Chapter 7

Non-extensive Solution to Cosmological Lithium Problem

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Abstract The standard Big-Bang model predicts the primordial abundances of ^{2}H(D), ^{3}He, ^{4}He in excellent agreement with observations, except for ^{7}Li that is overpredicted by a factor of about three. Despite many attempted solutions to this discrepancy using conventional nuclear physics over the past decades, the lithium enigma persists. Here we present an investigation of Big Bang nucleosynthesis (BBN)

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A. Formicola et al. (eds.), *Nuclei in the Cosmos XV*, Springer
Proceedings in Physics 219, https://doi.org/10.1007/978-3-030-13876-9_7

predictions when we adopt a non-extensive distribution to describe the velocity profile of the nuclides in the primordial plasma. We find excellent agreement between predicted and observed primordial abundances of D, ^{4}He, ^{7}Li for the case of $1.069 \leq q \leq 1.082$, which indicates a possible new solution to the cosmological lithium problem.

7.1 Introduction

Big Bang theory is regarded as one of the most successful explanation for the origin of our Universe. However, the Big Bang theory still face what has turned out to be an intractable issue: Big Bang nucleosynthesis (BBN) predictions and their corresponding astronomical observables are consistent only for abundances of D,^{3}He,^{4}He, while the abundance of ^{7}Li is anamalously overpredicted by most present theories by a factor of about three [1, 2]. Attempts to resolve this discrepancy from the perspective of conventional nuclear physics have been unsuccessful for several decades [3, 4].

In this work we propose a solution to the lithium problem that arises naturally from a simple modification of the velocity distributions of nuclei during the BBN epoch [5]. In the BBN model, the predominant nuclear-physics inputs are thermonuclear reaction rates (derived from cross sections). A key assumption in all thermonuclear rate determinations is that the velocities of nuclei may be described by the classical Maxwell-Boltzmann (MB) distribution [6]. However, this assumption might be violated as non-thermal BBN processes may plausibly take place, for example, as a result of dark matter or stochastic primordial magnetic field (PDF) [7, 8].

As derived from non-extensive statistics [9], the Tsallis distribution can be used to describe particle velocity profile as a deviation from classical MB distribution by introducing a real parameter q and reduces to the nominal MB distribution when non-extensive parameter $q = 1$. In the following sections, we will introduce the derivation of an expression of thermonuclear reaction rates using the Tsallis distribution, and we subsequently investigate the impact of this new distribution on primordial nucleosynthesis.

7.2 Non-extensive Reaction Rate

It is well-known that thermonuclear rate for a typical $1 + 2 \rightarrow 3 + 4$ reaction is usually calculated by folding the cross section $\sigma(E)_{12}$ with a MB distribution [6]

$$\langle \sigma v \rangle_{12} = \sqrt{\frac{8}{\pi \mu_{12} (kT)^3}} \int_0^\infty \sigma(E)_{12} E \exp\left(-\frac{E}{kT}\right) dE, \tag{7.1}$$

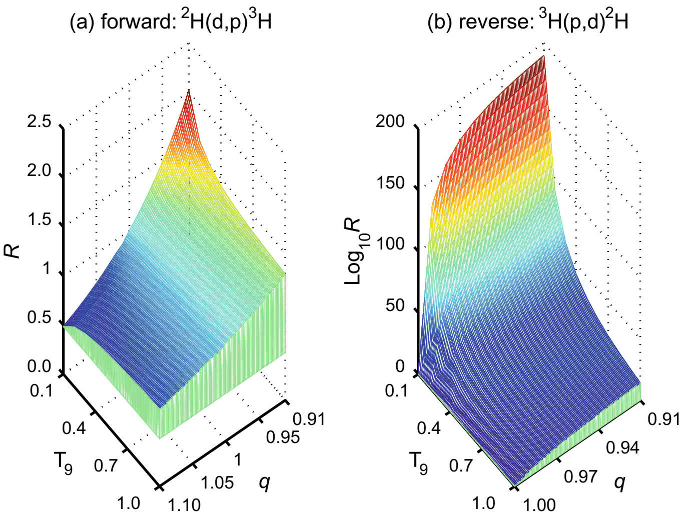

Fig. 7.1 Ratio between rates calculated using Tsallis and MB distributions for the ^{2}H(d,p)^{3}H reaction as functions of temperature T_9 and q values, **a** for forward reaction (in linear scale), and **b** for reverse reaction (in logarithmic scale)

with k the Boltzmann constant, μ_{12} the reduced mass of particles 1 and 2. In Tsallis statistics, the q-Gaussian velocity distribution can be expressed by Silva et al. [10]

$$f_q(\mathbf{v}) = B_q \left(\frac{m}{2\pi kT}\right)^{3/2} \left[1 - (q-1)\frac{m\mathbf{v}^2}{2kT}\right]^{\frac{1}{q-1}}, \tag{7.2}$$

