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**LETTER TO THE EDITOR**

# Density dependence of in-medium nucleon–nucleon cross sections

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**Abstract**
The lowest-order correction of the density dependence of in-medium nucleon–nucleon cross sections is obtained from geometrical considerations of Pauli-blocking effects. As a by-product, it is shown that the medium corrections imply an $1/E$ energy dependence of the density-dependent term.

The nucleon–nucleon cross section is a fundamental input in theoretical calculations of nucleus–nucleus collisions at intermediate and high energies ($E/A \gtrsim 100$ MeV). One expects to obtain information about the nuclear equation of state by studying global collective variables in such collisions (see, e.g., [1]). Transport equations, such as the BUU equation, are often used as tools for the analysis of experimental data and as a bridge to the information concerning the equation of state (see, e.g., [2]). The nucleon–nucleon cross sections are building blocks in these transport equations.

In previous theoretical studies of heavy-ion collisions at intermediate energies ($E/A \simeq 100$ MeV) the nucleon–nucleon cross section was multiplied with a constant scaling factor to account for in-medium corrections [3, 4]. As pointed out in [2], this approach fails in low-density nuclear matter where the in-medium cross section should approach its free-space value. A more realistic approach uses a Taylor expansion of the in-medium cross section in the density variable. One obtains [5]

$$\sigma_{NN} = \sigma_{NN}^{free}\,(1 + \alpha\bar{\rho}) \tag{1}$$

where $\bar{\rho} = \rho/\rho_0$, $\rho_0$ is the normal nuclear density and $\alpha$ is the logarithmic derivative of the in-medium cross section with respect to the density, taken at $\rho = 0$,

$$\alpha = \rho_0 \frac{\partial}{\partial\rho}\,(\ln\sigma_{NN})\,|_{\rho=0}. \tag{2}$$

This parametrization is motivated by Brückner $G$-matrix theory and is basically due to Pauli-blocking of the cross section for collisions at intermediate energies [6]. Values of $\alpha$ between $-0.4$ and $-0.2$ yield the best agreement with involved $G$-matrix calculations using realistic nucleon–nucleon interactions [6].

  

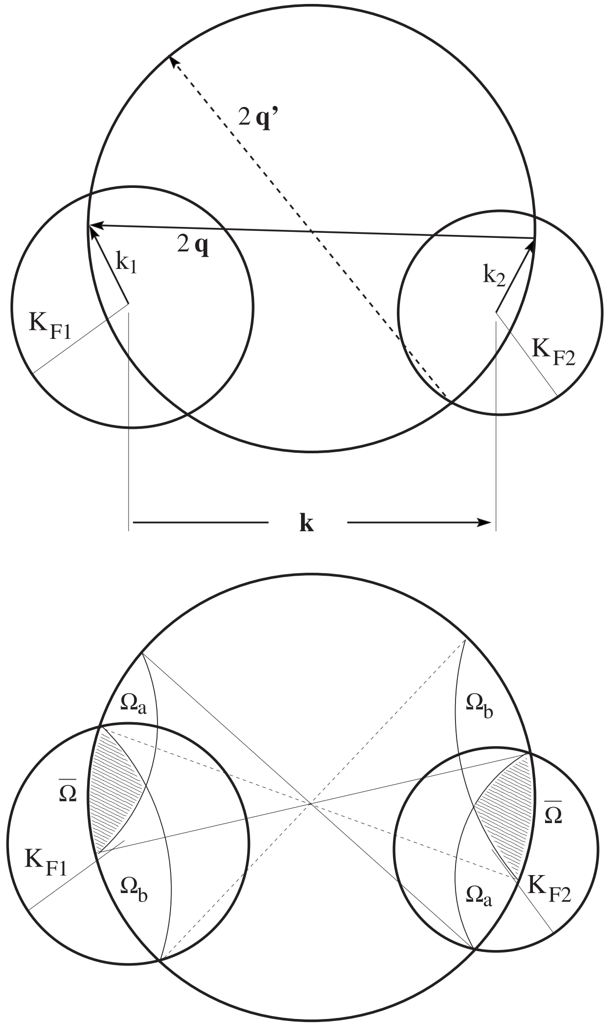


**Figure 1.** The geometric description of Pauli blocking, in momentum space, for binary collisions of target and projectile nucleons.

In this paper we give a simple and transparent derivation of the lowest-order expansion of the in-medium nucleon–nucleon cross section in terms of the nucleon density. In our approach the leading term of the expansion turns out to be $\alpha'\rho^{2/3}$ with $\alpha'$ proportional to $1/E$. This energy dependence agrees with experimental results on total nucleus–nucleus cross sections.

