Is the tetraneutron a bound dineutron–dineutron molecule?

C A Bertulani and V Zelevinsky

National Superconducting Cyclotron Laboratory, and Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824-1321, USA

E-mail: bertulani@nscl.msu.edu and zelevinsky@nscl.msu.edu

Abstract
In light of a new experiment which claims a positive identification, we discuss the possible existence of the tetraneutron. We explore a model based on a dineutron–dineutron molecule. We show that this model is not able to explain the tetraneutron as a bound state, in agreement with other theoretical models already discussed in the literature.

In a recent experiment [1], the existence of bound neutron clusters was studied by fragmentation of intermediate energy (30–50 MeV/nucleon) ^{14}Be nuclei. In particular, the fragmentation channel ^{10}Be $+^4$n was observed and the ^{4}n system was tentatively described as a bound tetraneutron system.

The possible existence of the tetraneutron has been discussed theoretically by numerous authors already in the 1960s (e.g. [2–9]). The experimental search at that time [10–13], including the experiments on double pion charge exchange reactions with ^{4}He [14, 15], gave negative results. The general conclusion, based on experiment and theory, was that the tetraneutron cannot be bound. More recent experiments [16–18, 22] and theoretical calculations [19–21, 23, 24] give support to this assertion, except perhaps [25] where a hyperspherical function method was used and led the authors to the conclusion that the tetraneutron may exist as a resonance in the four-body continuum at energy of about 1–3 MeV. These theoretical models have recently been reconsidered in [26] where it was concluded that a too strong four-nucleon force is needed to bind the tetraneutron and that this force would unreasonably bind ^{4}He by about 100 MeV.

It thus seems very unlikely that the tetraneutron can be explained within any standard theoretical model. In this brief report we consider the possibility that the tetraneutron can be described as a composite dineutron–dineutron molecular system. To our knowledge this is a hitherto unexplored model for the possible binding of neutron matter. In this model, the dineutron is considered to be slightly bound due to a polarization mechanism induced by the presence of the other dineutron. Although the dineutron system is not bound (it has a virtual singlet state at $E = 66$ keV), there is some theoretical [27–29] and experimental [30] evidence that it might become bound in the presence of another nuclear system.
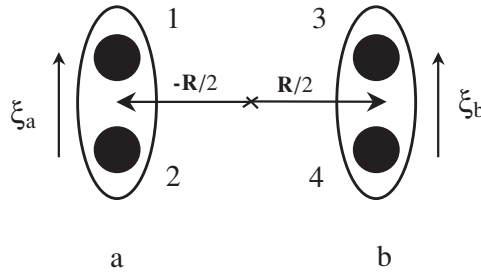
Figure 1. Tetraneutron as a dineutron–dineutron molecule.

Another reason to believe that neutrons clusterize, or form correlated bound pairs inside a nucleus, is the existence of the Borromean nuclear systems, e.g. ^{6}He and ^{11}Li, or nultineutron configuration systems in light nuclei, e.g. ^{8}He. The Borromean systems [31] are three-body systems in which the particles are not bound in pairs, but the three-body system is bound. For example, although ^{10}Li and the two-neutron systems are not bound, ^{11}Li is bound. It is the presence of ^{10}Li which induces the neutron–neutron binding correlation, as first suggested by Migdal [27]. The authors of [32] also suggest that two of the three most likely ground-state configurations in ^{8}He are compatible with an $\alpha + ^4$n cluster system.

The model for the tetraneutron is based on two clusters of dineutron molecules as displayed in figure 1. A similar model was used in the framework of the generator coordinate method for the scattering problem in [33]. The molecular models, similar to those used in quantum chemistry, were successfully applied to the chain of Be isotopes [34]. The dineutron wavefunction can be written as

$$\varphi_{12} = \sqrt{2}\mathcal{A}_{12}\left\{N_0 \exp\left[-\frac{\xi_a^2}{2b^2}\right]\chi_1^{(+)}\chi_2^{(-)}\right\}, \tag{1}$$

where $\xi_a = 2|\mathbf{r}_1 + \mathbf{R}/2|$ is the (relative) intrinsic coordinate and $\mathbf{r}_1$ is the position of the neutron 1 with respect to the centre of mass of the tetraneutron. The relative motion spatial wavefunction is taken as a Gaussian with $N_0 = (b^3\pi^{3/2})^{-1/2}$, and b is the oscillator parameter. The dineutron is assumed to be in a spin-0 state, described by the spinors $\chi_1^{(+)}$ (spin-up) and $\chi_2^{(-)}$ (spin-down), respectively. The wavefunction is antisymmetrized by the operator $\mathcal{A}_{12} = \frac{1}{2}(1 - P_{12})$, where P_{12} exchanges neutrons 1 and 2. An analogous wavefunction, φ_{34}, is written for the second dineutron molecule. Since the spatial part of the dineutron internal wavefunctions is even under inversion, the operator $\mathcal{A}_{12}$ only acts over the spin variables and we can write equation (1) as

