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Electrornagnetic Production of Lepton Pairs in Relativistic Heavy Ion Collisions

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Abstract We evaluate the probabilities and cross sections for the production of lepton pairs in relativistic heavy ion collisions in the first order time dependent perturbation theory and using the Sommerfeld-Maue wave functions for the lepton pair. We obtain useful analytical expressions for the production of low and highly energetic pairs. We compare our results with those existent in the literature. discuss new physical aspects, up to now unexplored. and make applications to some heavy ion reactions.

1. INTRODUCTION

Soon after the discovery of the positron in 1932, many theoretical works were performed which aimend to evaluate the cross sections for the production of electron-positron pairs in collisions of light (or a fast charged particle) with a nucleus. This was expected to be present in collisions originated by cosmic rays reaching the earth's surface and this process would be an experimental check of the validity of the positron theory of Dirac which had just been proposed. Most of the earlier theoretical works on that subject were done at about the same time. and in the special case of pair production in the collisions of relativistic charged particles, there were works by Furry and Carlson[1]. Landau and Lifshitz[2]. Bhabha[3]. Racah[4], and Nishina, Tomonaga, and Kobayasi[5]. Except in the work by Furry and Carlson where the final result was shown to be wrong by a missing logarithmic factor[2], all other works reproduced the same results as that of Landau and Lifshitz[2] which studied e^+e^- production in a collision of two fast nuclei in the Born approximation and treating the projectile motion semiclassically.

It was only recently, with the csnstruction of relativistic heavy ion accelerators. that a new interest in this field appeared (see e.g. refs. 6-12). The cross sections for pair-production in a collision between two charged particles are roughly proportional to $Z_1^2 Z_2^2$ and for heavy systems like $^{238}U + ^{238}U$ they will

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be very large, up to many kilobarns. This can be a cause of many difficulties in the study of experiments with relativistic heavy ions (RHI). For example. in RHI colliders they can lead to a beam loss due to the capture of slow electrons in an inner orbit of one of the ions (see e.g. refs. 6 and 9); this could even be useful in order to keep control of the beam luminosity. as was pointed out by Anholt and Gould[6].

Among the newest works on this subject[7-12], the most exact approach is the one followed by Becker. Grun and Scheid[8,9] in the semiclassical approximation. They expanded the interaction potential in multipoles and used Coulomb-Dirac wavefunctions for the eiectron and the positron. In this way. they obtaíned the impact parameter dependence. as well as the cross sections. for e^+e^- production for any energy of the pair and for RHI beams up to 100 GeV/nucleon. One of the difficulties of the calculation is the evaluation of the multipole sums for beam energies around 100 GeV/nucleon and greater, because it relies strongly on long numerical computing. Another very useful approach is the so-called equivalent photon method. which was used in refs. 7, 11 and 12. Besides being very simple to calculate, this method provides good quantitative derivation of the total cross sections, although it lacks a more complete description of the process.

Although it seems to be an old subject. we feel that more work is necessary in the description of pair production in RHI collisions and that some physical aspects of it are still unexplored. In this paper, we shall also use the semiclassical approach (which is appropriate for RHI collisions) to deduce the lepton pair (also muon and tau pairs) production probabilities and cross sections in RHI collisons. but instead of using the Coulomb-Dirac wave functions we shall use the Sommerfeld-Maue wavefunctions for the pair (see e.g. ref. 13 and references therein). These wavefunctions are equal to the Coulomb-Dirac ones for the spatial region around the nuclei which most contributes to the cross sections. In this way one can avoid the multipole expansion used by Becker, Grun. and Scheid (actually. this had already been suggested by those authors in that paper). Since this process is very similar to the production of pairs by a real photon, we can use many of the integrals that were evaluated by Bethe, Maximon, Davies and Nordsiek[13]. Our principal aim is to deduce expressions that are as simple as possibie, and to keep track of the physics of the process. They can be very useful for a fast estimate of the process in the regions where they are valid. We will show that analytical expressions can be obtained only in special cases of the pair energy. If we call these energies e_+ and ϵ_-, we show that we can deduce analytically the pair production probabilities and cross sections when (we use here natural units. with $\hbar = 1$, and $c = 1$)

elow paire

$$E_-, \epsilon_+ \simeq m \tag{1.1a}$$

fast paire

$$m << e_-, \epsilon_+ << \gamma m \tag{1.1b}$$

and

ultra-fast paire

$$\epsilon_-, \epsilon_+ \simeq \gamma m \tag{1.1c}$$

where

$$7 = 1/(1-v^2)^{1/2} \tag{1.2}$$

in the relativistic Lorentz factor associated with the heavy ion beam.

The results of Landau and Lifshitz are valid when the condition eq.(1.1b) is valid. Indeed, that is the energy region. which gives the greatest contribution to the total cross section, integrated over the energy of the pair. We show that for heavy ions there will be a correction to their results in a similar way as that found for pair production by a real photon in the field of a large Z nucleus[13]. Analogous study has also been done by Nikishov and Pichkurov[10] in the energy region eq.(1.1b). but slightly different final results were obtained. The energy region inferred by the condition eq.(1.1c) is easily studied by means of a Lorentz transformation of the results obtained in the energy region eq.(1.1a), and it is also important since it can originate a cloud of pairs surrounding the projectile in RHI colliders.

In section 2 we evaluate the differential probabilities and cross sections for lepton pair production. and we apply it in section 2.1 to the case of slow and ultra-fast lepton pairs, and in section 2.3 fast lepton pairs, which is the most important case for e^+e^- pairs. In section 2.4 we extend the calculations to include the case for which the target (or the projectile) is not completely naked but stil has a part (or all) of its atomic electrons.

Since their masses are much higher, the production of $\mu^+\mu^-$ and $\tau^+\tau^-$ pairs depends much more on the energy of the heavy ion beams, as we show in section 3. There we show that if the heavy ion beam energy is not very high ($\gamma >> 16$ for $\mu^+\mu^-$ production, and 7 >> 270 for $\tau^+\tau^-$ production), there is a big difference from the results for e^+e^- production. In section 4 we present our conclusions.

2. LEPTON PAIR PRODUCTION IN RHI COLLISIONS

2.1 - Probability amplitudes

In the following we shall calculate the electromagnetic production amplitude of lepton pairs in the field of a target nucleus with mass and charge number A_2 and Z_2, respectively. by means of a relativistic projectile with velocity v, impact parameter b. and mass and charge number A_1 and Z_1. The calculation is valid for impact parameters such that $b > R = R_1 + R_2$ and R_1 and R_2 are the respective nuclear radii. We shall consider the target nucleus as fixed, neglecting its recoil, and we place the origin of our coordinate system at its center of mass.

In the semiclassical approach the projectile is assumed to move in a straight-line and will generate a time-dependent electromagnetic field which will lead to the production of pairs in the field of the target. Since the probability amplitude for pair production is, generally. smaller than unity. we can calculate it in the first order time-dependent perturbation theory (as soon as we take into account the distortion of the wavefunctions of the pair due to the field of the target nucleus). It is given by

$$a_{\ell^+\ell^-} = \frac{1}{i}\int_{-\infty}^{\infty} dt\, e^{i\omega t} < \Psi_{\ell^-}|V(\mathbf{r},t)|\Psi_{\ell^+} > \tag{2.1.1}$$

where

$$w = G + c- \tag{2.1.2}$$

and Ψ_{ℓ^+} (Ψ_{ℓ^-}) is the wavefunction of the positive (negative) lepton. The interaction potential $V(\mathbf{r},t)$ is given by *

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* Here we use the notation A, $= (A_0, \mathbf{A})$, and the surn convention $A_\mu B_\mu = A_0B_0 - \mathbf{A}.\mathbf{B}$.

$$V(\mathbf{r},t) = \int d^3r A_\mu(\mathbf{r},t) j_\mu(\mathbf{r}) \qquad (2.1.3)$$

where $j_\mu \equiv (\rho,\mathbf{j})$ is the transition four current and $A_\mu \equiv (1,\mathbf{v})\phi$ where $\phi(\mathbf{r},t)$ is the Lienard-Wiechert potential (see e.g. ref. 14, p.654)

$$\phi(\mathbf{r},t) = \frac{Z_1 e\gamma}{\left[(x-b_x)^2 + (y-b_y)^2 + \gamma^2(z-vt)^2\right]^{1/2}} \qquad (2.1.4)$$

for a charged particle moving in a straight line with an impact parameter $b = \sqrt{b_x^2 + b_y^2}$. The z-axis is taken along the beam direction and we use for the coordinate of the pair the notation $\mathbf{r} = (x, y, z)$. We can also write eq.(2.1.4) in the integral form

$$\phi(\mathbf{r},t) = \frac{Z_1 e\gamma}{2\pi^2} \int d^3p \frac{e^{i\mathbf{p}\cdot\{\mathbf{S}-\mathbf{S}'(t)\}}}{p^2} \qquad (2.1.5)$$

where

$$\mathbf{S} = (x, y, \gamma z), \qquad \mathbf{S}' = (b_x, b_y, \gamma vt) \qquad (2.1.6)$$

Inserting this in eq.(2.1.1), the integral in t yields

$$\frac{2\pi}{\gamma v}\delta(p_z - \omega/\gamma v) \qquad (2.1.7)$$

and we obtain

$$a_{\ell^+\ell^-} = \frac{Z_1 e}{i\pi v} \int d^2p_T \frac{H(\mathbf{p}')}{p_T^2 + (\omega/\gamma v)^2} e^{i\mathbf{p}_T\cdot\mathbf{b}} \qquad (2.1.8)$$

where

$$\mathbf{p}' = (\mathbf{p}_T, \omega/v) \qquad (2.1.9)$$

and

$$H(\mathbf{p}') = \int d^3r < \Psi_{\ell^-} | v_\mu j_\mu(\mathbf{r}) e^{i\mathbf{p}'\cdot\mathbf{r}} | \Psi_{\ell^+} > \qquad (2.1.10)$$

with $v_\mu = (1,\mathbf{v})$. The index T means an arbitrary direction, perpendicular to the beam. Using the continuity equation for the transition current and eq.(2.1.9), we can express the above matrix element in terms of the longitudinal and transversal components of the transition current as

$$H(\mathbf{p}') = \int d^3r \Big\langle \Psi_{\ell^-} \Big| \Big[\frac{j_z}{v\gamma^2} + \frac{\mathbf{p}_T \cdot \mathbf{j}_T}{\omega} \Big] e^{i\mathbf{p}'\cdot\mathbf{r}} \Big| \Psi_{\ell^+} \Big\rangle \tag{2.1.11}$$

For 7 >> 1 we can neglect the first term inside brackets in eq. (2.1.11). as it is done quite generally in the equivalent photon approximation. Then **eq.(2.1.8)** reduces to

$$a_{\ell^+\ell^-} = \frac{Z_1 e^2}{i\pi\omega v} \int\int d^2p_T\, d^3r \frac{\mathbf{p}_T \cdot < \Psi_{\ell^-} | \vec{\alpha}_T e^{i\mathbf{p}'\cdot\mathbf{r}} | \Psi_{\ell^+} >}{p_T^2 + (\omega/\gamma v)^2} e^{i\mathbf{p}_T\cdot\mathbf{b}} \tag{2.1.12}$$

where we used $\mathbf{j}_T = e\vec{\alpha}_T$, and $\vec{\alpha}_T$ is a Dirac matrix of component perpendicular to the beam direction.

