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Stellar oscillations induced by the passage of a fast stellar object

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In this paper, we investigate induced oscillations by the gravitational field of a fast stellar object, such as a neutron star or a black-hole in a near miss collision with another star. Nonadiabatic collision conditions may lead to large amplitude oscillations in the star. We show that for a solar-type star a resonant condition can be achieved by a fast moving stellar object with velocity in the range of 100 to 1000 km/s, passing at a distance of a few multiples of the star radius. Although such collisions are rare, they are more frequent than head-on collisions, and their effects could be observed through a visible change of the star luminosity occurring within a few hours.

1. Introduction

Stellar oscillations, commonly known as pulsations, are understood in terms of modulation geared by the interaction of radiation with matter on its way from the center of the star.[1] Little is known about how other kinds of oscillations can be generated in stellar encounters, although an effort has been made in studying tidal oscillations due to the gravitational interaction with a companion in a binary system (see e.g. Refs. 2–4). Stellar encounters are often investigated in the context of gravitational waves which could, in principle, be detected by ground and space-based laser interferometers.[5] Gamma-ray bursts arising from tidal disruption of neutron stars (NS) in NS–NS or NS–Black-hole (BH) binary coalescence have also attracted interest.[6–10] Stellar oscillations excited by the tidal field of a compact object in binary systems have been the subject of several publications.[11–15]

The kinetic energy of a star with $m_\odot$ and velocity $v = 1000$ km/s is about 10^{49} ergs, which is enough energy to power the luminosity of the Sun for one billion years.

A central collision between such objects would be nothing less than spectacular. But if only a small fraction of this energy is transformed into the internal energy of a star during a relatively short time, the consequences would also be dramatic. This can be achieved in near miss encounter, with impact parameters larger than the sum of the radii of the collisional partners. There is plenty of available space for near misses, but very little room for central collisions and far collisions are much more frequent. It is therefore relevant to find out what are their consequences. Such processes have been previously studied for the specific purpose of assessing tidal oscillations in NS.[2,16] Here we show that a resonant condition arises for solar-like stars within the range of possible velocities, leading to effects within our observational reach.

2. Collision Model

We consider the internal response of a star due to the passage of a fast stellar object (FSO), e.g. an NS or a BH, at large impact parameters. This response can be modeled by considering the tidal force on a mass element dm_s of the star, roughly given by[17,18] $dF \sim 2GM dm_s x/R^3$, where M is the mass of the FSO, G the gravitational constant, R is the distance between the center of mass of the FSO and the star, and x is the distance of the mass element dm_s from the center of the star. The tidal force is best described by expanding the gravitational field of the FSO into multipoles, yielding at a position $\mathbf{x}$ inside the star[19]

$$V(\mathbf{x}, t) = -GM \sum_{lm} \frac{4\pi}{2l+1} Y_{lm}(\hat{\mathbf{R}}(t)) \frac{x^l}{R^{l+1}(t)} Y_{lm}^*(\hat{\mathbf{x}}). \tag{1}$$

In the center of mass of the star, the force on a mass element at $\mathbf{x}$ is obtained from the derivative of Eq. (1) with respect to x. The distance $R(t)$ is a function of time, and $t = 0$ is taken when the two stars are at the periapsis, or distance of closest approach. In the frame of reference of the star, the tidal force acts in opposite sides from its center, trying to elongate it and leading to a time-dependent ellipsoidal shaped oscillation. To lowest order, the passage of a FSO will induce quadrupole shaped vibrations, as seen in Fig. 1. Higher multipole vibrations such as octupole oscillations are also possible, but are orders of magnitude smaller and have been neglected here.

The time dependence of $R(t)$ and $\theta(t)$ is described by a hyperbolic Kepler trajectory, parametrized by an orbital eccentricity $\epsilon > 1$, where $\theta(t)$ is the angular position of the FSO measured from the center of mass of the system and with respect to the line joining it to the star so that, at $t = 0$, $\theta = 0$, and $R = a$, the distance at the periapsis. For an encounter with impact parameter b, one has $a = 2b/(\alpha + \sqrt{\alpha^2 + 4})$ where $\alpha = GM m_s/(Eb)$, $E = \mu v^2/2$ is the collision energy and $\mu = M m_s/(m_s + M)$ is the reduced mass. The relation between the angular position and time can be obtained solving coupled equations for R and t along the trajectory (see e.g. Ref. 20).

