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Pair Production with Atomic-shell Capture in Relativistic Heavy Ion Collisions

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Abstract We evaluate the probabilities and cross sections for the production of electron-positron pairs in relativistic heavy ion collisions in the case for which the electron is captured in an atomic orbit in the projectile, or in the target. Analytical expressions are obtained, with help of which one may gain a useful insight into the characteristic features of the process. Such a process will also play a role in future relativistic heavy ion colliders since it changes the charge of the ions. The obtained formulas are expected to be very useful for the study of such phenomena.

1. INTRODUCTION

The cross sections for electron-positron pair production in relativistic heavy ion (RHI) collisions are very large in comparison with the ones related to other processes of atomic or nuclear origin (see e.g. refs. 1 and 2). The pair production in the collisions of fast nuclei was studied in the thirties (refs. 3-7) and recently it has regained interest with the construction of new relativistic heavy ion machines (see refs. 8-11). With the obtained large values of the cross sections for production of free electron-positron pair. it is also of interest to study those pair production processes where the electron is captured in a bound atomic orbit in the projectile. or in the target. The first theoretical work on this subject was carried out by Becker. Grun and Scheid[12]. There they used a partial wave expansion of the electron and the positron in terms of exact Dirac-Coulomb wave functions. and numerically calculated the probabilities and cross sections for the process in the first order semiclassical theory. We pursue here a different calculation of the same process by trying to avoid the partial wave expansion and to obtain analytical formulas for the important cases. This can be accomplished by using approximate wave functions for the final bound state of the electron and dor the free positron. Since this process has a close relationship to the photo-electric effect, we find that the main integrals which appear in our calculations were already performed

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in the past. Here we use the same approach as in our previous calculation[2] of production of free pairs in the collisions of fast nuclei.

Our semiclassical calculations are given in section 2 and explicit analytical results are given for the capture of the created electron in a K-shell of the projectile, or in the target. There we also show how the limiting cases can be obtained by using the equivalent photon method (see e.g. ref. 13. and references therein). The cross sections for the capture in higher atomic orbits, being of much less importance, are studied in section 3. Our conclusions are given in section 4.

2. PAIR PRODUCTION WITH K-SHELL CAPTURE

In the following we shall calculate the electromagnetic production probabilities and cross sections of electron-positron pairs in the field of a target nucleus with mass and charge number A_2 and Z_2, respectively. by means of a relativistic projectile with velocity v (which we take parallel to the z-axis), impact parameter b. and mass and charge number A_1 and Z_1. We shall consider the case of pair production with simultaneous capture of the electron in a bound atomic orbit in the target nucleus. The same results can be applied for the case of capture of the electron by the projectile. The calculation is valid for impact parameters such that $b > R = R_1 + R_2$, where R_1 and R_2 are the respective nuclear radii. We shall consider the target nucleus as fixed, neglecting its recoil, and we place the origin of our coordinate system at its center of mass.

In the semiclassical approach the projectile is assumed to move in a straight-line and will generate a time-dependent electromagnetic field which will lead to the production of pairs in the field of the target. Since the probability amplitude for pair production is ,generally, smaller than unity. we can calculate in the first order time-dependent perturbation theory (as soon as we take into account the distortion of the wavefunctions of the pair due to the field of the target nucleus). In a previous work we have shown that the probability amplitude to create an electron-positron pair, with respective energies equal to ϵ_+, ϵ_-, in a RHI collision with $\gamma >> 1$ is given by[2] (we use $\hbar = c = 1$)

$$a_{e^+e^-} = \frac{Z_1 e}{i\pi v} \int d^2 p_T \frac{H(\mathbf{p}')}{\mathbf{p}_T^2 + (\omega/\gamma v)^2} e^{i\mathbf{p}_T \cdot \mathbf{b}} \tag{2.1}$$