For $\Psi_{\ell\pm}$ we use the Sommerfeld-Maue wave functions which were also used by Davies, Bethe. and **Maximon**[13] (see also ref. 15. p. 143) to calculate pair production by means of a real photon (see ref. 13 for a complete discussion about these wavefunctions), namely

$$\Psi_{\ell^-} = N_- e^{i\mathbf{k}_-\cdot\mathbf{r}} \Big[1 - \frac{i}{2\epsilon_-} \vec{\alpha}\ \nabla \Big] u\ -\quad a\mathbf{1}, -ik_-r - i\mathbf{k}_- \cdot \mathbf{r}) \tag{2.1.13a}$$

and

$$\Psi_{\ell^+} = N_+ e^{-i\mathbf{k}_+\cdot\mathbf{r}} \Big[1 + \frac{i}{2\epsilon_+} \vec{\alpha} \cdot \nabla \Big] wF(-ia_+, 1, -ik_+r + i\mathbf{k}_+ \cdot \mathbf{r}) \tag{2.1.13b}$$

where u and w are the Dirac spinors corresponding to the **negative** and positive leptons with moments $\mathbf{k}_-$ and $\mathbf{k}_+$, respectively, F is the confluent hypergeometric fuction and

$$a_\pm = \frac{Z_2 e^2}{v_\pm}, \qquad N_\pm = exp\Big[\mp \frac{\pi a_\pm}{2} \Big] \Gamma(1 + ia_\pm) \tag{2.1.13c}$$

with $v_\pm$ equal to the respective velocities of the created pair.

Inserting eqs.(2.1.13) in (2.1.12) we find

$$a_{\ell^+\ell^-} = \frac{Z_1 e^2}{i\pi\omega v} N_+ N_- \sum_{\lambda=1,2} u^* \big[\alpha^\lambda G_{1\lambda} + \alpha^\lambda (\vec{\alpha} \cdot \mathbf{G}_{2\lambda}) + (\vec{\alpha} \cdot \mathbf{G}_{3\lambda}) \alpha^\lambda \big] w \tag{2.1.14}$$

where $\lambda = 1,2$ represent the two orthogonal components transverse to the beam. The tensors $G_{1\lambda}, \mathbf{G}_{2\lambda}$ and $\mathbf{G}_{3\lambda}$ are given by

$$[G_{1\lambda}, \mathbf{G}_{2\lambda}, \mathbf{G}_{3\lambda}] = \int d^2p_T \frac{p_T^\lambda[I_1, \mathbf{I}_2, \mathbf{I}_3]}{p_T^2 + (\omega/\gamma v)^2} e^{i\mathbf{p}_T\cdot\mathbf{b}} \tag{2.1.15}$$

where

$$I_1 = \int e^{i\mathbf{q}\cdot\mathbf{r}} F_1 F_2 d^3r \tag{2.1.16a}$$

$$\mathbf{I}_2 = \frac{i}{2\epsilon_+} \int e^{i\mathbf{q}\cdot\mathbf{r}} F_1 \nabla F_2 d^3r \tag{2.1.16b}$$

$$\mathbf{I}_3 = \frac{i}{2\epsilon_-} \int e^{i\mathbf{q}\cdot\mathbf{r}} F_2 \nabla F_1 d^3r \tag{2.1.16c}$$

with

$$\mathbf{q} = \mathbf{p}' - \mathbf{k}_+ - \mathbf{k}_- \tag{2.1.16d}$$

and

$$F_1 \equiv F(ia_-, 1, ik_-r + i\mathbf{k}_- \cdot \mathbf{r})$$

$$F_2 \equiv F(-ia_+, 1, ik_+r + i\mathbf{k}_+ \cdot \mathbf{r}) \tag{2.1.16e}$$

The integrals eq.(2.1.16) were calculated analytically by Nordsiek, Bethe. and Maximon[13].

The differential probability for the production of lepton pairs is obtained from eq.(2.1.14) as

$$dP_{\ell^+\ell^-} = \sum_{spins} |a_{\ell^+\ell^-}|^2 \rho_f \tag{2.1.17}$$

where

$$\rho_f = \frac{k_+k_-}{(2\pi)^6} \epsilon_+\epsilon_- d\epsilon_+ d\epsilon_- d\Omega_+ d\Omega_-$$

is the density of final states of the pair.

Using the properties of the Dirac matrices we find

$$dP_{\ell^+\ell^-} = \left(\frac{Z_1 e^2}{\pi\omega v}\right)^2 |N_+N_-|^2 \frac{\rho_f}{\epsilon_+\epsilon_-} \sum_\lambda \Big\{ \left[\epsilon_+\epsilon_- - k_{+z}k_{-z} + m^2\right] |G_{1\lambda}|^2$$

$$+ \left[\epsilon_+\epsilon_- + k_{+z}k_{-z} - m^2\right] \left[|\mathbf{G}_{2\lambda}|^2 + |\mathbf{G}_{3\lambda}|^2\right.$$

$$\left. - 2(G^*_{3\lambda})_z (G_{2\lambda})_z\right] - 2(\mathbf{k}_- \cdot \mathbf{G}^*_{2\lambda})(\mathbf{k}^T_+ \cdot \mathbf{G}^T_{3\lambda})$$

$$+ 2(\mathbf{k}^T_+ \cdot \mathbf{k}^T_-)\left[(\mathbf{G}^T_{3\lambda})^* \cdot \mathbf{G}^T_{2\lambda}\right] + 2(\mathbf{k}_- \cdot \mathbf{G}^*_{3\lambda})\left[\mathbf{k}_+ \cdot \mathbf{G}_{2\lambda} - k_{+z}(G_{3\lambda})_z\right]$$

$$+ 2G^*_{1\lambda}\left[\epsilon_-\left\{\mathbf{k}_+ \cdot \mathbf{G}_{2\lambda} - k_{+z}(G_{3\lambda})_z\right\} + \epsilon_+\left\{\mathbf{k}_- \cdot \mathbf{G}_{3\lambda} - k_{-z}(G_{2\lambda})_z\right\}\right]$$

$$+ 2k_{-z}(G^*_{2\lambda})_z\left[\mathbf{k}_+ \cdot (\mathbf{G}_{3\lambda} - \mathbf{G}_{2\lambda})\right] + complex\ conjugate \Big\} \qquad (2.1.18)$$

In the approxirnations we are going to rnake. the integrals $(G_{1\lambda}, \mathbf{G}_{2\lambda}, \mathbf{G}_{3\lambda})$ will be zero for one of the cornponents. say $\lambda = 2$ if we choose b along x-axis, and the surn λ reduces to only one terrn.

2.2 - Slow and ultra-fast electron-positron pairs

2.2.1 - Slow pairs

We now consider the production of low energetic lepton pairs okying the condition eq.(1.1a). We use the analytic expressions for the integrals eq.(2.1.16) as given by the equations (6.13) of the work of Maximon and Bethe[13] and keep only the terms of lowest order in k_+/m and k_-/m. Since only values of p_T up to $\omega/\gamma v << m$ will contribute to the integrals eq.(2.1.15). we also put $p_T = 0$ in the nurnerators of those expressions. Inserting the obtained results for $I_1, \mathbf{I}_2$ and $\mathbf{I}_3$ in eq. (2.1.15). we find

$$G_{12}, \mathbf{G}_{22}, \mathbf{G}_{32} = 0 \qquad (2.2.1a)$$

and

$$G_{11} = \frac{C}{\omega^3}\left[2(k_{-z} - k_{+z}) - i\omega Z_2 e^2 \frac{(k_+k_{+z} + k_-k_{-z})}{k_+k_-}\right] M(b,\omega,\gamma) \qquad (2.2.1b)$$

$$\mathbf{G}_{21} = -\frac{C}{\omega^2}\left[\hat{\mathbf{z}} + iZ_2e^2\frac{(k_+k_{+z}\hat{\mathbf{z}} - \mathbf{P}/2)}{k_+k_-}\right]M(b,\omega,\gamma) \tag{2.2.1c}$$

$$\mathbf{G}_{31} = \frac{C}{\omega^2}\left[\hat{\mathbf{z}} - iZ_2e^2\frac{(k_-k_{-z}\hat{\mathbf{z}} + \mathbf{P}/2}{k_+k_-}\right]M(b,\omega,\gamma) \tag{2.2.1d}$$

with

$$P = k_-\mathbf{k}_+ - k_+\mathbf{k}_- \tag{2.2.2}$$

and where $\hat{\mathbf{z}}$ is a unit vector in the RHI **beam** direction. The function $M(b,w,7)$ is given by

$$M(b,\omega,\gamma) = \omega\int d^2p_T\frac{p_T\cos\theta e^{ip_Tb\cos\theta}}{[p_T^2+(\omega/\gamma v)^2][p_T^2+\omega^2]}$$

$$= \frac{2\pi i}{1-1/\gamma^2v^2}\left[\frac{1}{\gamma v}K_1\left(\frac{\omega b}{\gamma v}\right) - K_1(\omega b)\right] \tag{2.2.3}$$

where K_1 is the modified Bessel function of first order. Inserting **eq.(2.2.1)** in (2.1.8) and keeping only the **lowest** order terms in k_+/m and k_-/m, we find

$$dP_{\ell^+\ell^-}(b) = \frac{8}{(2\pi)^8}Z_1^2Z_2^2e^4\frac{k_+k_-}{\omega^6v^2}|N_+N_-C|^2|M(b,\omega,\gamma)|^2$$

$$\times\left\{\left[k_+^2\sin^2\theta_+ + k_-^2\sin^2\theta_-\right]\left[1-(Z_2e^2)^2\right]\right.$$

$$\left.+\,2\epsilon_+\epsilon_-(Z_2e^2)^2\right\}d\Omega_+d\Omega_-d\epsilon_+d\epsilon_- \tag{2.2.4}$$

The impact parameter **dependence** of **eq.(2.2.4)** is embedded in the function $M(b,\omega,\gamma)$. which we plot in **fig.1** as a function of wb and $7 = 100$. We observe that M tends rapidly to its asymptotic value for $w \gtrsim 1$. This asymptotic value is obtained by neglecting the second term inside brackets in the numerator, and the second term in the denominator of **eq.(2.2.3)**, **i.e.** we can set

$$M \simeq \frac{2\pi i}{7''}K_1\left(\frac{wb}{7''}\right), \qquad for \qquad b \gtrsim \frac{1}{m} \tag{2.2.5}$$

where we used the approximation $w \simeq 2m$.