Nonradial stellar oscillation modes can be described with hydrodynamical models to high accuracy (see e.g. Ref. 21). We adopt a simple model including only (a) a
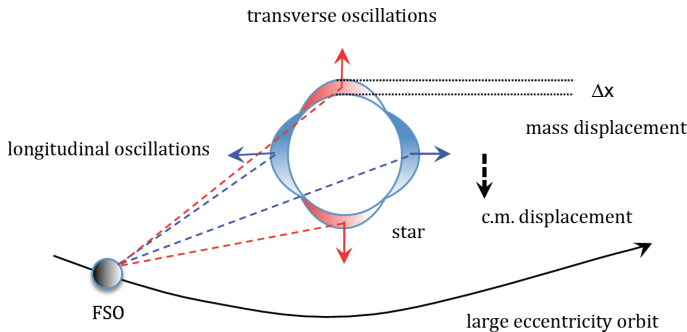
Fig. 1.   (Color online) Tidal oscillations induced in a star by the passage of a fast stellar object.

single inertia parameter, (b) a linear restoring force, and (c) a damping parameter. The model should be useful for a rough estimate of the induced oscillations. The simplest model of this kind was developed by Lord Kelvin, described, e.g. in Refs. 2, 22–25. For quadrupole oscillations in a spherically homogeneous self-gravitating star with radius r_s, mass m_s, and average density $\rho_0 = m_s/(4\pi r_s^3/3)$, the assumption of an incompressible fluid yields a natural oscillation frequency

$$\omega_0^2 = \frac{\mathcal{K}_2}{\mathcal{M}_2} = \frac{16\pi G\rho_0}{15} = \frac{4Gm_s}{5r_s^3},\tag{2}$$

where $\mathcal{M}_2$ and $\mathcal{K}_2$ are the respective quadrupole inertia and stiffness parameters. For small amplitude quadrupole oscillations, the inertia parameter has been deduced in Ref. 26 from which one also obtains the stiffness parameters, namely

$$\mathcal{M}_2 = \frac{3m_s r_s^2}{10} \quad \text{and} \quad \mathcal{K}_2 = \frac{6Gm_s^2}{25r_s}.\tag{3}$$

The stiffness arises from increase of gravitational energy due to the quadrupole deformation from a spherical star shape. Stellar oscillation damping is difficult to model as it can arise from "gas", "radiation" and "turbulence" contributions, each of them varying wildly over temperature, density, and other properties of the stellar interior. For a gas the viscosity varies as $\gamma_g \sim T^{5/2}$, whereas for radiation $\gamma_r \sim T^4$. In the presence of turbulence, $\gamma_t = R_e\gamma_r/3$, where R_e is the Reynolds number and the viscosity is several orders of magnitude larger than the radiative (or Jeans) viscosity.[27] In the absence of turbulence, radiation damping dominates over gas viscosity. To avoid dealing with specific stellar conditions, we assume a friction coefficient of the form $\gamma = A_\gamma \mathcal{M}_2\omega_0$, with A_γ taken as a free parameter.

The velocity distribution of nearby stars ($\lesssim 100\,\text{pc}$), obtained with the Hypparcos satellite, shows a non-negligible number of stars moving at speeds in excess of $100\,\text{km/s}$.[28] Hypervelocity stars, with $v \gtrsim 1000\,\text{km/s}$ are rare, and able to escape the galaxy, but have already been observed.[29] To maximize the effect, we are looking after we consider an FSO moving at a high speed, $v = 1000\,\text{km/s}$, relative to the star. For an encounter with an impact parameter b, the "collision time", i.e. the time during which the gravitational force is most effective, is $t_{\text{coll}} \sim b/v$. For

an encounter with $b = 5r_\odot$ and $v = 1000\,\text{km/s}$, one gets $t_{\text{coll}} \sim 1\,\text{h}$. The period of oscillations associated with Eq. (2) for a ST star is $t_{\text{osc}} = 2\pi/\omega_0 \sim 3$ h. Hence, we expect a resonating response of oscillations in this system for impact parameters in the range of a few times r_s.