where

$$P' = (\mathbf{p}_{T\perp}\omega/v) \tag{2.2}$$

and

$$H(\mathbf{p}') = \frac{e}{\omega}\,\mathbf{p}_T \cdot \int \mathrm{d}^3\mathbf{r} < \Psi_{e^-}|\vec{\alpha}_T e^{i\mathbf{p}'\cdot\mathbf{r}}|\Psi_{e^+} > \tag{2.3}$$

where. Ψ_{e^-} and (Ψ_{e^+}) are. respectively. the wavefunctions of the created electron and positron. $\vec{\alpha}_r$ is a Dirac matrix of component perpendicular to the beam direction. and

$$\omega = \epsilon_+ + m - I_K \tag{2.4}$$

with I_K equal to the ionization energy of the K-shell electron (in the following we shall use $I_K = 0$. which is appropriate only for small Z nuclei.)

For Ψ_{e^+} we use the Sommerfeld Maue wave function (see ref. 14 for a complete discussion of these wavefunctions), namely

$$\Psi_{e^+} = N_+ e^{-i\mathbf{k}_+\cdot\mathbf{r}}\left[1 + \frac{i}{2\epsilon_+}\vec{\alpha}\cdot\nabla\right] wF(-ia_+, 1, ik_+ r + i\mathbf{k}_+\cdot\mathbf{r}) \tag{2.5}$$

and for the captured electron we use the bound K-shell wavefunction. valid for $Z_2 e^2 << 1$,

$$\Psi_{e^-} = N_-\left[1 + \frac{i}{2}Z_2 e^2 \vec{\alpha}\cdot\frac{\mathbf{r}}{r}\right] e^{-\eta r} u_0 \tag{2.6a}$$

where

$$u_0^{1/2} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \quad u_0^{-1/2} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \quad and \quad \eta = mZ_2e^2 \tag{2.6b}$$

In the above wavefunctions u_0 and w are the Dirac spinors corresponding to the bound electron in a K-shell and a to a free positron with momentum $\mathbf{k}_+$, respectively. F is the confluent hypergeometric function and

$$N_- = \sqrt{(Z_2me^2)^3/\pi}, \quad N_+ = \left[\frac{2\pi a_+}{e^{2\pi a_+} - 1}\right]^{1/2}, \ with \ a_+ = \frac{Z_2e^2}{v_+} \qquad (2.6c)$$

where v_+ equal to the velocity of the positron. The components $u_0^{+1/2}$ and $u_0^{-1/2}$ of the spinor u_0 correspond to the electron with spin up and down, respectively (see ref. 15. eq. 31.11).

Although. due to the use of the wavefunctions eqs.(2.5) and (2.6). the following resultã may be appropriate for collisions for which one (or both) nuclei satisfy the condition $Ze^2 << 1$, we expect that even for every heavy nuclei they will be useful since the main ingredients are contained in the calculations.

Inserting eqs. (2.5) and (2.6) in eq.(2.1) we find

$$a^K_{e^+e^-} = \frac{Z_1e^2}{i\pi\omega v} N_+N_- \sum_{\lambda=1,2} u_0^*\left[\alpha^\lambda G_{1\lambda} + (\vec{\alpha}\cdot\mathbf{G}_{2\lambda})\alpha^\lambda + \alpha^\lambda(\vec{\alpha}\cdot\mathbf{G}_{3\lambda})\right] w \qquad (2.7)$$

where $\lambda = 1,2$ represents the two orthogonal components transverse to the beam. The tensors $G_{1\lambda}, \mathbf{G}_{2\lambda}$ and $\mathbf{G}_{3\lambda}$ are given by

$$[G_{1\lambda}, \mathbf{G}_{2\lambda}, \mathbf{G}_{3\lambda}] = \int d^2p_T \frac{p_T^\lambda [J_1, \mathbf{J}_2, \mathbf{J}_3]}{p_T^2 + (\omega/\gamma v)^2} e^{i\mathbf{p}_T\cdot\mathbf{b}} \qquad (2.8)$$

whcre

$$J_1 = \int e^{i\mathbf{q}\cdot\mathbf{r} - \eta r} F_2 d^3r \qquad (2.9a)$$

$$\mathbf{J}_2 = -\frac{iZ_2e^2}{2} \int e^{i\mathbf{q}\cdot\mathbf{r}-\eta r} \frac{\mathbf{r}}{r} F_2 d^3r \qquad (2.9b)$$