$$\begin{aligned}\varphi_{12} &= N_0 \exp\left[-\frac{\xi_a^2}{2b^2}\right]\frac{1}{\sqrt{2}}\left\{\chi_1^{(+)}\chi_2^{(-)} - \chi_1^{(-)}\chi_2^{(+)}\right\} \\ &= N_0 \exp\left[-\frac{\xi_a^2}{2b^2}\right]\chi_{12}^{(0)}.\end{aligned} \tag{2}$$

Accordingly, the wavefunction for the second dineutron is

$$\varphi_{34} = N_0 \exp\left[-\frac{\xi_a^2}{2b^2}\right]\chi_{34}^{(0)}. \tag{3}$$

The total wavefunction of the tetraneutron is

$$\Psi = \sqrt{6}\mathcal{A}_{12,34}[\varphi_{12}\varphi_{34}]\Phi(\mathbf{R}), \tag{4}$$

where the operator $\mathcal{A}_{12,34} = \frac{1}{6}(1 - P_{13} - P_{14} - P_{23} - P_{24} + P_{13}P_{24} + P_{14}P_{23})$ implements the antisymmetrization between the dineutrons. The factors $\sqrt{2}$ in equation (1) and $\sqrt{6}$ in equation (4) account for the proper normalization. The function $\Phi(\mathbf{R})$ is the dineutron–dineutron molecular wavefunction.

Using equations (1)–(4), we obtain (here we drop the upper index (0) in the singlet spinors)

$$\Psi = \sqrt{6}N_0^2\{f(\xi_a, \xi_b)\chi_{12}\chi_{34} - g(\xi_a, \xi_b, \mathbf{R})\chi_{14}\chi_{32} - h(\xi_a, \xi_b, \mathbf{R})\chi_{13}\chi_{24}\}\Phi(\mathbf{R}), \tag{5}$$

where

$$f(\xi_a, \xi_b) = \frac{1}{2}\exp\left[-\frac{\xi_a^2 + \xi_b^2}{2b^2}\right], \tag{6}$$

$$g(\xi_a, \xi_b, \mathbf{R}) = \frac{2}{3}f(\xi_a, \xi_b)\exp\left[-\frac{R^2}{b^2}\right]\exp\left[\frac{\xi_a \cdot \xi_b}{2b^2}\right], \tag{7}$$

and

$$h(\xi_a, \xi_b, \mathbf{R}) = \frac{2}{3}f(\xi_a, \xi_b)\exp\left[-\frac{R^2}{b^2}\right]\exp\left[-\frac{\xi_a \cdot \xi_b}{2b^2}\right]. \tag{8}$$

The total Hamiltonian for the tetraneutron system is

$$H = -\frac{\hbar^2}{2m_N}\sum_{i=1}^{4}\Delta_i + V. \tag{9}$$

The neutron–neutron potential was taken as a two-body Volkov potential [36],

$$V = \sum_{i\leqslant j}^{4}\left(1 - M + MP_{ij}^x\right)V_{ij}, \qquad V_{ij}(r) = V_\alpha\exp(-r^2/\alpha^2) + V_\beta\exp(-r^2/\beta^2), \tag{10}$$

where P_{ij}^x is the Majorana exchange operator and the parameters V_α, V_β, α and β are chosen to reproduce the scattering length and effective range in low-energy nucleon–nucleon collisions, as well as the binding energy of ^{4}He [36].

The total Hamiltonian H can be presented as

$$H = T_a + T_b + T_R + V, \tag{11}$$

with

$$T_{a,b} = -\frac{\hbar^2}{4m_N}\Delta_{\xi_{a,b}}, \qquad T_R = -\frac{\hbar^2}{2m_N}\triangle_{\mathbf{R}}. \tag{12}$$

Since in our model the mean positions of the clusters are taken to be at $\mathbf{R}/2$ and $-\mathbf{R}/2$, respectively, and since the position of each nucleon is symmetric with respect to the origin of each cluster, there is no admixture of spurious motion of the centre of mass.

