

# Influence of damping on the excitation of the double giant resonance

G. Baur<sup>1,\*</sup>, C.A. Bertulani<sup>2,\*\*</sup>, D. Dolci<sup>2</sup><sup>1</sup> IKP, Forschungszentrum Jülich, Postfach 1913, 52425 Juelich, Germany<sup>2</sup> Instituto de Física, Universidade Federal do Rio de Janeiro 21945-970 Rio de Janeiro, RJ, Brazil

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**Abstract.** We study the effect of the spreading widths on the excitation probabilities of the double giant dipole resonance. We solve the coupled-channels equations for the excitation of the giant dipole resonance and the double giant dipole resonance. Taking  $Pb+Pb$  collisions as example, we study the resulting effect on the excitation amplitudes, and cross sections as a function of the width of the states and of the bombarding energy.

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## 1 Introduction and theoretical background

Double giant dipole resonances have been mainly studied in heavy ion Coulomb excitation experiments at high energies (for a recent review, see [1]). The feasibility of such experiments has been predicted in 1986 [2,3] where the magnitude of the cross sections for the excitation of the Double Giant Dipole Resonance (DGDR) was calculated (see also [4]). In [3] a recipe was given for treating the effect of the width of the giant resonances on the excitation probabilities and cross sections. In this paper we make a quantitative prediction of this effect using a realistic coupled-channels calculation for the excitation amplitudes. The coupling interaction for the nuclear excitation  $i \rightarrow f$  in a semiclassical calculation for a electric ( $\pi = E$ ), or magnetic ( $\pi = M$ ), multipolarity, is given by (6)–(7) of [5])

$$W_C = \frac{V_C}{\epsilon_0} = \sum_{\pi\lambda\mu} W_{\pi\lambda\mu}(\tau), \quad (1)$$

where

$$W_{\pi\lambda\mu}(\tau) = (-1)^{\lambda+1} \frac{Z_1 e}{\hbar v b^\lambda} \frac{1}{\lambda} \times \sqrt{\frac{2\pi}{(2\lambda+1)!!}} Q_{\pi\lambda\mu}(\xi, \tau) \mathcal{M}(\pi\lambda, \mu). \quad (2)$$

Above,  $b$  is the impact parameter,  $\gamma = (1 - \beta^2)^{-1/2}$ ,  $\beta = v/c$ ,  $\tau = \gamma vt/b$  is a dimensionless time variable,  $\epsilon_0 = \gamma \hbar v/b$  sets the energy scale and  $Q_{\pi\lambda\mu}(\xi, \tau)$ , with  $\xi \equiv \xi_{if} = (E_f - E_i)/\epsilon_0$ , depends exclusively on the properties of the projectile-target relative motion. The multipole operators, which act on the intrinsic degrees of freedom are, as usual,

$$\mathcal{M}(E\lambda, \mu) = \int d^3r \rho(\mathbf{r}) r^\lambda Y_{1\mu}(\mathbf{r}), \quad (3)$$

and

$$\mathcal{M}(M1, \mu) = -\frac{i}{2c} \int d^3r \mathbf{J}(\mathbf{r}) \cdot \mathbf{L}(r Y_{1\mu}), \quad (4)$$

We treat the excitation problem by the method of Alder and Winther [6]. We solve a time-dependent Schrödinger equation for the intrinsic degrees of freedom in which the time dependence arises from the projectile-target motion, approximated by the classical trajectory. For relativistic energies, a straight line trajectory is a good approximation. We expand the wave function in the set  $\{|k\rangle; k = 0, N\}$  of eigenstates of the nuclear Hamiltonian, where 0 denotes the ground state and  $N$  is the number of intrinsic excited states included in the coupled-channels (CC) problem. We obtain a set of coupled equations.