$$\mathbf{J}_3 = \frac{i}{2\epsilon_+} \int e^{i\mathbf{q}\cdot\mathbf{r}-\eta r} \nabla F_2 d^3r \qquad (2.9c)$$

with

$$\mathbf{q} = \mathbf{p}' - \mathbf{k}_+ \qquad (2.9d)$$

and

$$F_2 \equiv F(-ia_+, 1, ik_+r + i\mathbf{k}_+\cdot\mathbf{r}) \qquad (2.9e)$$

Th᷊ integrals eq.(2.9) are very similar to the ones involved in the photoelectric effect and can be carried out analytically (see e.g. ref. 15, p. 435 and 436).

Since only values of p_T up to $\omega/\gamma v << m$ will contribute to the integrals eq.(2.8) we put $\mathbf{p}_T = 0$ in eq.(2.9), which amounts to using $\mathbf{q} = (\omega/v)\hat{\mathbf{z}} - \mathbf{k}_+$ in them, where $\hat{\mathbf{z}}$ is equal to a unit vector in the beam direction. In this way. the integral in $\mathbf{p}_T$ of eq.(2.8) can be done exactly in terms of the modified Bessel function of first order. As we have shown in our previous calculations on production of free electron-positron pairs[2]. this approximation is good for RHI collisions with impact parameters larger than the Compton wavelength of the electron, i.e. for $b \gtrsim 1/m$ which are the impact parameters which most contribute to the total cross sections. Indeed, for $b \gtrsim 1/m$ the probability amplitude for pair production tends to a constant value (see the exact numerical calculations of Becker, Grun and Scheid'), while for $b \gtrsim 1/m$ it decays proportionally to $1/b^2$, up to a cutoff limit given by $b \gtrsim \gamma/\omega$. This has a consequence that the contribution of impact parameters $b \gtrsim 1/m$ to the total cross sections increases logarithmically with the RHI-beam energy. while the contribution from smaller impact parameters gives a constant and small quantity.

With these approximations the integrals eq.(2.8) can be solved analytically and we obtain (we choose **b** along the s-axis)

$$G_{12}, \mathbf{G}_{22}, \mathbf{G}_{32} = 0 \tag{2.10a}$$

and

$$G_{11} = -i\frac{4\pi^2 Z_2 e^2}{\gamma v}\frac{k_+(k_+ - \epsilon_+ \cos\theta_+)}{\epsilon_+(\epsilon_+ + m)(\epsilon_+ - k_+ \cos\theta_+)^2}K_1\left(\frac{\omega b}{\gamma v}\right) \tag{2.10b}$$

$$\mathbf{G}_{21} = i\frac{2\pi^2 Z_2 e^2}{\gamma v^2}\frac{\omega\hat{\mathbf{z}} - v\mathbf{k}_+}{(\epsilon_+ + m)(\epsilon_+ - k_+ \cos\theta_+)^2}K_1\left(\frac{\omega b}{\gamma v}\right) \tag{2.10c}$$

$$\mathbf{G}_{31} = i\frac{2\pi^2 Z_2 e^2}{\gamma v^2}\frac{\omega\hat{\mathbf{z}} - v\mathbf{k}_+}{m(\epsilon_+ + m)(\epsilon_+ - k_+ \cos\theta_+)}K_1\left(\frac{\omega b}{\gamma v}\right) \tag{2.10d}$$

where K_1 represents the modified Bessel function of first order. and ϕ_+ and θ_+ are, respectively, the azimuthal and the polar angle of emission of the created positron with respect to the beam direction.