We use equation (5) as a starting point of a variational procedure. To obtain an effective Schrödinger equation for $\Phi(\mathbf{R})$, we multiply the full four-body Schrödinger equation $H\Psi = E\Psi$ by $\Psi^\dagger/\Phi(\mathbf{R})$ and integrate over ξ_1 and ξ_2. We will assume the variational function $\Phi(\mathbf{R})$ to be independent of angles (i.e., an s-wave state),

$$\Phi(\mathbf{R}) = \frac{1}{\sqrt{4\pi}}\frac{u(R)}{R}, \tag{13}$$

so that the operator T_R in equation (12) becomes

$$T_R = -\frac{\hbar^2}{2m_N}\frac{\mathrm{d}^2}{\mathrm{d}R^2}. \tag{14}$$

The integrals over ξ_1 and ξ_2 can be performed analytically. One obtains an effective Schrödinger equation for $u(R)$ in the form

$$-\frac{\hbar^2}{2m_N}u''(R) + RV_3(R)u'(R) + \{V_1(R) + V_2(R) - E\}u(R) = 0 \tag{15}$$

where

$$V_1(R) = \frac{\hbar^2}{m_N b^2}\left(\frac{R^2}{b^2} - \frac{7}{4}\right)\exp(-R^2/b^2), \tag{16}$$

$$V_2(R) = 2\left[M - 1 - M\frac{1 - \exp(-2R^2/b^2)}{1 - \exp(-R^2/b^2)}\right][f(\alpha) + f(\beta)], \tag{17}$$

$$f(\alpha) = \frac{V_\alpha \alpha^3}{(\alpha^2 + b^2)^{3/2}}\left\{1 + \exp\left[-\frac{R^2}{\alpha^2 + b^2}\right]\right\}, \tag{18}$$

and

$$V_3(R) = -\frac{2\hbar^2}{m_N b^2}\exp(-R^2/b^2). \tag{19}$$

Since we are interested only in the relative energy between the clusters, we extract from the total Hamiltonian the internal kinetic energy of each cluster: $T^{\text{int}} = 3\hbar^2/(4mb^2)$ and $V^{\text{int}} = V_\alpha(1 + b^2/\alpha^2)^{-3/2} + V_\beta(1 + b^2/\beta^2)^{-3/2}$ that do not depend on R.

Equation (15) can be put in a conventional form of a Schrödinger equation:

$$-\frac{\hbar^2}{2m_N}v''(R) + \{U_1(R) + U_2(R) - E\}v(R) = 0, \tag{20}$$

where

$$v(R) = u(R)\exp\left[\frac{1}{2}\int_0^R f(R')\,\mathrm{d}R'\right], \qquad f(R) = -\frac{\hbar^2}{2m_N}RV_3(R). \tag{21}$$

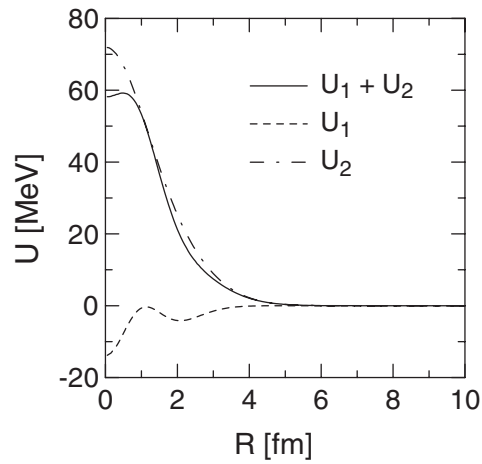The effective potentials in equation (20) are given by

$$U_2(R) = V_2(R), \tag{22}$$

and

$$U_1(R) = V_1(R) - \frac{RV_3'(R)}{2} - \frac{V_3(R)}{2} + \frac{m_N}{2\hbar^2}[RV_3(R)]^2$$