We adopt the idea that the main effect of medium corrections is due to the Pauli-blocking of nucleon–nucleon scattering. Pauli-blocking prevents the nucleons from scattering into final occupied states in binary collisions between the projectile and target nucleons. This is best seen in momentum space, as shown in figure 1. We see that energy and momentum conservation, together with the Pauli principle, restrict the collision phase space to a complex geometry involving the Fermi-spheres and the scattering sphere. In this scenario, the in-medium cross section corrected by Pauli-blocking can be defined as

$$\sigma_{NN}(k, K_{F1}, K_{F2}) = \int \frac{\mathrm{d}^3 k_1\, \mathrm{d}^3 k_2}{(4\pi K_{F1}^3/3)(4\pi K_{F2}^3/3)} \frac{2q}{k} \sigma_{NN}^{free}(q) \frac{\Omega_{Pauli}}{4\pi} \tag{3}$$

where $k$ is the relative momentum per nucleon of the nucleus–nucleus collision (see figure 1), and $\sigma_{NN}^{free}(q)$ is the free nucleon–nucleon cross section for the relative momentum $2\boldsymbol{q} = \boldsymbol{k}_1 - \boldsymbol{k}_2 - \boldsymbol{k}$, of a given pair of colliding nucleons. Clearly, Pauli-blocking enters through the restriction that $|\boldsymbol{k}_1'|$ and $|\boldsymbol{k}_2'|$ lie outside the Fermi spheres. From energy and momentum conservation in the collision, $\boldsymbol{q}'$ is a vector which can only rotate around a circle with centre at $\boldsymbol{p} = (\boldsymbol{k}_1 - \boldsymbol{k}_2 - \boldsymbol{k})/2$. These conditions yield an allowed scattering solid angle given by [7]

$$\Omega_{Pauli} = 4\pi - 2(\Omega_a + \Omega_b - \bar{\Omega}) \tag{4}$$

where $\Omega_a$ and $\Omega_b$ specify the excluded solid angles for each nucleon, and $\bar{\Omega}$ represents the intersection angle of $\Omega_a$ and $\Omega_b$ (see figure 1).

The solid angles $\Omega_a$ and $\Omega_b$ are easily determined. They are given by

$$\Omega_a = 2\pi(1 - \cos\theta_a) \qquad \Omega_b = 2\pi(1 - \cos\theta_b) \tag{5}$$

where $\boldsymbol{q}$ and $\boldsymbol{p}$ were defined above, $\boldsymbol{b} = \boldsymbol{k} - \boldsymbol{p}$, and

$$\cos\theta_a = (p^2 + q^2 - K_{F1}^2)/2pq \qquad \cos\theta_b = (p^2 + q^2 - K_{F2}^2)/2bq. \tag{6}$$

The evaluation of $\bar{\Omega}$ is tedious but can be done analytically. The full calculation was done in [7] and the results have been reproduced in the appendix of [8]. To summarize, there are two possibilities:

(1) $$\bar{\Omega} = \Omega_i(\theta, \theta_a, \theta_b) + \Omega_i(\pi - \theta, \theta_a, \theta_b) \qquad \text{for} \quad \theta + \theta_a + \theta_b > \pi \tag{7}$$

(2) $$\bar{\Omega} = \Omega_i(\theta, \theta_a, \theta_b) \qquad \text{for} \quad \theta + \theta_a + \theta_b \leqslant \pi \tag{8}$$

where $\theta$ is given by

$$\cos\theta = (k^2 - p^2 - b^2)/2pb. \tag{9}$$

The solid angle $\Omega_i$ has the following values:

(a) $$\Omega_i = 0 \qquad \text{for} \quad \theta \geqslant \theta_a + \theta_b \tag{10}$$

(b) $$\begin{aligned} \Omega_i = 2\Bigg[ & \cos^{-1}\left(\frac{\cos\theta_b - \cos\theta\cos\theta_a}{\sin\theta_a(\cos^2\theta_a + \cos^2\theta_b - 2\cos\theta\cos\theta_a\cos\theta_b)^{1/2}}\right) \\ & + \cos^{-1}\left(\frac{\cos\theta_a - \cos\theta\cos\theta_b}{\sin\theta_b(\cos^2\theta_a + \cos^2\theta_b - 2\cos\theta\cos\theta_a\cos\theta_b)^{1/2}}\right) \\ & - \cos\theta_a \cos^{-1}\left(\frac{\cos\theta_b - \cos\theta\cos\theta_a}{\sin\theta\sin\theta_a}\right) \\ & - \cos\theta_b \cos^{-1}\left(\frac{\cos\theta_a - \cos\theta\cos\theta_b}{\sin\theta\sin\theta_b}\right)\Bigg] \end{aligned} \tag{11}$$

for

$$|\theta_b - \theta_a| \leqslant \theta \leqslant \theta_a + \theta_b \tag{12}$$