To simplify the expression we introduce the dimensionless parameter  $\Theta_{kj}^{(\lambda\mu)}$  by the relation

$$\Theta_{kj}^{(\lambda\mu)} = (-1)^{\lambda+1} \frac{Z_1 e}{\hbar v b^\lambda} \frac{1}{\lambda} \sqrt{\frac{2\pi}{(2\lambda+1)!!}} \mathcal{M}_{kj}(E\lambda) \quad (5)$$

\* e-mail: G.Baur@fz-juelich.de

\*\* e-mails: bertu@if.ufrj.br and dolci@if.ufrj.br

Then we write the coupled channels equations for the excitation amplitudes  $a_k(\tau)$  in the form [5]

$$\begin{aligned} \frac{da_k(\tau)}{d\tau} = & -i \sum_{j=0}^N \sum_{\pi\lambda\mu} Q_{\pi\lambda\mu}(\xi_{kj}, \tau) \Theta_{kj}^{(\lambda\mu)} \\ & \times \exp(i\xi_{kj}\tau) a_j(\tau). \end{aligned} \quad (6)$$

In what follows we concentrate on the  $E1$  excitation mode. In this case, we have

$$\begin{aligned} Q_{E10}(\xi, \tau) &= \gamma\sqrt{2} \left[ \tau\phi^3(\tau) - i\xi \left(\frac{v}{c}\right)^2 \phi(\tau) \right]; \\ Q_{E1\pm 1}(\xi, \tau) &= \mp\phi^3(\tau), \end{aligned} \quad (7)$$

where  $\phi(\tau) = (1 + \tau^2)^{-1/2}$ .

Following [3] the inclusion of damping leads to the coupled-channels equations with damping terms, i.e.,

$$\begin{aligned} \frac{da_k(\tau)}{d\tau} = & -i \sum_{j=0}^N \sum_{\mu} Q_{E1\mu}(\xi_{kj}, \tau) \Theta_{kj}^{(1\mu)} \\ & \times \exp(i\xi_{kj}\tau) a_j(\tau) - \frac{\Gamma_k}{2\epsilon_0} a_k(\tau). \end{aligned} \quad (8)$$

These equations lead to master equations for the occupation probabilities,  $\tilde{P}_j(\tau) = |a_j(\tau)|^2$ , in the form

$$d\tilde{P}_k(\tau)/d\tau = G_k(\tau) - L_k(\tau) \quad (9)$$

where

$$\begin{aligned} G_k(\tau) &= 2 \Im m \sum_j \sum_{\mu} Q_{E1\mu}(\xi_{kj}, \tau) \Theta_{kj}^{(1\mu)} \\ & \times \exp(i\xi_{kj}\tau) a_k(\tau) a_j^*(\tau) \end{aligned} \quad (10)$$

and

$$L_k(\tau) = -\frac{\Gamma_k}{\epsilon_0} \tilde{P}_k(\tau). \quad (11)$$

These equations can be integrated, yielding the conservation law,

$$\begin{aligned} \sum_k \left( \tilde{P}_k(\tau) + \tilde{F}_k(\tau) \right) &= 1, \quad \text{where} \\ \tilde{F}_k(\tau) &= \frac{\Gamma_k}{\epsilon_0} \int_{-\infty}^{\tau} \tilde{P}_k(\tau') d\tau' \end{aligned} \quad (12)$$

Due to the exponential decay of the states with  $k \geq 1$ , we have for  $t \rightarrow \infty$  the limit  $\tilde{P}_k(\infty) = \delta_{j0} \tilde{P}_0(\infty)$  and

$$\tilde{P}_0(\infty) + \sum_k \tilde{F}_k(\infty) = 1. \quad (13)$$

This means that for  $t \rightarrow \infty$  there is a probability to find the system in the ground state given by  $\tilde{P}_0(\infty)$  and a probability that it has been excited and decayed through the channel  $j$  which is given by  $\tilde{F}_j(\infty)$ . Thus, the set of

equations 8 are shown to correctly describe the contribution to the excitation through channel  $j$ .

The excitation probability of an intrinsic state  $|j\rangle$  in a collision with impact parameter  $b$  is obtained from an average over the initial orientation and a sum over the final orientation of the nucleus, as

$$P_j(b) = \frac{1}{2I_0 + 1} \sum_{M_0, M_j} |\tilde{F}_j(\infty)|^2, \quad (14)$$