The differential probability for pair production with K-shell capture is obtained frorn eq.(2.7) as

$$dP^K_{e^+e^-} = \sum_{spins} |a^K_{e^+e^-}|^2 \frac{k_+\epsilon_+}{(2\pi)^3} d\epsilon_+ d\Omega_+ \tag{2.11}$$

where the surnmation is taken over different spin orientations of the electron and the positron. Using the properties of the Dirac matrices we find

$$\begin{aligned} dP^K_{e^+e^-} = \left(\frac{Z_1 e^2}{\pi\omega v}\right)^2 |N_+N_-|^2 \frac{k_+}{(2\pi)^3} \Big\{ &(\epsilon_+ + m)|G_{11}|^2 \\ &+ 2G^*_{11}[\mathbf{k}_+ \cdot (\mathbf{G}_{21} - \mathbf{G}_{31}) - 2\mathbf{k}^T_+ \cdot \mathbf{G}^T_{21}] \\ &+ (\epsilon_+ - m)[|\mathbf{G}_{21}|^2 + |\mathbf{G}_{31}|^2 + 2(2\mathbf{G}^T_{21} \cdot \mathbf{G}^T_{31} - \mathbf{G}_{21} \cdot \mathbf{G}_{31})]\Big\} d\epsilon_+ d\Omega_+ \end{aligned} \tag{2.12}$$

where $\mathbf{k}^T_+(\mathbf{G}^T_{\mu\lambda})$ denotes the transverse component of $\mathbf{k}_+(\mathbf{G}_{\mu\lambda})$.

Now we insert the expressions (2.10) in (2.12). putting v $\simeq$ c $= 1$ overall. and we find

$$\begin{aligned} dP^K_{e^+e^-} = &\frac{2}{\pi} Z_1^2 Z_2^6 \alpha^8 \frac{1}{(e^{2\pi a_+} - 1)} \frac{m^3}{\epsilon_+(\epsilon_+ + m)^3} \frac{\sin^2\theta_+}{\left(1 - \frac{k_+}{\epsilon_+}\cos\theta_+\right)^4} \\ &\times \left[\chi(\chi^3 + \chi^2 - \chi - 1) + (4 - 6\chi + 2\chi^3)\cos^2\phi_+ \right. \\ &\left. - (1 + \chi + 2\cos^2\phi_+)(\chi^2 - 1)^{3/2}\cos\theta_+\right] \frac{1}{\gamma^2} K_1^2\left(\frac{\omega b}{\gamma}\right) d\epsilon_+ d\Omega_+ \end{aligned} \tag{2.13}$$

where $\chi = \epsilon_+/m$, and a $=$ e^2/Ac $\simeq 1/137$ is the fine structure constant.

The angular distribution for the direction of ernission of the created positron rnay be expressed in terrns of the adirnensional function

$$W^K_{e^+e^-} = \frac{2\pi\sin^2\theta_+}{\left(1 - \frac{k_+}{\epsilon_+}\cos\theta_+\right)^4} \left[2 - 4\chi - \chi^2 + 2\chi^3 + \chi^4 - (2 + \chi)(\chi^2 - 1)^{3/2}\cos\theta_+\right] \tag{2.14a}$$

which is obtained by integration of the angular part of eq.(2.13) over the azimuthal angle ϕ_+.

When $(\epsilon_+ - m) << m$, then

$$W^K_{e^+e^-} \simeq 4\pi \frac{k_+^2}{m^2} sin^2 \theta_+ \tag{2.14b}$$

which means that low energetic positrons will be emitted preferentially in the direction perpendicular to the RHI-beam. For $\epsilon_+ >> m$. the angular distribution is approximately

$$W^K_{e^+e^-} \simeq 4\pi \left(\frac{\epsilon_+}{m^4}\right)^4 \frac{sin^2\theta_+}{\left(1 - \frac{k_+}{\epsilon_+} cos\,\theta_+\right)^4} \simeq 64\pi \left(\frac{\epsilon_+}{m}\right)^4 \frac{\theta_+^2}{\left[\left(\frac{m}{\epsilon_+}\right)^2 + \theta_+^2\right]^4} \tag{2.14c}$$

which implies that highly energetic positrons will be created with their momenta directed very forwardly. up to a maximum angle $\theta_+^{max} \simeq m/\epsilon_+$.