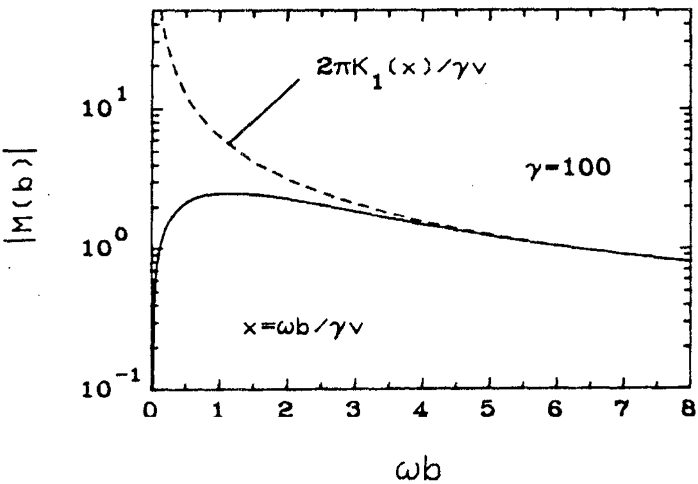

Fig.1 - Impact parameter dependence of the production probability of slow electron-positron pairs in RHI collisions expressed in terms of the adimensional function $M(b)$ as given by **eq.(2.2.3)**. The dashed line corresponds to its asymptotic limit. as given by **eq.(2.2.5)**.

Since the Compton wavelength of the muon (or tau) is much smaller than the nuclear dimensions. this approximation is very good for describing the impact parameter dependence of $\mu^+\mu^-$ and r + ~pair production. Nonetheless, in the case of e^+e^- it will only be appropriate for impact parameters larger than the Compton wavelength of the electron. which is much larger than the nuclear dimensions. As we will soon see, this will have as a consequence that the total cross section, integrated over all impact parameters will depend on the nuclear dimensions in the case of muon and tau pair production, but not in the case of electron pair production. This will lead to very different behaviour of the cross sections in the two cases. Let us therefore study first the case of e^+e^- production and postpone the study of $\mu^+\mu^-$ and $\tau^+\tau^-$ production to the section 3.

In the case of e^+e^- production one can have impact parameters much smaller than the Compton wavelength, for which we see in figure 1 that $M \rightarrow 0$, which seems to be an unrealistic behaviour. In fact, the probability to produce an electron-positron pair should go to a constant value as $b \to 0$, which was indeed shown in the calculations of Becker. Grun and Scheid[8]. We would obtain the same in our calculations if we had not neglected the first term inside bracket in eq.(2.1.11) which although not contributing appreciably to the cross section, has a finite. non-zero contribution for the differential probability as $b \to 0$. But for $w \gtrsim 1$ the impact parameter dependences is very well reproduced by using the approximation eq.(2.2.5). Moreover. the differential probability decreases very slowly until impact parameters much larger than the Compton wavelength of the electron and the uncertainty about the impact parameter dependence for $b \simeq 1/m$ is not very important for the total cross section. specially for RHI collisions.

The modified Bessel function of first order has the following property

$$\left(\frac{\omega b}{\gamma v}\right) K_1\left(\frac{\omega b}{\gamma v}\right) \simeq \begin{cases} 1, & for \quad \frac{\omega b}{\gamma v} < 1 \\ 0, & for \quad \frac{\omega b}{\gamma v} > 1 \end{cases} \tag{2.2.6}$$

This implies that the pair production probability decays like $1/b^2$ for impact parameter b larger than the Compton wavelength, i.e. for $b > 1/m$, up to a cutoff limit given by $b \simeq \gamma v/\omega$. Above this cutoff limit it will decay exponentially, which will guarantee the convergence of the cross section. Indeed, with these simplifications the differential cross section can be easily obtained by using

$$g(\varsigma) = 2\pi \int_{1/m}^{\infty} b\left(\frac{\omega}{\gamma v}\right)^2 K_1^2\left(\frac{\omega b}{\gamma v}\right) db = \pi \varsigma^2 \left[K_0^2 - K_1^2 + \frac{2}{\varsigma} K_0 K_1\right]$$

$$\simeq 2\pi\, ln\left(\frac{\delta}{\varsigma}\right) \quad \text{for} \quad \varsigma << 1 \tag{2.2.7a}$$

where the Bessel functions K_N are functions of the parameter

$$\varsigma = \frac{\omega}{\gamma m v} \tag{2.2.7b}$$

and $\delta = 0.681...$ is a number related to the Euler's constant. We can write the result as (putting $v = 1$)

$$d\sigma_{e^+e^-} = \frac{4}{\pi}(Z_1 Z_2 r_e \alpha)^2 \frac{k_+ k_-}{\omega^6} \frac{a_+ a_-}{(e^{2\pi a_+} - 1)(1 - e^{-2\pi a_-})} \ln\left(\frac{\gamma \delta m}{\omega}\right)$$

$$\times \left\{\left[k_+^2 \sin^2\theta_+ + k_-^2 \sin^2\theta_-\right]\left[1 - (Z_2\alpha)^2\right]\right.$$

$$\left. + 2\epsilon_+\epsilon_-(Z_2\alpha)^2\right\} d\Omega_+ d\Omega_- d\epsilon_+ d\epsilon_- \tag{2.2.8}$$

where $r_e = e^2/mc^2 = 2.817...fm$ is the classical electron radius. $\alpha = \frac{e^2}{\hbar c} \simeq 1/137$ is the fine structure constant. and we used

$$|N_+ N_- C|^2 = \frac{4(2\pi)^4 \alpha^2 a_+ a_-}{(e^{2\pi a_+} - 1)(1 - e^{-2\pi a_-})} \tag{2.2.9}$$

which can be inferred from the definitions eq.(2.1.13c). From eq.(2.2.8) one can calculate the invariant mass of the e^+e^- pairs for a given experimental setup. We observe that the angular distribution of the slow pairs is symmetric around 90^0 degrees due to the presence of the sine functions inside brackets of the eq.(2.2.8), i.e. slow pairs are created preferentially with respective velocities perpendicular to the beam direction.

The angular integrations can be carried out easily and we get

$$\frac{d^2\sigma_{e^+e^-}}{d\epsilon_+ d\epsilon_-} = \frac{128\pi}{3}(Z_1 Z_2 r_e \alpha)^2 \frac{a_+ a_-}{(e^{2\pi a_+} - 1)(1 - e^{-2\pi a_-})} \frac{\sqrt{(\epsilon_+ - m)(\epsilon_- - m)}}{\omega^4}$$

$$\times \left[(\omega - 2m) + (Z_2\alpha)^2\left(\frac{7}{2}m - w\right)\right] \ln\left(\frac{\gamma\delta m}{\omega}\right) \tag{2.2.10}$$

For heavy ions, and for pair energies such that $(\epsilon_\pm - m) << m$, we have in most cases

$$a_\pm = Z_2\alpha\sqrt{\frac{m}{2(\epsilon_\pm - m)}} >> 1 \tag{2.2.11}$$

Then eq.(2.2.10) simplifies to

$$\frac{d^2\sigma_{e^+e^-}}{d\epsilon_+ d\epsilon_-} = 32\pi Z_1^2 Z_2^6 \alpha^6 r_e^2 \frac{m^2}{\omega^4} \ln\left(\frac{\gamma\delta m}{\omega}\right) e^{-2\pi a_+} \tag{2.2.12}$$

In figure 2 we plot the adimensional function $(m/r_e)^2 d^2\sigma/d\epsilon_+ d\epsilon_-$ obtained from eq.(2.2.10) as a function of $(\epsilon_- - m)/m$ for $(\epsilon_+ - m)/m = 0.01$. and as a function of $(\epsilon_+ - m)/m$ for $(\epsilon_- - m)/m = 0.01$. The dashed lines correspond to the approximation eq.(2.2.12). We observe that while it increases rapidly as a function of ϵ_+, it is approximately constant as a function of ϵ_-. This is a consequence of the different behaviour of the electron and the positron wavefunctions in the Coulomb field of the target. The positrons are very unlikely to be produced with small kinetic energies due to the Coulomb repulsion in the field of the target nucleus. For targets with small charge this effect diminishes because a_+ gets smaller and the energy distribution for positrons and electrons tends to be a symmetric function of ϵ_+ and ϵ_- (see eq.2.2.15).