The stellar oscillations can be disentangled into a mixture of transverse and lon-gitudinal oscillations, as displayed in Fig. 1. For collisions with impact parameters equal to $5r_s$ and larger and for velocities $v \sim 1000\,\text{km/s}$, the orbital eccentricity is large for ST stars, and the hyperbolic orbits become nearly straight lines. There-fore, we can safely consider transverse (t) and longitudinal (l) oscillations as being those transverse and along the asymptotic velocity, respectively. The equations of motion for small forced harmonic oscillations can be derived from Eq. (1) in terms of the inertia and stiffness parameters of Eqs. (2) and (3), yielding

$$f_i(t) = \ddot{x}_i(t) + \beta\dot{x}_i(t) + \omega_0^2 x_i(t), \quad i = t, l, \tag{4}$$

along the two directions, where $f_i(t)$ is the driving tidal force per unit mass, and $\beta = A_\gamma\omega_0$. For a straight line trajectory ($R^2 = b^2 + v^2 t^2$) with no coupling among the orthogonal oscillations, this problem is solvable in analytical form. For hyperbolic trajectories with large eccentricities, our simulations show that accurate results can be obtained replacing the impact parameter b by $b' = a$ in the analytical solutions below, with a equal to the distance of closest approach at the periapsis.

The solution of Eq. (4) is expressed in terms of the Fourier transform $x_i(t) = (2\pi)^{-1/2}\int \tilde{x}_i(\omega)\exp(i\omega t)d\omega$. For a straight line trajectory with effective impact parameter a, the amplitudes $\tilde{x}_i(\omega)$ are given by

$$\tilde{x}_t(\omega) = \left(\frac{8}{\pi}\right)^{1/2}\frac{GM}{av}\frac{\xi K_1(\xi)}{[(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2]^{1/2}}, \tag{5}$$

where K_1 is the first order modified Bessel function, and

$$\tilde{x}_l(\omega) = i\left(\frac{8}{\pi}\right)^{1/2}\frac{GM}{av}\frac{\xi K_0(\xi)}{[(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2]^{1/2}}, \tag{6}$$

with K_0 the corresponding zeroth order modified Bessel function. The "adiabacity" parameter $\xi = \omega a/v$ measures the degree to which the star responds adiabatically to the driving tidal force. The function $\xi K_1(\xi)$ is nearly constant ($K_1 \sim 1/\xi$) for $\xi < 1$, and decays exponentially $K_1 \sim \exp(-\xi)$ for $\xi > 1$. Hence, oscillation modes with frequencies up to $\omega \sim v/a$ will be preferred and those with larger frequencies will be suppressed exponentially.

3. Results and Discussion

In Fig. 2, we plot the amplitudes $|\tilde{x}_i(\omega)|^2$ (multiplied by $10^{-22}\,\text{m}^2\text{s}^2$) as a function of frequency, for longitudinal (dashed line) and transverse (solid line) oscillations. We used $M = 2m_\odot$, $b = 5r_\odot$, $v = 1000$ km/s, $m_s = m_\odot$, $r_s = r_\odot$, and $A_\gamma = 0.1$. Radii and masses are taken in units of the solar mass, $m_\odot$, and radius, $r_\odot$, respectively. We
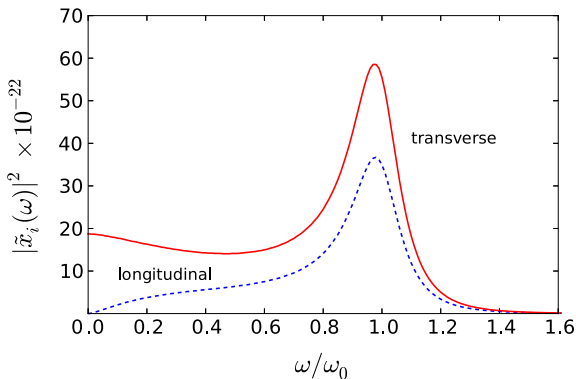
Fig. 2. (Color online) The amplitudes $|\tilde{x}_i(\omega)|^2$ multiplied by 10^{-22} (m^2s^2 units) as a function of the ratio of the frequency ω and the natural oscillation frequency ω_0, for longitudinal (dashed line) and transverse (solid line) oscillations.

observe a remarkable resonant condition for these choice of parameters. As expected, the resonance peak decreases as the stellar viscosity increases (increasing A_γ). The resonance peak also decreases with the stellar radius because the star gets softer as the radius decreases, if its mass is kept constant. In this case the natural oscillation frequency ω_0 becomes large and the resonance matching condition $b/v \sim 1/\omega_0$ does not take place, except for very high FSO velocities, beyond reasonable expectations from present observations. Keeping the same parameters above but varying r_s, we conclude that for white dwarfs (WD) ($r_s \sim r_\odot/10^2$) and NS ($r_s \sim r_\odot/10^5$) the natural oscillation frequency is too high to match the resonant condition.