where B_q denotes the q-dependent normalization constant. Thus, the non-extensive reaction rate becomes

$$\langle\sigma v\rangle_{12} = B_q\sqrt{\frac{8}{\pi\mu_{12}}} \times \frac{1}{(kT)^{3/2}} \times \int_0^{E_{\max}} \sigma_{12}(E)E\left[1-(q-1)\frac{E}{kT}\right]^{\frac{1}{q-1}} dE, \tag{7.3}$$

with $E_{\max}=\frac{kT}{q-1}$ for $q>1$, and $+\infty$ for $0< q <1$. The corresponding reverse rate is expressed as the following equation:

$$\langle\sigma v\rangle_{34} = c \times B_q\sqrt{\frac{8}{\pi\mu_{12}}} \times \frac{1}{(kT)^{3/2}} \times \int_0^{E_{\max}-Q} \sigma_{12}(E)E\left[1-(q-1)\frac{E+Q}{kT}\right]^{\frac{1}{q-1}} dE. \tag{7.4}$$

As an example, Fig. 7.1 shows the impact of q values on the forward and reverse rates as functions of temperature for ^{2}H(d,p)^{3}H, one of the most important reactions involved in the BBN. In the region of $0.1 \leq T_9 \leq 1.0$ and $0.91 \leq q \leq 1.1$, the forward rate calculated with the Tsallis-distribution deviates from the MB rates by relatively modest factor of 2 at most. However, the reverse rate is *super*sensitive to deviations of q from unity. For $0.91 \leq q \leq 1$ (i.e., $q < 1$), the corresponding Tsallis reverse rate deviates tremendously from the MB rates by about 200 *orders of magnitude*. For instance, even with a very small deviation ($q = 0.999$), the Tsallis reverse rate of ^{2}H(d,p)^{3}H is about 10^{10} times larger than the MB reverse rate at 0.2 GK. As for the $q > 1$ case, the Tsallis reverse rate is negligible small in comparison to the MB rates.

7.3 BBN Calculation

We have for the first time used a non-extensive velocity distribution to determine thermonuclear reaction rates of primary importance to BBN in a consistent manner, details seen in [5]. Figure 7.2 shows predicted abundances as a function of the non-extensive parameter q along with observed primordial abundances of D, ^{4}He and ^{7}Li, which can be found in [5]. For these species, agreement at the 1σ level is found

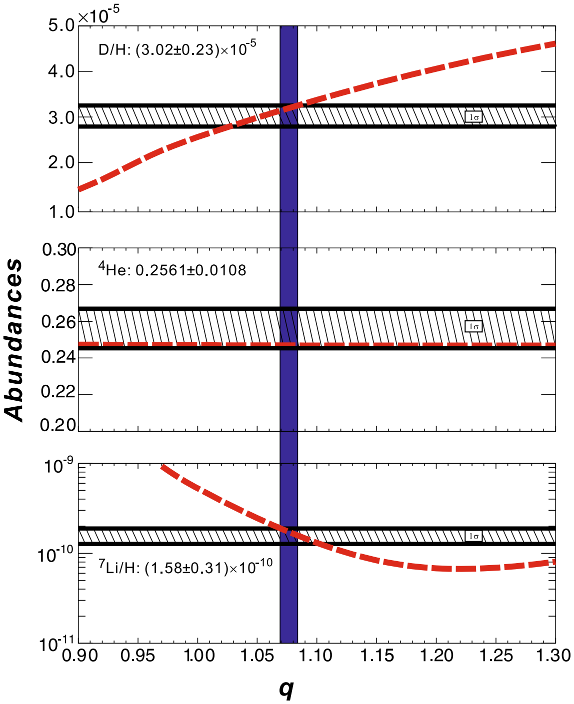

Fig. 7.2 Predicted primordial abundances versus the non-extensive parameter q (in red dashed lines). The observed primordial abundances with 1σ uncertainty for D, ^{4}He, and ^{7}Li are drawn. The vertical (dark blue) band constrains the range of q parameter, i.e., $1.069 \leq q \leq 1.082$, which reconcile the predicted D, ^{4}He, and ^{7}Li abundances with the observed ones

for $1.069 < q < 1.082$ as shown in dark blue band. Thus, we find that deviations from the MB distribution of baryon velocities during BBN offer a new solution to the cosmological lithium problem. This solution might be suited for scenario where stochastic PMF fluctuations is taken into account [8]. We encourage extensions of the present study to astrophysical sites of higher density to further interrogate and test the usual assumptions of classical statistics.

Acknowledgements This work was financially supported by the Major State Basic Research Development Program of China (2016YFA0400503) and the the National Natural Science Foundation of China (Nos. 11705244, 11490562, 11135005, 11321064).

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