and the cross section is obtained by the classical expression

$$\sigma_j = 2\pi \int P_j(b) T(b) b db. \quad (15)$$

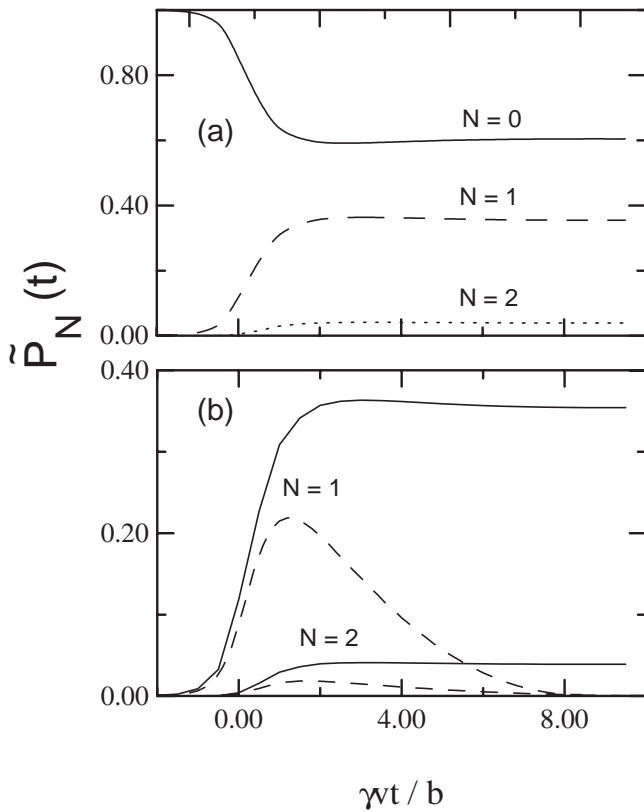
Above,  $T(b)$  accounts for absorption according to the prescription of [5], using the nucleon-nucleon cross sections and the ground state density of  $Pb$  from experimental data.

## 2 Results and discussion

We consider the excitation of giant resonances in  $^{208}\text{Pb}$  projectiles, incident on  $^{208}\text{Pb}$  targets at 640 A·MeV. This reaction has been studied at the GSI/SIS, Darmstadt [7]. For this system the excitation probabilities of the isovector giant dipole ( $GDR$ ) at 13.5 MeV are large and, consequently, higher order effects of channel coupling should be relevant. To assess the importance of the damping effects, we calculate the matrix elements assuming that the  $GDR$  is an isolated state depleting 100% of the energy-weighted sum-rule. The matrix element for the  $GDR \rightarrow DGDR$  transition incorporates the boson factor  $\sqrt{2}$ , as usual [3]. The energy location of the  $DGDR$  state is taken as 27 MeV, consistent with the experimental data. The spin and parities of the states are given by  $1^-$  for the  $GDR$ , and  $0^+$  and  $2^+$  for the  $DGDR$ , respectively. The distribution of the strength among the  $0^+$  and  $2^+$   $DGDR$  states are simply obtained from Clebsch-Gordan coefficients [8].

In Fig. 1 we plot the time-dependent occupation probabilities of the ground state, in  $Pb$ ,  $N = 0$ , of the  $GDR$  state,  $N = 1$ , and of the  $DGDR$  state,  $N = 2$ , respectively. Figure 1(a) shows the occupation probabilities with the widths equal to zero,  $\Gamma_N = 0$ . In Fig. 1(b) we plot the occupation probabilities of the  $GDR$  state,  $N = 1$ , and of the  $DGDR$  state,  $N = 2$ , with  $\Gamma_N = 0$  (full lines), and with  $\Gamma_{GDR} = 4$  MeV (experimental), and  $\Gamma_{DGDR} = 5.7$  MeV (dashed lines). The width of the  $DGDR$  is set to  $\Gamma_{DGDR} = \sqrt{2}\Gamma_{GDR}$ , following the apparent trend of the experimental data [7]. Note, that  $\Gamma_{DGDR} = 2\Gamma_{GDR}$  has a better (and simpler) theoretical explanation [9]. But, the time integrated population of the  $DGDR$  state will not be much influenced by using the later parametrization.

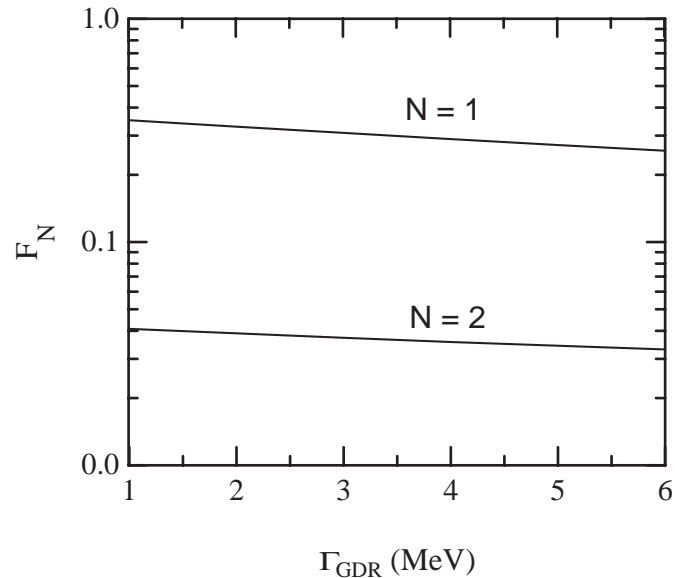
We observe that the inclusion of damping leads to strong modifications in the time-dependent occupation probabilities of the  $GDR$  and  $DGDR$  states. One might wrongly deduce from Fig. 1 that the excitation probabilities are reduced proportionally to the difference between