For very low energies. the electrons can be even caught in an orbit around the nucleus and we expect that $d\sigma/d\epsilon_-$ must go to a constant for $\epsilon \to m$ which could be extrapolated to $\epsilon < m$ in order to find and approximate value of the cross section for pair production with capture of the electron in an orbit around the target. An approximate value for this constant can be found by integrating eq.(2.1.12) from $\epsilon_+ = m$ to $2m$, up to which we expect it may be a reasonable approximation. We obtain

$$\frac{d\sigma_{\epsilon^+\epsilon^-}}{d\epsilon_-} \simeq (2\pi)^3 Z_1^2 Z_2^8 \alpha^8 r_e^2 \frac{1}{m} a_m^{-3/2} W_{-3/2,-1}(a_m)\, ln\left(\frac{\gamma\delta}{2}\right) e^{-a_m/2}$$

$$\simeq 2\sqrt{2} Z_1^2 Z_2^5 \alpha^5 r_e^2 \frac{1}{m}\, ln\left(\frac{\gamma\delta}{2}\right) e^{-\sqrt{2}\pi Z_2 \alpha} \qquad (2.2.13)$$

where $W_{\lambda,\mu}$ is the Whittaker function (see ref. 16, p. 1059). and $a_m = \sqrt{2}\pi Z_2\alpha$. If we now assume that this constant behaviour of $d\sigma/d\epsilon_-$ will continue for $\epsilon_- < m$. we can make a rough estimate of the cross section for pair production in which the electron is captured in an orbit around the target by

$$\sigma_{e^+e^-}^{\text{capt}} \simeq 2\sqrt{2} Z_1^2 Z_2^5 \alpha^5 r_e^2 \frac{I}{m}\, ln\left(\frac{\gamma\delta}{m}\right) e^{-\sqrt{2}\pi Z_2 \alpha} \qquad (2.2.14)$$

where I is a quantity of order of the ionization energy of the K-shell electron. By using this rough approximation. and putting $I \simeq m$ to simplify. we find that the cross section for electron-positron pair production with capture of the electron by the target ion in a uranium-uranium collision with projectile energy equal

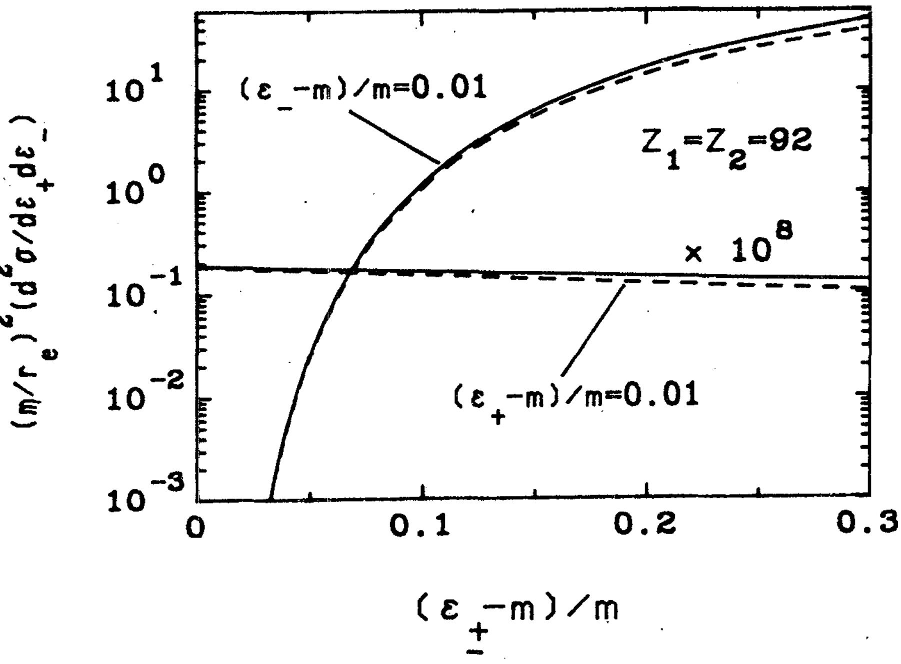

Fig.2 - The double differential cross section $d^2\sigma/d\epsilon_+d\epsilon_-$ in units of r_e^2/m^2, for $Z_1 = Z_2 = 92$, and as a function of $(\epsilon_+ - m)/m$ for $(\epsilon_+ - m)/m = 0.01$ (increasing curve). Also shown is the dependence of this function with respect to $(\epsilon_- - m)/m$ for $(\epsilon_+ - m)/m = 0.01$ (flat curve). This curve is multiplied by 10^8 in order to be shown in the same figure. The dashed lines correspond to the approximation eq.(2.2.12).

to 100 Gev/nucleon is about 47 barns. Becker, Grun and Scheid found the value of 68 barns for the same reaction. That this approximate agreement is not accidental can be easily verified by comparing the values inferred from figure 4 of that reference with the estimates based on eq.(2.2.14). Nonetheless, besides the approximations made on the integrations from eqs.(2.2.10) to (2.2.14), when the condition eq.(2.2.11) is attained there must be corrections to the Sommerfeld-Maue wave functions (2.1.13), and the use of exact Coulomb-Dirac wavefunctions for the final state of a free positron and a bound electron will be important. The cross sections for annihilation of a positron with a bound $(K, L, ...)$ electron are **well** known (see, **e.g.** ref. 15, p. 463). Since this proces. by detailed balance, is related to the inverse one. **i.e.** $7 + Z \rightarrow e^+ + (Z + e^-)_{K,L,...}$, one can **perform** a more exact derivation of the above mentioned process. by using the equivalent photon **method**[11] Work in **this direction is** in progress

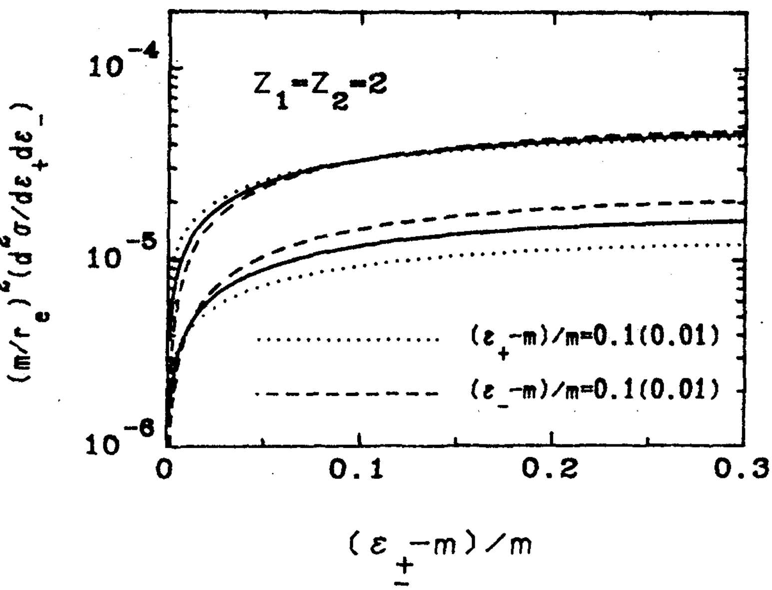

Fig.3 - The double differential cross section $d^2\sigma/d\epsilon_+ d\epsilon_-$ **in** units of r_e^2/m^2 and as a function of $(\epsilon_+ - m)/m$ for $(\epsilon_- - m)/m = 0{:}1)$ (upper **dashed** curve). and 0.01 (**lower** dashed curve) for $Z_1 = Z_2 = 2$. Also shown **is** the **dependence** of this function with respect tp $(\epsilon_- - m)/m$ for $(\epsilon_+ - m)/m = 0.01$ (upper dotted curve). and 0.01 (lower dotted curve). The **solid** curves correspond to the approximation **eq.(2.2.15)**.

In the **collisions** of **lightly** charged (like **e.g.** a! – a! collísions) and for paír energies such that $a_\pm << 1$, eq. (2.2.10) becomes

$$\frac{d^2\sigma_{\epsilon^+\epsilon^-}}{d\epsilon_+\epsilon_-} = \frac{32}{3\pi}(Z_1Z_2r_e\alpha)^2\sqrt{(\epsilon_+ - m)(\epsilon_- - m)}\frac{(\omega-2m)}{\omega^4}\, ln\left(\frac{\gamma\delta m}{\omega}\right) \quad (2.2.15)$$

which is symmetric in ϵ_+ and ϵ_-. In fig.3 we plot the same functions as in fig.2, but for $Z_1 = Z_2 = 2$. The solid curves correspond to the approximation eq.(2.2.15) for $(\epsilon_\pm - m)/m = 0.1$ (upper curve). and 0.01 (lower curve). The other curves are obtained from eq.(2.1.10) for $\epsilon_+ = constant$ (dotted curves). and for c− $= constant$ (dashed curves), and show the deviations from the approximations eq.(2.2.15).

As a last remark. we observe that when the relative velocity $v.$ of the created pair is very small, i.e. when

$$v, \gtrsim a = 1/137 \quad (2.2.16)$$

then one must take into account the Coulomb interaction between these particles. This was considered by Sacharov[17] in connection with the formation of a bound state of the electron-positron system (positronium). Since the main effect of considering the distortion of the Coulomb field is the presence of the terms containing $a_\pm$ in eq.(2.2.8), we can also make a correction to include the case eq.(2.2.16) by multiplying eq.(2.2.8) by the factor

$$\frac{2\pi\alpha/v_r}{1-e^{-2\pi\alpha/v_r}} \quad (2.2.17)$$

This correction will have as a consequence that the momentum of the electron and of the positron will be strongly correlated and that the cross section eq.(2.2.8) will have a sharp maximum when they are approximately equal in magnitude and in direction, i.e. for $\mathbf{k}_+ \simeq \mathbf{k}_-$ (see also ref. 18. p. 387).

2.2.2 - Ultra-fast pairs

The calculations of the last section can also be used to determine the probabilities and cross sections to produce slow pairs in the frame of reference of the projectile as soon as we make the exchange $Z_1 \leftrightarrow Z_2$ and evaluate the pair momenta and energies in that frame. However. in the laboratory frame of reference (target frame) these pairs will be very fast, with energies in the region given by eq.(1.1c).

Since the pairs are seen in the **projectile** frame moving **approximately** perpendicular to the **beam** direction, they **will** be observed in the **laboratory** frame moving very **forwardly** up to a maximum spreading **angle** of about $m/\epsilon_\pm \simeq 1/\gamma << 1$ (here we use the notation $\epsilon'_\pm$, etc. in the **projectile** frame. and $\epsilon_\pm$, etc. in the **laboratory** frame).