Integrating eq.(2.13) over the angular distribution of the positrons we find

$$\frac{dP^K_{e^+e^-}}{d\epsilon_+} = 8 Z_1^2 Z_2^6 \alpha^8 \frac{1}{(e^{2\pi a_+} - 1)} \frac{k_+ m}{(\epsilon_+ + m)^3} \frac{1}{\gamma^2} K_1^2\left(\frac{\omega b}{\gamma}\right)$$
$$\times \left[\frac{4}{3} + \frac{2\epsilon_+}{3m} + \left(\frac{\epsilon_+}{m}\right)^2 - \frac{\epsilon_+ + 2m}{k_+} ln\left(\frac{\epsilon_+ + k_+}{m}\right)\right] \tag{2.15}$$

The modified Bessel function of first order has the following property

$$\left(\frac{\omega b}{\gamma}\right) K_1\left(\frac{\omega b}{\gamma}\right) \simeq \begin{matrix} 1, & \text{for} & \frac{\omega b}{\gamma} < 1 \\ 0, & \text{for} & \frac{\omega b}{\gamma} > 1 \end{matrix} \tag{2.16}$$

This implies that the pair production probability decays as $1/b^2$ for impact parameter b larger than the Compton wavelength, i.e. $b > 1/m$, up to a cutoff limit given by $b \simeq \gamma/\omega$. Above this cutoff limit it will decay exponentially. which will guarantee the convergence of the cross section. Indeed. with these simplifications the differential cross section can be easily obtained by using

$$g(\varsigma) = 2\pi \int_{1/m}^{\infty} b\left(\frac{\omega}{\gamma}\right)^2 K_1^2\left(\frac{\omega b}{\gamma}\right) db = \pi\varsigma^2\left[K_0^2 - K_1^2 + \frac{2}{\varsigma}K_0K_1\right]$$
$$\simeq 2\pi\, ln\left(\frac{\delta}{\varsigma}\right) \qquad for \qquad \varsigma << 1 \tag{2.17a}$$

where the Bessel function K_N are functions of the parameter

$$\varsigma = \frac{\omega}{\gamma m} = \frac{\epsilon_+ + m}{\gamma m} \tag{2.17b}$$

and 6 = 0.681... is a number related to the Euler's constant. We can write the results as

$$\frac{d\sigma^K_{e^+e^-}}{d\epsilon_+} = 8Z_1^2Z_2^6\alpha^6r_e^2\frac{1}{(e^{2\pi a_+}-1)}\frac{k_+m^3}{(\epsilon_+ + m)^5}g(\varsigma)$$
$$\times\left[\frac{4}{3} + \frac{2\epsilon_+}{3m} + \left(\frac{\epsilon_+}{m}\right)^2 - \frac{\epsilon_+ + 2m}{k_+}\, ln\left(\frac{\epsilon_+ + k_+}{m}\right)\right] \tag{2.18}$$

where $r,$ = e^2/mc^2 = 2.817...fm is the classical electron radius.

For $\epsilon_+ \simeq m$,

$$\frac{d\sigma^K_{e^+e^-}}{d\epsilon_+} \simeq \frac{2\pi}{3}Z_1^2Z_2^6\alpha^6r_e^2\frac{k_+^3}{m^4}e^{-2\pi a_+}\, ln\left(\frac{\gamma\delta}{2}\right) \tag{2.19}$$

For $\epsilon_+ >> m$,

$$\frac{d\sigma^K_{e^+e^-}}{d\epsilon_+} \simeq 16\pi Z_1^2Z_2^6\alpha^6r_e^2\frac{m}{\epsilon_+^2}\frac{1}{e^{2\pi Z_2\alpha}-1}\, ln\left(\frac{\gamma\delta m}{\epsilon_+}\right) \tag{2.20}$$