**Fig. 1.** Occupation probabilities in Coulomb excitation of  $Pb$ , in  $Pb+Pb$  collisions at 640 MeV. A.  $N = 0$  for the ground state,  $N = 1$  for the GDR state,  $N = 2$ , for the DGDR state, respectively. Figure 1(a) shows the occupation probabilities with the widths equal to zero,  $\Gamma_N = 0$ . In Fig. 1(b) we plot the occupation probabilities of the GDR state,  $N = 1$ , and of the DGDR state,  $N = 2$ , with  $\Gamma_N = 0$  (full lines), and with  $\Gamma_{GDR} = 4$  MeV, and  $\Gamma_{DGDR} = 5.7$  MeV (dashed lines). The width of the DGDR is set to  $\Gamma_{DGDR} = \sqrt{2}\Gamma_{GDR}$ , according to the trend of the experimental data [7]

the maximum value of  $\tilde{P}_N(\tau)$ , with and without damping. However, the quantities shown in Fig. 1(b) includes the loss of occupation probabilities of a given state, *while* it is being populated by the time-dependent transitions. Thus, the reduction of the excitation probabilities of the GDR and the DGDR due to damping is smaller than deduced from Fig. 1. The relevant quantity to calculate the excitation cross sections are the quantities  $\tilde{F}_j(\infty)$ , which account for the time-integrated transition probability, followed by decay, of the state  $j$ .

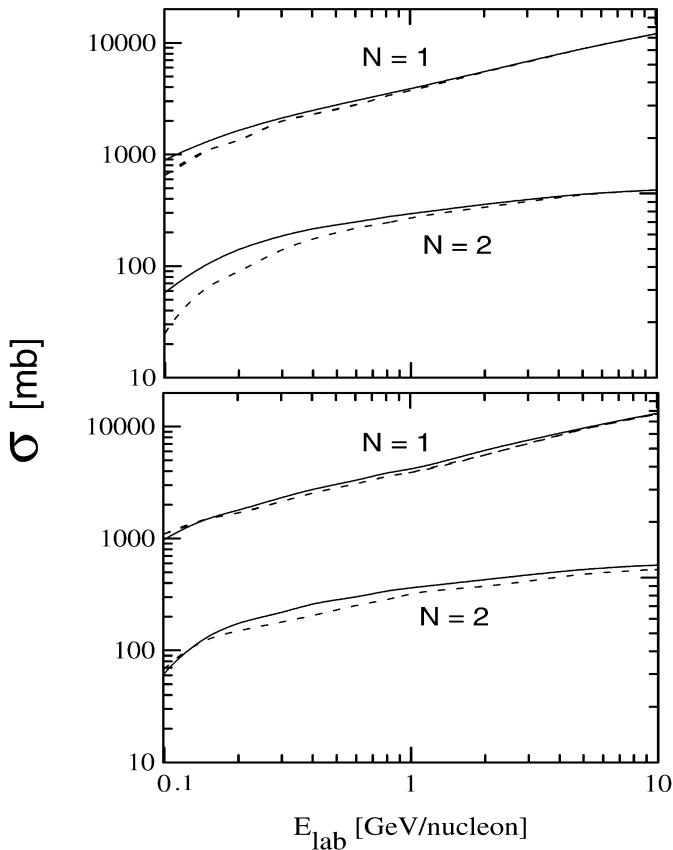
In Fig. 2 we plot the flux functions, or time-integrated transition probabilities to the GDR and the DGDR states as a function of the width of the collective state,  $\Gamma_{GDR}$ , keeping constant the ratio  $\Gamma_{DGDR}/\Gamma_{GDR} = \sqrt{2}$ . We keep the impact parameter fixed,  $b = 15$  fm. We note that, varying the width from 0 up to 4 MeV leads to a 10% decrease of the flux functions into GDR and DGDR states. A similar tendency is observed for the total cross section, integrated over impact parameters. The excitation probability decreases with about  $1/b^4$ , therefore the grazing collisions are weighted most strongly.