We can deduce the cross section for the production of **ultra-fast** pairs by **making** a Lorentz transformation of the expression (2.2.8) to the **laboratory** system. We use that $k_+k_-d\Omega_+d\Omega_-d\epsilon_+d\epsilon_-$ is a Lorentz invariant quantity. and that for $\gamma >> 1$ and $\theta << 1$ we have $\epsilon'_\pm \simeq (\epsilon_\pm/2\gamma)(1+\gamma^2\theta^2)$. We **also** use $k_\pm \simeq \epsilon_\pm$, and since the average **value** of $\gamma^2\theta^2$ **is of** order of unity. we set $\epsilon'_\pm \simeq \epsilon_\pm/\gamma$ where **possible**. Then the **angular** integration can be performed **easily** and we obtain

$$\frac{d^2\sigma_{\epsilon^+\epsilon^-}}{d\epsilon_+\epsilon_-} = \frac{64\pi}{5}(Z_1Z_2r_e\alpha)^2 \frac{a_+^T a_-^T}{(e^{2\pi a_+^T}-1)(1-e^{-2\pi a_-^T})}\frac{\epsilon_+\epsilon_-}{\omega^6}$$
$$\times \left\{(\epsilon_+^2+\epsilon_-^2)[1-(Z_1\alpha)^2]+2\epsilon_+\epsilon_-(Z_1\alpha)^2\right\} ln\left(\frac{\gamma^2\delta m}{\omega}\right) \tag{2.2.18}$$

where

$$a_\pm^T = \frac{Z_1\alpha}{v_\pm^T} \tag{2.2.19}$$

with $v_\pm^T$ equai to the transverse **velocity** of the pair. When $a_\pm^T << 1$ eq. (2.2.18) **simplifies** to

$$\frac{d^2\sigma_{\epsilon^+\epsilon^-}}{d\epsilon_+\epsilon_-} = \frac{16}{5\pi}(Z_1Z_2r_e\alpha)^2\frac{\epsilon_+\epsilon_-}{\omega^6}(\epsilon_+^2+\epsilon_-^2)\, ln\left(\frac{\gamma^2\delta m}{\omega}\right) \tag{2.2.20}$$

Although this **formula** is **only valid** for pair energies $\epsilon_\pm \simeq \gamma m$, it shows a **close resemblance** to the **results** for fast-pairs which we calculate **in** the next section. **Nonetheless**, since the energy region where the above equation can be **applied** is very restricted, most **of** the pairs **will** be created with energies obeying the condition **eq.(1.1b)**, as we **shall** see in the next section.

Fast **electron-positron** pairs

We now consider the production of energetíc lepton pairs obeying the condition eq.(1.1b). We use again the expressions (6.13) given in the work of Maximon and Bethe[13] and disregard terms of order $(m/\epsilon_\pm)^2$ and higher. We also put $p_T = 0$ in the numerators of that expressions. We find

$$I_1 = 2C\frac{\epsilon_+\epsilon_-}{\omega}\frac{1}{\left[m^2+(\mathbf{p}_T-\mathbf{k}_+^T)^2\right]\left[m^2+(\mathbf{p}_T-\mathbf{k}_-^T)^2\right]}$$
$$\times\left\{V_\pm(x)\frac{(\mathbf{k}_+^T)^2-(\mathbf{k}_-^T)^2}{\left[\mathbf{p}_T-(\mathbf{k}_+^T+\mathbf{k}_-^T)\right]^2}+i\frac{(Z_2\alpha)}{m^2}W_\pm(x)\left[m^2+(\mathbf{k}_+^T)^2+(\mathbf{k}_+^T)^2\right]\right\} \tag{2.3.1a}$$

$$\mathbf{I}_2 = C\frac{\epsilon_-}{\omega}\frac{1}{\left[m^2+(\mathbf{p}_T-\mathbf{k}_-^T)^2\right]}$$
$$\times\left\{V_\pm(x)\frac{(\mathbf{k}_+^T+\mathbf{k}_-^T)}{\left[\mathbf{p}_T-(\mathbf{k}_+^T+\mathbf{k}_-^T)\right]^2}+i\frac{(Z_2\alpha)}{m^4}W_\pm(x)\left[\mathbf{k}_+(\mathbf{k}_-^T)^2+m^2\mathbf{k}_+^T\right]\right\} \tag{2.3.1b}$$

$$\mathbf{I}_3 = C\frac{\epsilon_+}{\omega}\frac{1}{\left[m^2+(\mathbf{p}_T-\mathbf{k}_-^T)^2\right]}$$
$$\times\left\{-V_\pm(x)\frac{(\mathbf{k}_+^T+\mathbf{k}_-^T)}{\left[\mathbf{p}_T-(\mathbf{k}_+^T+\mathbf{k}_-^T)\right]^2}+i\frac{(Z_2\alpha)}{m^4}W_\pm(x)\left[\mathbf{k}_+(\mathbf{k}_-^T)^2+m^2\mathbf{k}_+^T\right]\right\} \tag{2.3.1c}$$

where

$$V_\pm(x) = F(-ia_+, ia_-; 1; x) \tag{2.3.2a}$$

$$W_\pm(x) = F(1-ia_+, 1+ia_-; 2; x) \tag{2.3.2b}$$

and

$$x = 1-\frac{(\mathbf{k}_+^T+\mathbf{k}_-^T)^2[\omega^2-(k_+-k_-)^2]}{4\omega^2(\epsilon_+-k_{+z})(\epsilon_--k_{-z})} \tag{2.3.2c}$$

Substituting these equations in the integrals (2.1.15) we will find that they are much more complicated than the ones in the section 2.2 due to the fact that the denominators contain the quantities $\mathbf{k}_+^T$ and $\mathbf{k}_-^T$ which are not negligible in comparison with m. Indeed. for fast pairs the angular distribution is very forward peaked and their transverse momenta will be of order

$$k_+^T, k_-^T \simeq m \tag{2.3.3}$$

But this implies that. again for b $\gtrsim$ 1/m we can take those denominators outside of the integrals over p_T by putting $p_T = 0$ in them. This simplifies the calculation enormously. since now we can calculate the integral in $\mathbf{p}_T$ analytically as in the case of slow pairs. and we obtain

$$G_{11} = 4\pi C\frac{\epsilon_+\epsilon_-}{\gamma v}K_1\left(\frac{\omega b}{\gamma v}\right)\frac{1}{\left[m^2+(\mathbf{k}_+^T)^2\right]\left[m^2+(\mathbf{k}_-^T)^2\right]}$$

$$\times\left\{V_\pm(x)\frac{(\mathbf{k}_+^T)^2-(\mathbf{k}_-^T)^2}{(\mathbf{k}_+^T+\mathbf{k}_-^T)^2}+i\frac{(Z_2\alpha)}{m^2}W_\pm(x)\left[m^2+(\mathbf{k}_+^T)^2+(\mathbf{k}_-^T)^2\right]\right\} \tag{2.3.4a}$$

$$\mathbf{G}_{21} = 2\pi C\frac{\epsilon_-}{\gamma v}K_1\left(\frac{\omega b}{\gamma v}\right)\frac{1}{\left[m^2+(\mathbf{k}_-^T)^2\right]}$$

$$\times\left\{V_\pm(x)\frac{(\mathbf{k}_+^T+\mathbf{k}_-^T)}{(\mathbf{k}_+^T+\mathbf{k}_-^T)^2}+i\frac{(Z_2\alpha)}{m^4}W_\pm(x)\left[\mathbf{k}_+(\mathbf{k}_-^T)^2-m^2\mathbf{k}_-^T\right]\right\} \tag{2.3.4b}$$

$$\mathbf{G}_{31} = 2\pi C\frac{\epsilon_+}{\gamma v}K_1\left(\frac{\omega b}{\gamma v}\right)\frac{1}{\left[m^2+(\mathbf{k}_+^T)^2\right]}$$

$$\times\left\{-V_\pm(x)\frac{(\mathbf{k}_+^T+\mathbf{k}_-^T)}{(\mathbf{k}_+^T+\mathbf{k}_-^T)^2}+i\frac{(Z_2\alpha)}{m^4}W_\pm(x)\left[\mathbf{k}_-(\mathbf{k}_+^T)^2-m^2\mathbf{k}_+^T\right]\right\} \tag{2.3.4c}$$

Inserting this result in eq.(2.1.18) we will find a complicated angular structure for the differential probability. But, this angular distribution is of the same form as that found by Bethe and Maximon[13] for e^+e^--production by a real photon. Therefore. using the same steps as they used for the evaluation of the angular integration, it is straightforward to show that the differential probability for the production of fast pairs is given by

$$\frac{d^2P_{e^+e^-}}{d\epsilon_+ d\epsilon_-} = \frac{4}{\pi^2}(Z_1Z_2\alpha r_e)^2\frac{1}{\gamma^2}K_1^2\left(\frac{\omega b}{\gamma}\right)\frac{1}{\omega^2}\left[\epsilon_+^2+\epsilon_-^2+\frac{2}{3}\epsilon_+\epsilon_-\right]$$

$$\times\left[ln\left(\frac{2\epsilon_+\epsilon_-}{m\omega}\right)-\frac{1}{2}-f(Z_2)\right] \tag{2.3.5}$$

where

$$f(Z) = Z^2\alpha^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+Z^2\alpha^2)} \tag{2.3.6}$$

Here we already find a crucial physical difficulty within this approach. Since the pairs with energies in the range given by eq. (1.1b) obey the same conditions in the projectile frame of reference, this expression should have the same structure if it were calculated in that frame of reference. But it is not so, because if we had calculated it in the frame of reference of the projectile, it would mean a simple exchange $Z_2 \leftrightarrow Z_1$ in eq.(2.3.5), which would lead to a'different result due to the presence of the functions $f(Z_1)$ in eq. (2.3.5). This difficulty arises because our approach is not symmetric in the nuclear charges from the very beginning. For example. the wave functions for the electron and positron are determined in the frame of reference of the nucleus at rest. neglecting the influence of the other nucleus on them. A solution to this problem by using a Lorentz covariant theory with Lorentz distorted wave functions for the electron-positron pair is, in no way simple, and to restore the required symmetry in the nuclear charges we postulate an average of the expressions obtained in the projectile and in the target system of reference as a reasonable result. This amounts to the replacement of the function $f(Z_2)$ by the averaged one

$$\bar{f}(Z_1,Z_2) = \frac{1}{(Z_1+Z_2)}[Z_1f(Z_1)+Z_2f(Z_2)] \tag{2.3.7}$$

When $Z_1 << Z_2$ (or $Z_2 << Z_1$), this modification is not relevant. since in eq.(2.3.5) only $f(Z_G)$ will appear. where Z_G is the greater from (Z_1, Z_2). But, when $Z_1 \simeq Z_2$ the approximation eq.(2.3.7) is rather speculative. because we do not know how the influence of both nuclear charges on the electron-positron wave functions will be. This point may be a source for future investigations.

