) (a) Combined photoabsorption cross sections in $^{48}\text{Ca}$ from the present work (blue circles) for $E_x \leq 25 \, \text{MeV}$ and from Ref. [53] (red squares) for $25 \leq E_x \leq 60 \, \text{MeV}$. (b) Running sum of the electric dipole polarizability in comparison to $\chi$EFT predictions, where the gray band is based on a set of two- plus three-nucleon interactions [17] and includes a partial uncertainty estimate from the many-body method.

**FIG. 5.** (Color online) Experimental electric dipole polarizability in $^{48}\text{Ca}$ (blue band) and predictions from $\chi$EFT (green triangles) and EDFs (red squares, for details on the functionals see [17], error bars from Ref. [63]). The green and black bar indicate the $\chi$EFT prediction selected to reproduce the $^{48}\text{Ca}$ charge radius [17] and the range of $\alpha_D$ predictions [13] from EDFs providing a consistent description of polarizabilities in $^{68}\text{Ni}$, $^{120}\text{Sn}$, and $^{208}\text{Pb}$, respectively.
experimental αD value with predictions from χEFT and state-of-the-art EDFs. Taking only the interactions and functionals consistent with the experimental range implies a neutron skin in 48Ca of 0.14 – 0.20 fm, to be compared with an estimate from Coulomb energies (0.14 fm) [15] and quasielastic charge-exchange reactions (0.19 – 0.21 fm) [65]. These values are significantly larger than the range 0.12 – 0.15 fm predicted from ab initio calculations [17] by exploiting correlations and experimental measurements of the charge radius.

**Summary.** We presented the first determination of the electric dipole polarizability of 48Ca using relativistic Coulomb excitation in the (p, p') reaction at very forward angles. The resulting dipole response of 48Ca is found to be remarkably similar to that of 40Ca, consistent with a small neutron skin in 48Ca. The result is in good agreement with predictions from χEFT and EDF calculations pointing to a neutron skin of 0.14 – 0.20 fm.

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[64] W. Nazarewicz, private communication.