**Fig. 2.** Flux functions, or time-integrated transition probabilities to the GDR and the DGDR states in  $Pb$  as a function of the width of the collective state,  $\Gamma_{GDR}$ , keeping constant the ratio  $\Gamma_{DGDR}/\Gamma_{GDR} = \sqrt{2}$ . We keep the impact parameter fixed,  $b = 15$  fm.  $N = 1$  for the GDR state,  $N = 2$ , for the DGDR state

In Fig. 3(a) we plot the effect of damping in the total cross sections, as a function of the bombarding energy. We take  $\Gamma_{GDR} = 0$  (full line) and  $\Gamma_{GDR} = 4$  MeV (dashed line), keeping constant the ratio  $\Gamma_{DGDR}/\Gamma_{GDR} = \sqrt{2}$ . We observe that the effect of damping disappears, as the bombarding energy increases. At high energies the reaction is fast, and the system does not have time to dissipate during its excitation. In this regime the sudden approximation is a valid approach to the calculation of the excitation amplitudes.

We note that our model is restricted to the excitation of isolated resonant states, including a time-dependent loss term on the far right of the coupled-channels equations 8. This is different from a study of the influence of the fragmentation of the resonances into many neighbouring states. In this case, the Coulomb excitation of the giant resonances is obtained as a superposition of excitations to states spread over an energy envelope, usually taken as a Lorentzian shape. States at lower energy are more easily excited than states at higher energies. Thus, the spreading of the resonances may lead to another kind of effect of the widths, not obtainable in the above treatment. To study this effect in a simple way, we use the harmonic model of [10]. The photo-nuclear cross sections which enter in these calculations are of Lorentzian shapes, with a cut at the low energy limit of 8 MeV, corresponding to the threshold for neutron emission, since most of the DGDR manifestation in experiments come from neutron emission after relativistic Coulomb excitation. The magnitude of the photo-nuclear cross sections are obtained by using a 100% depletion of the Thomas-Reiche-Kuhn energy-weighted sum-rule applied to the GDR.



**Fig. 3.** (a) Total cross sections, as a function of the bombarding energy. We take  $\Gamma_{GDR} = 0$  (full line) and  $\Gamma_{GDR} = 4$  MeV (dashed line), keeping constant the ratio  $\Gamma_{DGDR}/\Gamma_{GDR} = \sqrt{2}$ .  $N = 1$  for the GDR state,  $N = 2$ , for the DGDR state. (b) Results of the harmonic model [10] for the Coulomb excitation cross sections of the GDR ( $N = 1$ ) and the DGDR ( $N = 2$ ). The solid lines use  $\Gamma_{GDR} = \Gamma_{DGDR} = 0$ , while the dashed-lines are for  $\Gamma_{DGDR} = 5.7$  MeV and  $\Gamma_{GDR} = 4$  MeV

The harmonic model provides a simple analytical formula to calculate the excitation probabilities of the DGDR [10]. The resulting cross sections are shown in Fig. 3(b) where in the solid line we take  $\Gamma_{GDR} = \Gamma_{DGDR} = 0$ , while the dashed-lines are for  $\Gamma_{DGDR} = 5.7$  MeV and  $\Gamma_{GDR} = 4$  MeV. Except for the very low bombarding energies, we observe a similar effect as in Fig. 3(a). This is understood as a reduction due to the spread of states at energies above the energy centroid of the Lorentzian envelope. The excitation amplitude for these states are smaller, thus leading

to a net reduction of the energy integrated Coulomb excitation cross sections. A similar result has been obtained in [11].

### 3 Conclusion

In conclusion, we have obtained the dependence of the excitation amplitudes on the width of the giant resonance states. We show that the effect reduces excitation probabilities, and cross sections. We have developed an approach to solve this problem in realistic situations. It is demonstrated that the dynamical effect of the widths of the GR's in a time-dependent picture leads to a decrease of the cross sections, more accentuated for low energy collisions. The energy fragmentation of the giant resonances can be studied in a simple fashion within the harmonic model. The net effect is also to decrease the cross sections with increasing width, specially at low energy collisions.

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