Integrating eq.(2.3.5) over ϵ_- we find

$$\frac{d^2 P_{e^+e^-}}{d\epsilon_+} = \frac{112}{9\pi^2}(Z_1 Z_2 \alpha r_e)^2 \frac{1}{\gamma^2} K_1^2\left(\frac{2\epsilon_+ b}{\gamma}\right)\epsilon_+\left[ln\left(\frac{\epsilon_+}{m}\right) - \frac{1}{2} - \bar{f}(Z_1, Z_2)\right] \tag{2.3.8}$$

The same result can be obtained for $dP_{e^+e^-}/d\epsilon_-$ by exchanging the indice - and + in eq.(2.3.8). The respective expressions for the differential cross sections can be deduced from eqs.(2.3.5) and (2.3.8) by using the integral eq.(2.2.7). For example. the differential cross section $d\sigma/d\epsilon_+$ is equal to

$$\frac{d\sigma_{e^+e^-}}{d\epsilon_+} = \frac{56}{9\pi^2}(Z_1 Z_2 \alpha r_e)^2 \frac{1}{\epsilon_+}\left[ln\left(\frac{\epsilon_+}{m}\right) - \frac{1}{2} - \bar{f}(Z_1, Z_2)\right] ln\left(\frac{\gamma\delta m}{2\epsilon_+}\right) \tag{2.3.9}$$

In figure 4 we plot the differential cross section $d\sigma_{e^+e^-}/d\epsilon_+$ for production of e^+e^- pairs in uranium-uranium collisions and calcium-calcium collisions as a function of the positron energy ϵ_+. and for $\gamma = 100$ and 1000. Already here we see that the creation of positrons (and electrons) with small energies is strongly suppressed in comparison with the ones with higher energies. and when γ increases more and more positrons (and electrons) with higher energies are produced. Indeed. in this figure we see that the dashed curves ($\gamma = 100$) decrease faster with increasing energy of the positrons than the full curve ($\gamma = 1000$).

Mow we integrate eq.(2.3.8) over ϵ_+ and use the approximation eq.(2.2.6), in order to obtain the probability to create a e^+e^- pair in a RHI collision as a function of the impact parameter

$$P_{e^+e^-}(b) \simeq \frac{14}{9\pi^2}(Z_1 Z_2 \alpha r_e)^2 \frac{1}{b^2}\left[ln^2\left(\frac{\gamma\delta}{2mb}\right) - [1 + 2\bar{f}(Z_1, Z_2)]\, ln\left(\frac{\gamma\delta}{2mb}\right)\right] \tag{2.3.10}$$

valid for $\gamma\delta/m \gtrsim b \gtrsim 1/m$.

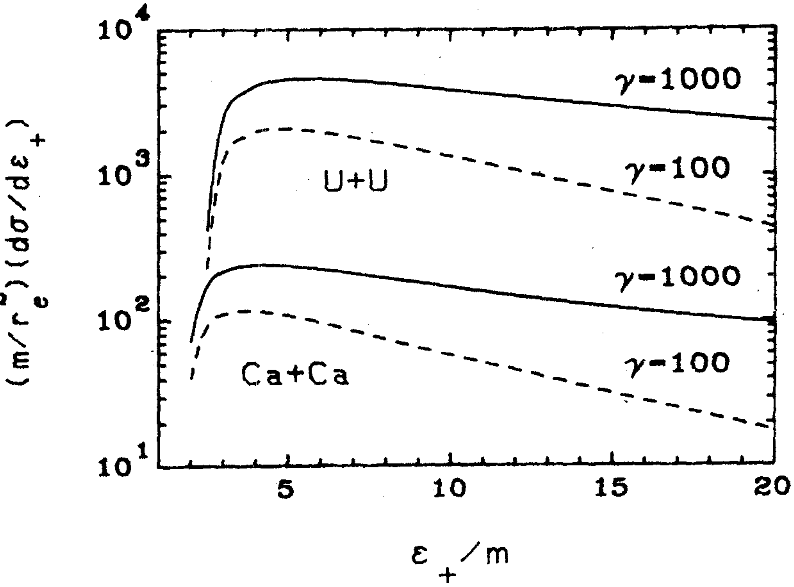

Fig.4 - The differential cross section $d\sigma_{e^+e^-}/d\epsilon_+$ for production of e^+e^- pairs in uranium-uranium collisions (upper curves) and calcium-calcium collisions (lower curves) as a function of the positron energy ϵ_+, and for $\gamma=$ 100 and 1000.

In figure 5 we plot the probability to create a e^+e^- pair in uranium-uranium and calcium-calcium collisions as a function of the Lorentz factor γ, and for impact parameter equal to the Compton wavelength $b = 1/m$. We note that for calcium-calcium collisions $P_{e^+e^-} << 1$, even for very large values of γ, which justifies the use of first order pertubation theory. Nonetheless. for uranium-uranium collision $P_{e^+e^-} > 1$ for $\gamma \gtrsim 500$, which violates the unitarity condition. This means that for extremely high energies, greater than several hundreds of GeV/nucleon, and for very heavy ions. it will be necessary to account for hígher order terms in the perturbation theory. In other words, one must consider the probability of creating two or more pairs in a single collision above those energies.

As we mentioned before $P_{e^+e^-}(b)$ goes to a constant, finite value for $b \gtrsim 1/m$, and diminishes very slowly (like $1/b^2$) as a function of b. up to a limit γ/m after

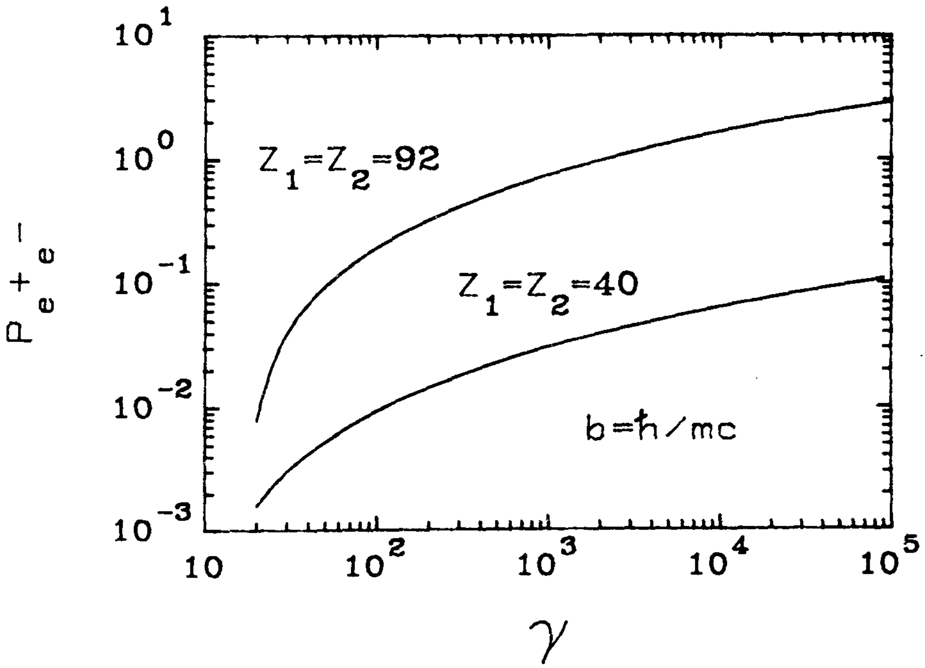

Fig.5 - The probability $P_{e^+e^-}$ for production of e^+e^- pairs in uranium-uranium collisions and calciurn-calcium collisions as a function of the relativistic Lorentz factor γ, and for impact parameter $b = 1/m$. Observe that for uranium-uranium collisions it becomes greater than one for 7 $\gtrsim$ 500.

which it decays exponentially. and this is the reason why the cross sections for pair production will be very large. In fact. integrating eq.(2.3.10) from $b = 1/m$ to $b = \gamma/m$ we find

$$\sigma_{e^+e^-} = \frac{28}{27\pi}(Z_1 Z_2 \alpha r_e)^2 \left[ln^3\left(\frac{\gamma\delta}{2}\right) - \frac{3}{2}(1+2\bar{f})\, ln^2\left(\frac{\gamma\delta}{2}\right)\right] \qquad (2.3.11)$$

Since the integration of eq.(2.3.5) over b can be done analytically by using eq.(2.2.7), a better result can be also found by integrating numerically $d^2\sigma/d\epsilon_+ d\epsilon_-$ over e+ and e−. But, for 7 $\gtrsim$ 100 the eq.(2.3.11) agrees very well with the numerical calculations. Except for the second term inside brackets and an irrelevant

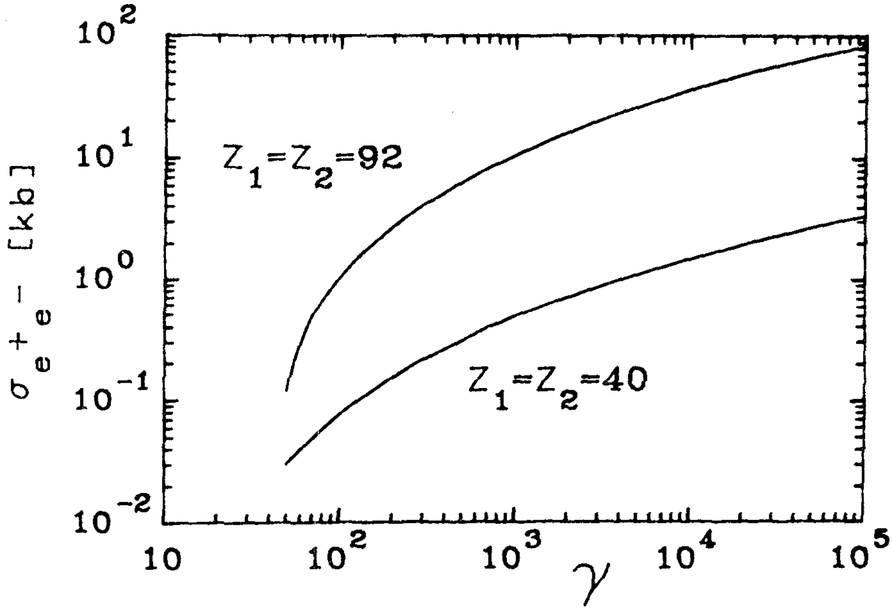

Fig.6 - The cross section $\sigma_{e^+e^-}$ for production of e^+e^- pairs in uranium-uranium collisions and calcium-calcium collisions as a function of the relativistic Lorentz factor 7. The ordinate is given in kilobarns.

factor in the logarithm which is not important for 7 >> 1, the above expressions agrees with the result found by Landau and Lifshitz[2] in the Born approximatin. The second term inside brackets is a correction due to the distortion of the electron-positron wavefunctions in the field of the nuclei.