$$= -\frac{\hbar^2}{m_N b^2}\left[\left(\frac{3}{4} + \frac{R^2}{b^2}\right)\exp(-R^2/b^2) - \frac{2R^2}{b^2}\exp(-2R^2/b^2)\right]. \tag{23}$$
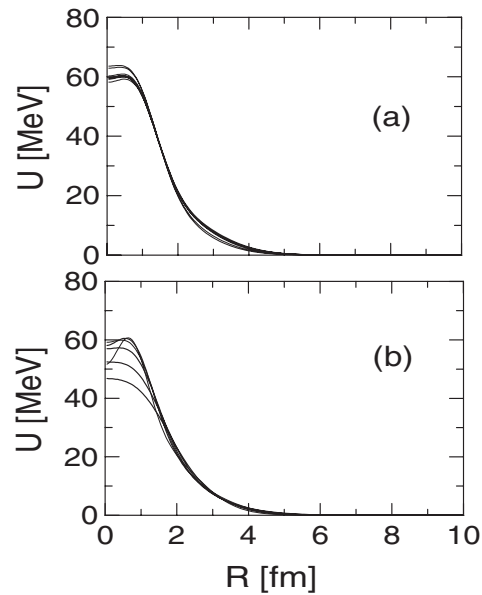
We use for the oscillator parameter $b = 1.5$ fm. A set of eight parameters of the Volkov potential were taken from [36]. In figure 2 we show the results for the Volkov V1 force. We note that the potential $U_2(R)$ is repulsive and the potential $U_1(R)$ is attractive. Their sum is dominated by the repulsive part.

In figure 3(a) we show the total potential $U_1(R) + U_2(R)$ for the eight sets of parameters in the Volkov potential, as taken from [36]. One sees that none of the parameter sets leads to a potential with an attractive pocket, which could be a sign for the existence of a bound state. But even in that case the pocket would have to be deep enough to allow for the appearance of the bound state.

Finally, we have varied the oscillator parameter from 1.2 fm to 2 fm for each set of parameters of the Volkov interaction. No pocket appeared in the effective potential $U_1(R) + U_2(R)$ within this range of variation. Figure 3(b) shows this for the specific case of the Volkov-1 interaction.

Figure 2. The effective potentials $U_1(R)$ and $U_2(R)$ entering equation (20). The solid curve is the sum of the two potentials.

Figure 3. (a) The total potential $U_1(R) + U_2(R)$ entering equation (20) for eight different parametrizations of the Volkov potential. The oscillator parameter $b = 1.5$ fm was used. (b) The same as in (a), but using the Volkov-1 interaction and varying b from 1.2 fm to 2.0 fm.

In summary, we have explored a model of the tetraneutron as a dineutron–dineutron molecule. Using a variational calculation we have found an effective Schrödinger equation for the relative motion of the dineutrons, after a proper account for the Pauli exclusion principle. An effective potential for the relative motion of the dineutron molecules was obtained. We showed that this potential does not have a pocket and thus the tetraneutron is very unlikely to be bound as a dineutron–dineutron molecule, although more complex variational approaches still can be explored. For example, one might consider a different spatial wavefunction for the

dineutron system. We have used Gaussian for convenience. But the asymptotic form of the wavefunction might not be correct. An Yukawa form might be more appropriate to achieve a lower energy for the system. Another improvement might be the use of more realistic interactions, other than the Volkov interaction. Interactions including tensor parts might lead to some modifications of our results. Finally, although less probable as an improvement, one might relax the assumption of singlet states for the dineutron allowing for the triplet configurations in the calculation.

Our study is complementary to other approaches and reinforces the commonly accepted idea that a tetraneutron is not a possible outcome of a theoretical calculation starting with underlying two-body nucleon–nucleon interactions. If the tetraneutron is bound, most probably it will be due to a special four-body attraction in $T = 2$ states or an exceptional fine tuning of the nucleon–nucleon interaction which does not seem to fit within present nuclear models. It might however be useful to recall an old phenomenological argument [5] against the stability of the tetraneutron. Adding a pair of neutrons to a nucleus one usually increases the separation energy of the proton. If this rule holds, a simple comparison of the particle-stable tritium and unstable ^{5}H immediately leads to the conclusion that $M(^4\text{n}) > 4M_n$.

After this paper was completed, a detailed variational Monte Carlo calculation for the tetraneutron was done in [37]. It was shown that it does not seem possible to change modern nuclear Hamiltonians to bind a tetraneutron without destroying many other successful predictions of those Hamiltonians. Otherwise, our understanding of nuclear forces would have to be significantly changed.

More elaborate QCD calculations based on lattice gauge theories cannot presently assess this problem. There are some lattice calculations for the H-dibarion state with not very reliable results (see [38, 39]). However, lattice calculations still cannot determine if, e.g., the deuteron is bound. State of the art lattice numerical calculations are aiming at $\mathcal{O}(10)$ MeV accuracy in binding energy, not $\mathcal{O}(1)$ MeV or less that is relevant for nuclear physics. Also, how the calculations depend on quark masses is an interesting but very difficult question.

Acknowledgment

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