(c) $$\Omega_i = \Omega_b \qquad \text{for} \quad \theta_b \leqslant \theta_a \quad \theta \leqslant |\theta_b - \theta_a| \tag{13}$$

(d) $$\Omega_i = \Omega_a \qquad \text{for} \quad \theta_a \leqslant \theta_b \quad \theta \leqslant |\theta_b - \theta_a|. \tag{14}$$

The integrals over $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ in (3) reduce to a fivefold integral due to cylindrical symmetry. Two approximations can be done which greatly simplify the problem: (a) on average, the symmetric situation in which $K_{F1} = K_{F2} \equiv K_F$, $\boldsymbol{q} = \boldsymbol{k}/2$, $\boldsymbol{p} = \boldsymbol{k}/2$ and $\boldsymbol{b} = \boldsymbol{k}/2$, is favoured; (b) the free nucleon–nucleon cross section can be taken outside of the integral in equation (3). Both approximations are supported by the studies of [8] and can be verified numerically [7]. The assumption (a) implies that $\Omega_a = \Omega_b = \bar{\Omega}$, which can be checked using equation (14). One obtains from (4) the simple expression

$$\Omega_{Pauli} = 4\pi - 2\Omega_a = 4\pi \left(1 - 2\frac{K_F^2}{k^2}\right). \tag{15}$$

Furthermore, assumption (b) implies that

$$\sigma_{NN}(k, K_F) = \sigma_{NN}^{free}(k)\,\frac{\Omega_{Pauli}}{4\pi} = \sigma_{NN}^{free}(k)\left(1 - 2\frac{K_F^2}{k^2}\right). \tag{16}$$

The above equation shows that the in-medium nucleon–nucleon cross section is about $\frac{1}{2}$ of its free value for $k = 2K_F$, i.e. for $E/A \simeq 150$ MeV, in agreement with the numerical results of [8].

The connection with the nuclear densities is accomplished through the local density approximation, which relates the Fermi momenta to the local densities as

$$K_F^2 = \left[\frac{3\pi^2}{4}\rho(r)\right]^{2/3} + \tfrac{5}{2}\xi\,(\nabla\rho/\rho)^2 \tag{17}$$

where $\rho(r)$ is the sum of the nucleon densities of each colliding nucleus at the position $r$. The second term is small and amounts to a surface correction, with $\xi$ of the order of 0.1 [8].

Inserting (17) into (16), and using $E = \hbar^2 k^2/2m_N$, we obtain (with $\bar{\rho} = \rho/\rho_0$)

$$\sigma_{NN}(E, \rho) = \sigma_{NN}^{free}(E)\left(1 + \alpha' \bar{\rho}^{2/3}\right) \qquad \text{where} \quad \alpha' = -\frac{48.4}{E\ (\text{MeV})} \tag{18}$$

where the second term of (17) has been neglected. This equation shows that the local density approximation leads to a density dependence proportional to $\bar{\rho}^{2/3}$. The Pauli principle yields a $1/E$ dependence on the bombarding energy. This behaviour arises from a larger phase space available for nucleon–nucleon scattering with increasing energy.

The nucleon–nucleon cross section at $E/A \lesssim 200$ MeV decreases with $E$ approximately as $1/E$. Thus we expect that, in nucleus–nucleus collisions, this energy dependence is flattened by the Pauli correction, i.e. the in-medium nucleon–nucleon cross section is flatter as a function of $E$, for $E \lesssim 200$, than the free cross section. For higher values of $E$ the Pauli blocking is less important and the free and in-medium nucleon–nucleon cross sections are approximately equal. These conclusions are in agreement with the experimental data for nucleus–nucleus reaction cross sections [9]. This was, in fact, well explained in [8].

Note that, for $E/A = 100$–$200$ MeV, and $\rho \simeq \rho_0$, equation (18) yields a coefficient $\alpha'$ between $-0.2$ and $-0.5$. This is in excellent agreement with the findings based on the BUU calculations, primarily intended to reproduce the experimental data on collective variables in intermediate energy nucleus–nucleus collisions.

In conclusion, we have presented a microscopic derivation of the lowest-order density correction for the in-medium nucleon–nucleon cross section. Despite its simplicity, the calculation shows that Pauli-blocking is able to explain almost entirely the magnitude of the correction term, although the power of the density-dependent term is slightly different from that commonly mentioned in the literature [2–6]. We also predict an energy dependence of the in-medium cross sections which was not accounted for previously. This calls for a further study of the consequence of this energy dependence in the transport equation analysis of collective variables in nucleus–nucleus collisions at intermediate energies.

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