In figure 1 we plot eq. (2.18), in units of r_e^2/m, as a function of ϵ_+/m, and for $Z_1 = Z_2 = 8$ and $7 = 100$. Also shown are the low (dashed line) and high (dotted line) positron energy approximations. We observe that the spectrum is strongly suppressed for $\epsilon_+/m \simeq 1$, which is due to the Coulomb repulsion in the field of the nucleus, which prevents creation of low energy positrons. It has a maximum around $\epsilon_+ \simeq 2m$, and decays as in eq.(2.20) after that. Since, as a function of ϵ_+, the differential cross for production of free pairs[2] decays proportionally to $1/\epsilon_+$, the total cross section (integrated over ϵ_+) for pair production with capture, besides an extra factor $(Z_2\alpha)^3$, will increase more slowly as a function of γ than that for production of free pairs.

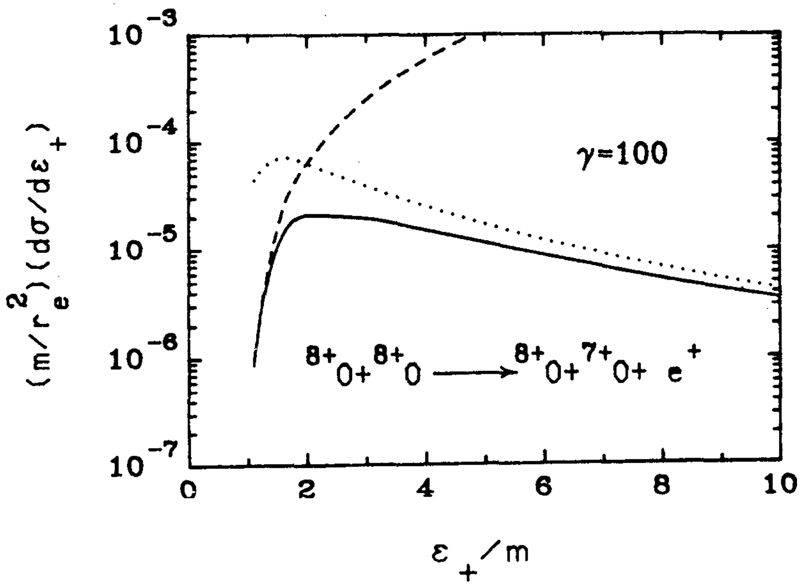

Fig.1 - The differential cross section $d\sigma^K_{e^+e^-}/d\epsilon_+$ in units of r_e^2/m for $Z_1 = Z_2 = 8$ (naked oxygen **ions**), $\gamma = 100$, and as a function of ϵ_+/m. The dotted curve corresponds to the approximation **eq.(2.19)** for low energy positrons. The dashed curve corresponds to the high energy approximation **eq.(2.20)**.

If we integrate **eq.(2.18) numerically** over the positron energy we obtain the **solid** curve as shown **in fig.2.** for $Z_1 = Z_2 = 8$ and as a function of γ. Also shown (dashed line) in that figure **is** the approximate expression

$$\sigma^K_{e^+e^-} \simeq \frac{33\pi}{10} Z_1^2 Z_2^6 \alpha^6 r_e^2 \frac{1}{e^{2\pi Z_2 \alpha} - 1}\left[ln\left(\frac{\gamma\delta}{2}\right) - \frac{5}{3}\right] \qquad (2.21)$$

which can be obtained by setting $k_+ = \epsilon_+$ **overall** in **eq.(2.18)** and integrating it from $\epsilon_+ = 2m$ to ∞. **It will** be a good approximation for the numerical integration of eq. (2.18) for $\gamma \gtrsim 50$.

The ratio of the total cross section for the production of free pairs (see **e.g.**

ref.2) to that for which the electron is captured in a K-shell atomic orbit (which gives the biggest contribution. as we shall see in the next section) is approximately given by

$$\frac{\sigma^{capt}_{e^+e^-}}{\sigma^{free}_{e^+e^-}} \simeq \frac{33\pi}{20}(Z_2\alpha)^3\left[ln\left(\frac{\gamma\delta}{2}\right)\right]^{-2} \tag{2.22}$$

This means that, compared to the production of free pairs, pair production with capture will be more important for ions with larger charges and for lower energies. For $Z_2 = 8$ and $\gamma = 100$, we find $\sigma^{capt}/\sigma^{free} \simeq 10^{-4}$.