In figure 6 we plot the cross section for production of e^+e^- pairs in relativistic uranium-uranium collisions and calcium-calcium collisions as a function of the Lorentz factor γ, based on eq.(2.3.11). These cross sections are about one to two orders of magnitude smaller than the ones calculated in ref.11 where the equivalent photon method was used. This occurs because there the equivalent photon numbers were integrated from a minimum impact parameter equal to R (sum of the nuclear radii). As we saw, the minimum impact parameter that should be used is equal to the Compton wavelength, below which the contribution to the total cross section for pair production is negligible. That error makes the cross section in ref. 11 much bigger than it should be. But the results agree quite **well** with the ones obtained here if we make the substitution $R \rightarrow 1/m$ in that calculation. **In** view of our previous discussion about the pair production probability. there must exist some corrections to eq. (2.3.11) for uranium-uranium **collisions** with *7* $\gtrsim$ **500.** There one must take also into account the probability of creating two or more pairs in a single **RHI** collision. This may change the **dependence** of the cross section on *7.*

From the previous results one observes that the probability for the production of fast e^+e^- pairs in the collision of two fast **nuclei** in comparison with slow (or ultra-fast) ones scales like

$$\frac{P^{fast}_{e^+e^-}}{P^{slow}_{e^+e^-}} \simeq ln^2\left(\frac{\gamma}{mb}\right) \tag{2.3.12a}$$

and the ratio between the cross sections scales like

$$\frac{\sigma^{fast}_{e^+e^-}}{\sigma^{slow}_{e^+e^-}} \simeq ln^2\,\gamma \tag{2.3.12b}$$

which means that for *7* >> 1, most of the e^+e^- pairs will be fast ones. **i.e.** will have energies in the range given by **eq.(1.1.b).** Therefore. we can say that the total probability or cross section for producing e^+e^- pairs in RHI are given accurately enough by eqs. (2.3.10) and (2.3.11). or by the respective numerical integration of eq. (2.3.5).

2.4 - Effects of screening

The above cross sections were evaluated under the assumption that the RHI were naked, without their electron **cloud.** Let **us,** for simplicity. assume that only one of the ions is screened by the atomic electrons. say the target. Then. the correction to the previous results can be performed in a completely analogous way as in the case of pair production by a real **photon**[13]. Therefore. we only present the final results. which for **partial** screening are *(7* $\gtrsim$ **100)**

$$\frac{d\sigma_{e^+e^-}}{d\epsilon_+} = \frac{2}{9\pi}(Z_1 Z_2 \alpha r_e)^2 \frac{1}{\epsilon_+}\left[6\Phi_1(\chi) + \Phi_2(\chi) - \frac{28}{3}\,ln\,Z_2 - 28\bar{f}\right] ln\left(\frac{\gamma\delta m}{2\epsilon_+}\right) \tag{2.4.1}$$

where Φ_1 and Φ_2 are the Bethe functions for atomic screening[19] as function of the parameter $\chi = (2m\omega/\epsilon_+\alpha)Z_2^{-1/3}$ (see also ref.15, p.395).

In case of complete screening. i.e. when $\epsilon_\pm >> m/(Z_2^{-1/3}\alpha)$, then we can use $\Phi_1(0) = 4\,ln\,183$ and $\Phi_2(0) = 4\,ln\,183 - 2/3$ and eq.(2.4.1) reduces to ($\gamma \gtrsim 100$)

$$\frac{d\sigma_{e^+e^-}}{d\epsilon_+} = \frac{56}{9\pi}(Z_1 Z_2 \alpha r_e)^2 \frac{1}{\epsilon_+}\left[ln\left(\frac{183}{Z_2^{1/3}}\right) - \frac{1}{42} - \bar{f}\right] ln\left(\frac{\gamma\delta m}{2\epsilon_+}\right) \qquad (2.4.2)$$

The total cross sections for e^+e^--pair production in RHI when one of the ions is completely screened is obtained by integrating eq.(2.4.2) from $\epsilon_+ = m/(Z_2^{1/3}\alpha)$ to γm, i.e. ($\gamma \gtrsim 100$)

$$\sigma_{e^+e^-} = \frac{28}{9\pi}(Z_1 Z_2 \alpha r_e)^2 \frac{1}{\epsilon_+}\left[ln\left(\frac{183}{Z_2^{1/3}}\right) - \frac{1}{42} - \bar{f}\right]\left[ln^2\left(\frac{\gamma\delta Z_2^{1/3}\alpha}{2}\right) - ln^2\left(\frac{\delta}{2}\right)\right] \qquad (2.4.3)$$

In the case of partial screening a numerical integration of eq.(2.4.1) will be necessary.

In figure 7 we show the cross section for pair production in oxygen-calcium and oxygen-uranium collisions as a function of the Lorentz factor γ. The solid lines correspond to the case of no-screening of the target and of the projectile, as it could be the situation in a RHI collider. The dashed lines correspond to the case of complete screening of the target. When screening is present the cross sections will always be smaller by at less a factor 2-4, also for very high beam energies.

3. PRODUCTION OF HEAVY LEPTONS

The same previous calcolations can be applied for $\mu^+\mu^-$ and $\tau^+\tau^-$-pair production in RHI. but care must be taken with the following facts. First, since the Compton wavelength of these leptons satisfies the condition

$$\frac{\hbar}{mc} << R = R_1 + R_2 \qquad (3.1a)$$

where

$$R_{1,2} \simeq 1.2 A_{1,2}^{1/3} \qquad (3.1b)$$

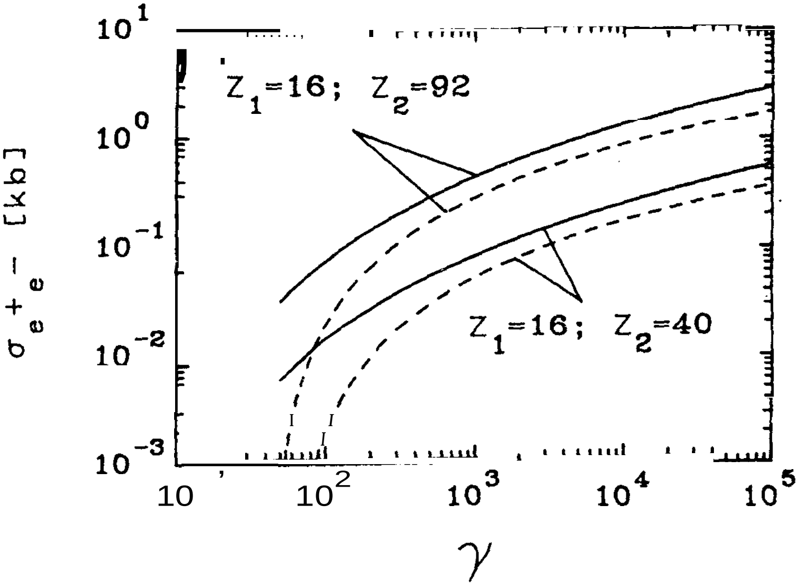

Fig.7 - The cross section $\sigma_{e^+e^-}$ for production of e^+e^- pairs in oxygen-calcium collisions as a function of the relativistic Lorentz factor γ. The solid lines refer to completely naked projectile and target and the dashed lines refer to completely naked projectile. but completely screened target.

are the nuclear radii of the ions, the impact parameter dependence of the pair production probabilities is accurately enough described by expressions given in section 2, but in the cross sections one must replace the variable ζ as given in eq. (2.2.7b) by another one given by

$$\xi = \frac{\omega R}{\gamma v} \tag{3.2}$$

This means that for

$$\gamma >> 16, \qquad for \qquad \mu^+\mu^- \text{ pair production} \tag{3.3a}$$

and

$$\gamma >> 270, \qquad for \qquad \tau^+\tau^- \text{ pair production} \tag{3.3b}$$

we can replace

$$ln\left(\frac{\gamma\delta m}{\omega}\right) \qquad by \qquad ln\left(\frac{\gamma\delta}{\omega R}\right) \tag{3.4}$$

in the equations for the cross sections given in section 2 to obtain the respective cross sections for $\mu^+\mu^-$ and $\tau^+\tau^-$ production.