Note that the collision mechanism discussed here is different than the stellar tidal disruption or breakup in a head-on collision binary coalescence of either a BH or a NS with another NS,[6] or those induced in mergers in binary systems.[31–34] A distant encounter with a FSO (unless its mass is very large) is unable to yield a tidal disruption of either a WD or a NS, unless maybe for very small impact parameters (see below and also Ref. 16).

In Fig. 3, we show the time-dependent oscillation displacements from equilibrium in units of the stellar radius with the same parameters used in Fig. 2. At the periapsis the oscillation amplitudes can reach 10% of the star radius. This is a large amplitude oscillation, unprecedented by any known observation. Evidently, for large amplitudes one expects a nonlinear behavior of the oscillations, requiring a more sophisticated model than adopted here. The stellar oscillations start well before the FSO reaches the periapsis ($t = 0$) and are largest at $t = 0$. The results displayed in Fig. 3 are close to resonance. An even larger effect would be obtained for a grazing impact parameter, when the stars nearly touch each other at the periapsis. As expected, induced longitudinal oscillations are smaller than transverse ones, but not by much. The difference between oscillations along the two directions increases for conditions off the resonance region. By increasing the star radius by
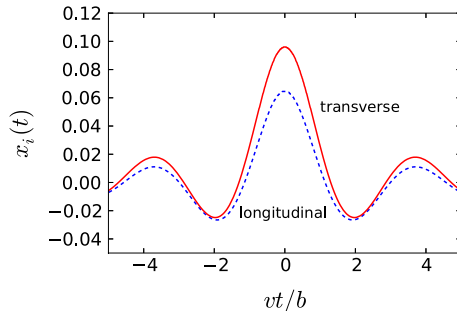
Fig. 3. (Color online) Oscillation amplitudes in units of the stellar radius as a function of time. The same parameters were used as in Fig. 2.

a factor of 10, resonance conditions can be achieved even for a distant collision, $b = 100 r_s$ and larger.

The energy transferred to the stellar oscillations can be obtained from $\Delta E = \sum_{j=l,t} \int_{-\infty}^{\infty} F_j(t) \dot{x}_j(t)$ or $\Delta E = -2\mathcal{R}e \sum_j \int_0^{\infty} i\omega \tilde{x}_j(\omega) \tilde{F}_j^*(\omega) d\omega$. The momentum transferred to the recoil, or center of mass motion, of the star is approximately given by $\Delta p_{cm} = 2 G M m_s / bv$ and the recoil energy by $\Delta E_{cm} = (\Delta p)^2_{cm} / 2 m_s$. For an encounter with $M = 2 m_\odot$, $b = 5 r_\odot$, $v = 1000\,\text{km/s}$, $m_s = m_\odot$, $r_s = r_\odot$, we get $\Delta E_{cm} = 5.5 \times 10^{46}$ ergs, e.g. 0.3% of the FSO bombarding energy is transferred to recoil. A much smaller energy is transferred to tidal oscillations. Using $A_\gamma = 0.1$, one obtains 0.49×10^{41} ergs and 1.6×10^{41} ergs transferred to longitudinal and transverse stellar oscillations, respectively. This is larger than energies emitted in X-ray bursts from accretion in binary systems. However, this energy is transferred to the star (and possibly released in form of radiation) in a much larger time scale: A few hours instead of seconds as in X-ray bursts. Only a fraction of 2.8×10^{-6} of the recoil energy goes into internal excitation of the star. But assuming this star radiates all this energy in form of light with the sun's luminosity $(3.9 \times 10^{26}\,\text{W})$, it would be enough for 1.3 years of steady solar luminosity. An appreciable amount of this energy may be emitted in long wavelength radiation of long duration, i.e. within few hours. The characteristics of this radiation depend on many intrinsic stellar properties.

Figure 4 shows the energy in ergs transferred to longitudinal (dashed line) and transverse (solid line) oscillations in a solar mass star as a function of the impact parameter in units of the star radius. We use the same parameters for M, r_s, m_s and A_γ as in Fig. 2. The thin lines show the results obtained with exact hyperbolic trajectories. Only for small impact parameters there is a visible deviation from the results using straight-line trajectories with recoil correction. The inset shows that the ratio between the two energies increases in the same impact parameter range. For large b the longitudinal contribution becomes as relevant as the transverse one.