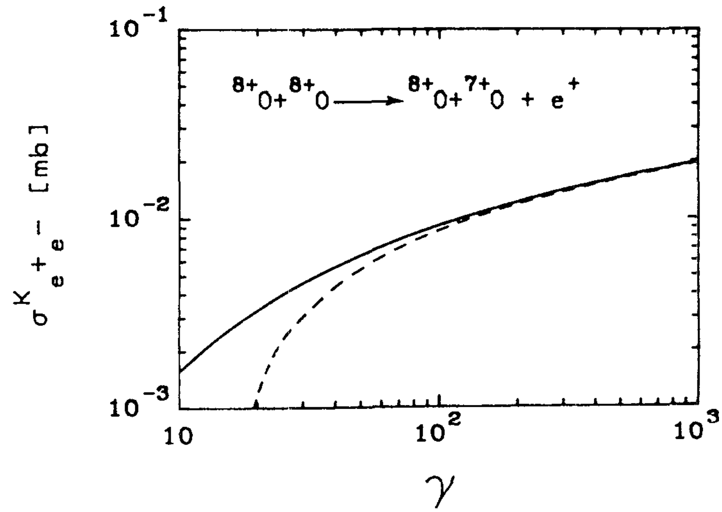

Fig.2 - The cross section $\sigma^K_{e^+e^-}$, for pair production with simultaneous capture of the electron in the K-shell of one the nucleus. in a RHI collision with $Z_1 = Z_2 = 8$ (naked oxygen ions). and as a function of 7.

3. CONTRIBUTION FROM HIGHER ORBITS

In principle, it is possible to calculate the cross section for capture of the electron to higher atomic orbits in a way similar to the K-shell capture. However, since the calculations become more cumbersome and the contributions are much smaller than that for capture in the K-shell, we prefer to use another method which is based on the previous works for the one-photon annihilation of positrons with atomically bound electrons[15-17].

First we mention how some of the results of the previous section can be obtained with the equivalent photon method. In this approach one needs the cross section for the photoproduction process

$$\gamma + Z_2 \rightarrow e^+ + (Z_2 + e^-)_{K,L,..} \tag{3.1}$$

By time reversal. this process is related to the one-photon annihilation process

$$e^+ + (Z_2 + e^-)_{K,L,...} \rightarrow \gamma + Z_2 \tag{3.2}$$

the theoretical cross sections for which are known since the thirties, like e.g. in the calculations of Fermi and Uhlenbeck[16], Nishina, Tomonaga and Tamaki[17], and others (see e.g. ref. 15, p. 463, and references therein). In the extreme relativistic (E.R.). $\epsilon_+ >> m$. and non-relativistic (N.R.). $\epsilon_+ \simeq m$. cases we have[17]

$$\sigma^{ann} = 8\pi^2 Z_2^6 \alpha^5 r_e^2 \begin{cases} \frac{k_+}{3m} \frac{1}{e^{2\pi a_+} - 1} & \text{N.R.} \\ \frac{m}{\epsilon_+} \frac{1}{e^{2\pi Z_2 \alpha} - 1} & \text{E.R.} \end{cases} \tag{3.3}$$

By means of the detailed balance theorem. these cross sections are related to the photoproduction cross sections by

$$\sigma_\gamma^{prod} = 8\pi^2 Z_2^6 \alpha^5 r_e^2 \begin{cases} \frac{k_+^3}{12m^3} \frac{1}{e^{2\pi a_+} - 1} & \text{N.R.} \\ \frac{m}{\epsilon_+} \frac{1}{e^{2\pi Z_2 \alpha} - 1} & \text{E.R.} \end{cases} \tag{3.4}$$