The conditions eq.(3.3) are quite severe and only for RHI accelerators working at extremely large energies they will be useful, specially for $\tau^+\tau^-$ production. Therefore, we consider the opposite case, i.e. when $\gamma \gtrsim 16$ for $\mu^+\mu^-$ and $\gamma \gtrsim 270$ for $\tau^+\tau^-$ production. Then the function in eq.(2.2.7a) becomes

$$g(\xi) \simeq \pi^2 e^{-2\varsigma} \tag{3.5}$$

The expressions (2.2.8) and (2.2.10) will be correct if we replace

$$ln\left(\frac{\gamma\delta m}{\omega}\right) \qquad by \qquad \frac{\pi}{2}e^{-\frac{2\omega R}{\gamma}} \tag{3.6}$$

This means that the double differential cross section for $\mu^+\mu^-$ and $\tau^+\tau^-$ production with beam energies satisfying the above condition is

$$\frac{d^2\sigma_{\ell^+\ell^-}}{d\epsilon_+ d\epsilon_-} = \frac{64\pi^2}{3}(Z_1Z_2r_\ell\alpha)^2\frac{a_+a_-}{(e^{2\pi a_+}-1)(1-e^{-2\pi a_-})}\frac{\sqrt{(\epsilon_+-m)(\epsilon_--m)}}{\omega^4}$$

$$\times\ [(w-2m)+(Z_2\alpha)^2(\frac{7}{2}m-\omega)]e^{-\frac{2\omega R}{\gamma}} \tag{3.7}$$

where the subscript $\ell^+\ell^-$ is used in this section for muon or tau pairs.

We could also make a rough estimate of the cross section for muon (or tau) production in which the negative muon (or tau) is captured in an orbit around the target by making the exchange eq.(3.6) in eq.(2.2.14). But. by using the resulting expression to calculate the cross section for muon pair production with capture of the negative muon in a uranium-uranium collision with projectile energy equal to 10 GeV/nucleon, we find a value about a thousand times bigger than the one obtained by Momberger et al.[20] for the same reaction. which is 2.9 x $10^{-7}mb$. This occurs because. besides the approximation made to obtain the estimate eq.(2.2.14), the Compton wavelength of the muon captured in an orbit around a heavy nucleus is smaller than the nuclear radius and its wavefunction will be strongly influenced by the charge distribution inside the nucleus. This makes unrealistic the above treatment based on a point center of charge, and gives a higher value than that obtained by Momberger et *al.* because in this approximation the muon wavefunction extends farther away from the nucleus than it should, giving a bigger contribution to the matrix element eq.(2.1.1). These effects were considered by those authors in that paper. Speculations about tau-atomic orbits in RHI collisions were recently made by Weiss[21], where such finite size effects are very decisive.

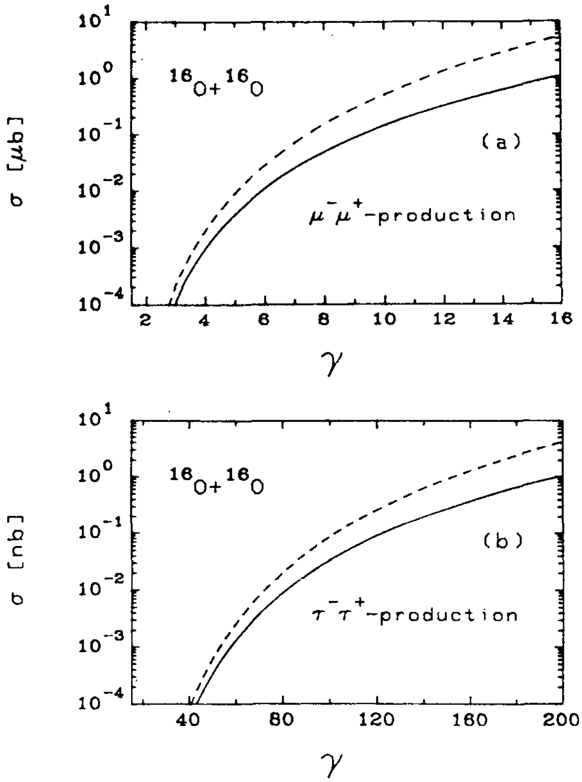

Fig.8 - The cross section $\sigma_{e^+e^-}$ for production of $\mu^+\mu^-$ and $\tau^+\tau^-$ pairs in oxygen-oxygen collisions as a function of the Lorentz factor γ. The solid lines correspond to the eq.(3.9) and the dashed ones to its asymptotic limit.

When the charges of the ions are small, such that the approximation $a_\pm << 1$ can be used, we can integrate eq.(3.7) from $\epsilon_\pm = m$ to $2m$ and obtain

$$\frac{d\sigma_{\ell^+\ell^-}}{d\epsilon_+} = \frac{1}{3}\Big(Z_1Z_2\alpha r_\ell\Big)^2\frac{1}{m}\Big(\frac{\gamma}{2mR}\Big)^{3/2}\Big(\frac{\epsilon_+}{m}-1\Big)^{3/2}$$

$$\times\Big[\Gamma\Big(\frac{3}{2}\Big)-\Gamma\Big(\frac{3}{2};\frac{2mR}{\gamma}\Big)\Big]e^{-2(\epsilon_+ + m)R/\gamma}$$

$$\simeq \frac{\sqrt{\pi}}{6}(Z_1Z_2\alpha r_\ell)^2\frac{1}{m}\Big(\frac{\gamma}{2mR}\Big)^{3/2}\Big(\frac{\epsilon_+}{m}-1\Big)^{3/2}e^{-2(\epsilon_+ + m)R/\gamma} \quad (3.8)$$

where $\Gamma(\lambda; y)$ is the incomplete gamma function (see ref. 16. p.940). and the last equality corresponds to the asymptotic limit $mR/\gamma >> 1$.

We integrate eq.(3.8) over ϵ_+ in order to find the total cross section for muon (or tau) production under the condition that $mR/\gamma \gtrsim 1$, namely

$$\sigma_{\ell^+\ell^-} = \frac{1}{48}(Z_1Z_2\alpha r_\ell)^2\Big(\frac{\gamma}{mR}\Big)^4\Big[\Gamma\Big(\frac{3}{2}\Big)-\Gamma\Big(\frac{3}{2};\frac{2mR}{\gamma}\Big)\Big]$$

$$\times\Big[\Gamma\Big(\frac{5}{2}\Big)-\Gamma\Big(\frac{5}{2};\frac{2mR}{\gamma}\Big)\Big]e^{-\frac{4mR}{\gamma}}$$

$$\simeq \frac{\pi}{128}(Z_1Z_2\alpha r_\ell)^2\Big(\frac{\gamma}{mR}\Big)^4 e^{-\frac{4mR}{\gamma}} \quad (3.9)$$

This result is in good agreement with that of ref.12, where the cross section was calculated by using the equivalent photon method. There the cross section was given in terms of the exponential integral function $E_i(x)$ (see ref.16, p.312) and the asymptotic limit for $mR/\gamma >> 1$ is exactly the same as the one obtained above (see eq. 10 of that reference). In RHI collisions. for which the above approximations are not valid. a numerical integration of eq. (3.7) has to be performed.

Numerical values are plotted in figure 8 for $\sigma_{\ell^+\ell^-}$ in the collision $^{16}O + ^{16}O$ as a function of the Lorentz factor γ. The cross sections are much smaller than that for e^+e^- production in the same energy regime. This is due to the severe limitation imposed by the adiabatic cutoff in the cross section for projectile energies such that $mR/\gamma \gtrsim 1$, which strongly inhibits the creation of very massive particles when this condition is attained. When the projectile energy is very high. such that the conditions eq.(3.3) are valid than we can use eq.(2.3.11) also for $\mu^+\mu^-$ and $\tau^+\tau^-$ production. But even in that case the cross sections will be smaller by a factor $(m_e/m_\ell)^2$ (ie.. approximately 10^{-4} for $\mu^+\mu^-$ production. and aproximately 10^{-7} for $\tau^+\tau^-$ production) in comparison with that for e^+e^- production.

4. CONCLUSIONS

The construction of bigger and bigger accelerators seems to be a common trend in nuclear and particle physics (see e.g. refs. 6 and 22). Certainly. one of the most important purposes of these machines is the study of nuclear matter under extreme conditions. In central nucleus-nucleus collisions one hopes to observe new forms of nuclear matter. like the formation of the quark-gluon plasma (see e.g. ref.23). On the other hand. very strong electromagnetic fields for a very short time are present in distant collisions with no nuclear contact. This is essentially due to the Lorentz contraction of the fields. Such fields can also lead to interesting effects. many of which of atomic and nuclear origin. which where discussed in detail in ref.11.

In the present paper we have given a formulation of the eletromagnetic production of lepton pairs (electron-positron pairs being by far the most important) in RHI collisions, in a way which is as transparent as possible. We found tractable analytical results for the relevant kinematical situations. There are slow pairs produced around the target and projectile (which we called ultra-fast pairs) charges. respectilely. and dominantly. an intermediate energy region. This was the region studied in the thirties. In addition we give impact paramneter dependent results which point direcly to the limitation of the present approach for extremely relativistic collision of very heavy ions.

Lepton-pair production is suggested[24] as being a potentially efficient probe of quark matter formed in RHI collisions. As we saw in this article (and also in refs. 6-12). the electromagnetic production of leptons is by no means negligible, and although the multiplicity (i.e. the number of pairs) in a single collision is smaller than or about one. the cross sections for it are very high and êan be a source of experimental difficulties in the signature of that aspect of the quark-gluon plasma formation.

As a final remark. let us compare the electromagnetic production of leptons in RHI collisions with the corresponding process in reiativistic electron colliders. For a detailed theoretical study of that, see ref. 25. In such machines, the γ-values achieved are much higher than in the heavy ion case, therefore the cross sections are accurately enough given by eq. (2.3.11). We have astonishingly large e^+e^- production cross sections in RHI collisions. due to the large charge factor $Z_1^2Z_2^2$; however. heavy leptons pairs ($\mu^+\mu^-, \tau^+\tau^-$ pairs) are practically not produced unless the beam energy is very high (7 >> 16 for $\mu^+\mu^-$, and $\gamma >> 270$ for $\tau^+\tau^-$ pairs). Also. the electromagnetic production. in the two photon mechanism, of heavy quark-antiquark states (like the η_c, which was recently studied with the PLUTO detector at PETRA in high energy e^+e^- collisions[26]), will be negligible unless the beam energy is extremely large ($\gamma >> 600$ in case of η_c production).

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Resumo

Calculamos as probabilidades e seções de choque para a criação de pares elétron- pósitron em colisões nucleares relativlsticas utilizando as funções de onda de Sommerfeld-Maue para o par. Comparamos nossos resultados com os existentes na literatura. discutimos aspectos flsicos novos, até então inexplorados. e fazemos aplicações às reações relativlsticas com íons pesados.