The resonant conditions for induced oscillations by a FSO are ideal for ST stars. But it is worthwhile to investigate what happens in the case of an NS or a WD.
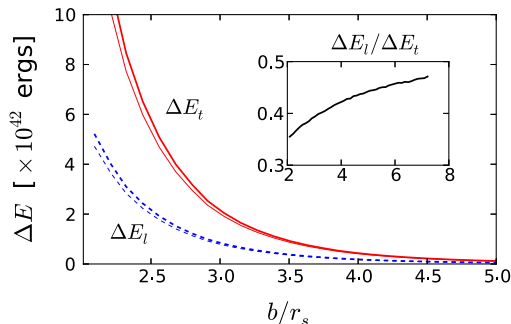
Fig. 4. (Color online) Energy, in ergs, transferred to oscillations in a solar-type star along (l: longitudinal) and perpendicular (t: transverse) the direction of incidence of a fast stellar object as a function of the impact parameter and in units of the star radius. The thin lines represent calculations obtained with parametrized hyperbolic trajectories. The inset shows the ratio between the two transferred energies as a function of the same impact parameter measure.

Table 1. The longitudinal and transverse energy transferred to a NS, WD and a solar-type (ST) star, all with masses $m_s = 1.4 m_\odot$, due to a collision with a fast stellar object with mass $M = 2 m_\odot$ passing by an impact parameter $b = 5 r_\odot$. The first column lists the assumed radius for the star. The last column gives the maximum tidal displacement in units of the assumed stellar radius.

r_s (km)	ΔE_l (ergs)	ΔE_t (ergs)	$x_{\max}/r_s$
10 (NS)	1.09×10^2	2.09×10^2	0.986×10^{-22}
7×10^3 (WD)	1.76×10^{32}	1.88×10^{32}	1.07×10^{-5}
7×10^5 (ST)	0.324×10^{42}	0.608×10^{42}	0.128

In Table 1, we show the longitudinal and transverse energy transferred to a NS, WD and a ST star, all with masses $m_s = 1.4 m_\odot$, due to a collision with a fast stellar object with mass $M = 2 m_\odot$ passing by an impact parameter $b = 5 r_\odot$. The first column lists the assumed radius for the star. The last column gives the maximum tidal displacement in units of the assumed stellar radius. One observes a dramatic change in the energy transfer due to the smaller star size in contrast to a ST star. For NS the energy transfer is negligible. The larger stiffness of a compact star corresponds to a large natural frequency, thus quenching the aforementioned resonant condition.

The results in Table 1 are for $b = 5 r_\odot$. But compact stars also allow closer encounters if the FSO is a WD, a NS, or a BH. Table 2 shows the same as in Table 1, but for closer encounters of the FSO with a WD and a NS. The collision impact parameter b is measured in units of 5 times the WD (rows 2 and 3) radius, or 5 times the NS radius (row 4). In these cases, the trajectories are significantly modified by the gravitational attraction and we solve Eq. (1) parametrized by a hyperbolic trajectory. A close encounter of a FSO and a NS might require the solution of general relativity equations for the trajectory, which we do not consider. Our results show that the energy emitted over a few hours is well below those of

Table 2.   Same as in Table 1, but for closer encounters of the FSO with an WD and a NS. The collision impact parameter b is measured in units of 5 times the WD (rows 2 and 3) radius, or 5 times the NS radius (row 4).

b	r_s (km)	ΔE_l (ergs)	ΔE_t (ergs)	x_{max}/r_s
$5r_{WD}$	10 (NS)	4.56×10^{27}	1.92×10^{28}	4.17×10^{-6}
$5r_{WD}$	7×10^3 (WD)	1.75×10^{43}	3.28×10^{43}	0.101
$5r_{NS}$	10 (NS)	1.19×10^{46}	2.27×10^{46}	0.102

known cosmic cataclysmic events, such as gamma-ray bursts,[33] but not worthless more investigation.

In the case of NS–NS collisions, our calculated energy transfer is $\sim 10^{46}$ ergs at $b = 5r_{NS}$. Note that we do not explore the equation of state of nuclear matter, relying solely on the physics of an incompressible fluid. According to Ref. 34, this energy would induce high frequency seismic oscillations in the NS which can couple to the magnetic field and spark a particle fireball burst. For solar-like stars and WDs, a close encounter with an ultrafast and ultramassive FSO can lead to *stellar fission*, similar to those occurring in a stretched water droplet. Although rare, such phenomena would be amenable to observation.

Acknowledgments

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