In the equivalent photon method (see e.g. refs. 10 and 13) it is assumed that the processes originated by the time-varying electromagnetic field generated by a relativistic charge are the same as those caused by a plane wave pulse of light containing $n(\omega)/\omega$ photons per unit energy. In this the same process originated by a real photon ω by means of the expression

$$\frac{d\sigma^{RHI}}{d\omega} = \frac{n(\omega)}{\omega} \sigma_\gamma^{prod} \tag{3.5}$$

In the case of pair production $d\omega = d\epsilon_+$ and [13]

$$n(\omega) \simeq \frac{2}{\pi} Z_1^2 \alpha \ln \left(\frac{\gamma \delta m}{\omega} \right) \tag{3.6}$$

Now it is a simple matter to show that inserting eqs.(3.4) in (3.5) and using eq.(3.6) we reproduce the eqs.(2.19) and (2.20).

In ref.17 the one-photon annihilation of L-shell electrons and positrons was also considered. It was found that the dominant contribution comes from the L■ subshell and that the cross section is given by 1/8 of the K-shell cross section. This is reiated to a general scaling law, which also appears in the photo-electric effect (see e.g. ref. 18, p. 303). given by

$$\sigma_{ns} = \frac{\sigma_{1s}}{n^3} \tag{3.7}$$

where the index ns denotes the spherically symmetric atomic subshells of order n. This relationship reflects the behaviour of the bound state wave functions in momentum space at large momenta. Assuming that this behaviour is valid for contribution of all atomic shells, one would obtain an increase of the total capture cross section (into s-orbits. which are the most important). as compared to that for capture in the K-shell, by a factor

$$\sum_{n=1}^{\infty} \frac{1}{n^3} = \xi(3) = 1.202 \tag{3.8}$$

where ξ is the Riemann-ξ-function. This means that eqs. (2.18) and (2.21) should be multiplied by eq.(3.8) if we want to have the contribution of all atornic orbits. which implies in a correction of about 20% for the total cross section.

4. CONCLUSIONS

We studied the electromagnetic cross sections for electron-positron pair production with simultaneous capture of the electron in an atomic shell of the projectile or of the target. We showed that simple analytical formulas can be obtained. by means of reâsonable approximations. This displays very clearly the iniportant features of the process. like the scaling law with the charges Z_1 and Z_2 of the heavy ions. the γ-dependence, and the energy shape of the positron spectrum.

This process could well be crucial for future relativistic colliders: the electron capture process changes the charge state of the circulating ions and leads to a beam-loss in further turns[8]. In a 100 GeV/nucleon uranium collider with a luminosity of $10^{27} cm^{-2} s^{-1}$ one can easily estimate the number of electron captures per second: this energy corresponds to an equivalent laboratory energy of 20400 GeV/nucleon, i.e. a value of $7 \simeq 2 \times 10^4$. From eq.(2.3.11) of ref. 2 we find that the total cross section for the production of free pairs is approximately $o \simeq 60$ kb. This means that approximately 10^8 pairs are produced per second. From eq.(2.21) we find that approximately 10^5 electrons per second will be captured in atomic orbits of the ions in the interaction region of the same beam. As pointed out by Anholt and Gould[8] this may impose an upper limit to the beam energy to be obtained in a RHI collider. or may even be used to control the beam luminosity by measuring the total amount of positrons created per unit time. The capture cross section should also be put into relation with other characteristic cross sections. For example. the nuclear geometric çross section is of order of one barn. i.e. they are a small fraction of the atomic capture cross section.

In principle, it is also possible to produce heavy lepton pairs (see e.g. refs. 2. 19 and 20). like $\mu^+\mu^-$ and $\tau^+\tau^-$ with a capture of the negative lepton. But. due to the much higher rnasses of these leptons, the corresponding cross sections are much smaller[19] and expected to be of minor importance.

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Resumo

Calculamos as probabilidades e seções de choque para a produção de pares de elétron- pósitron em reações relativisticas com núcleos pesados para o caso

em que o elétron é capturado em uma órbita atômica no projétil, ou no alvo. Tal processo possuirá um papel importante nos aceleradores nucleares a altas energias já que ele modifica a carga dos íons, levando a uma perda na luminosidade do feixe.