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HEAVY ION EXCITATION OF GIANT RESONANCES: A BRIDGE FROM THE ELASTIC SCATTERING TO THE INELASTIC DATA*

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We develop data-to-data relations involving the giant-resonance cross-sections and the elastic cross-section for heavy ion collisions at intermediate energies. The usefulness of this novel method is shown by applications to the $^{17}O + {}^{208}Pb$ at $E_{\rm Lab} =$ 84 MeV/nucleon.

The Coulomb excitation of giant multipole resonances in heavy ion collisions at intermediate energies allows a detailed study of multipolarity content, relative strength, decay branching ratios and other aspects of these collective states.[1] When compared to purely electromagnetic probes, heavy ions supply strong nuclear fields, which may complicate the analysis. Usually, one uses DWBA codes,[2] which, when compared to the experimental data, allow the extraction of the multipolarity content and relative strength of the excitation. Recently,[3] a simpler approach based on the use of the eikonal approximation to the distorted waves was shown to be quite adequate in describing the excitation of giant resonances in heavy ion induced reactions.

In this article we derive data-to-data (DTD) relations involving the inelastic cross-section for the excitation of giant resonances on the one hand and the elastic

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cross-section on the other. The competition between the Coulomb and the nuclear interaction implies that the deflection function reaches a maximum at the so-called rainbow angle, θ_r. The region $\theta > \theta_r$ is classically forbidden and is called the dark side of the Coulomb rainbow. For $\theta \gg \theta_r$ absorption sets in and the cross-sections for elastic scattering becomes very small. On the dark side of the Coulomb rainbow, where the DTD relations strictly hold, the inelastic cross-section for a given multipolarity is found to go as $C_\lambda q^2 \sigma_{\rm el}(q)$, where q is the momentum transfer. Thus we predict that in the relevant angular region the shape of the inelastic angular distribution does not depend on the multipolarity λ. These relations allow a reliable and easier analysis of the experimental data, as shown by an application to the $^{17}O + ^{208}Pb$ system. Details of the calculations will be presented elsewhere.[4] Data-to-data relations have been derived earlier for intermediate energy proton-nucleus scattering by Amado *et al.*[5] The fundamental difference between our work and Ref. 5 is the very important Coulomb effects in the heavy-ion system.

The amplitude for the transition of the nucleus from the ground state $|0\rangle$ to the excited state $|\lambda\mu\rangle$ is given, within the eikonal approximation,[3] by $[q = 2k\sin(\theta/2)]$

$$f_{N,C}^{\lambda\mu}(\theta) = \frac{ik}{2\pi\hbar v}\int e^{i\mathbf{q}\cdot\mathbf{r}+i\chi(b)}\langle \mathbf{r},\, \lambda\mu|U_{N,C}|\mathbf{r},\, 0\rangle d^2b\, dz\,, \tag{1}$$

where $N(C)$ stands for the nuclear (Coulomb) contribution, $U_{N,C}$ is the nucleus-nucleus interaction, and $\chi(b)$ is the total eikonal phase. For comparison, the elastic scattering amplitude is given by

$$f_{\rm el}(\theta) = \frac{ik}{2\pi}\int e^{i\mathbf{b}\cdot\mathbf{q}}\left[1 - e^{i\chi(b)}\right]d^2b = ik\int b\, J_0(qb)\left[1 - e^{i\chi(b)}\right]db\,. \tag{2}$$

The phase $\chi(b)$ is related to the potential by the usual eikonal formula

$$\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} dz\left[U_N\left(\sqrt{b^2+z^2}\right) + U_C\left(\sqrt{b^2+z^2}\right)\right]. \tag{3}$$

The nuclear potential, $U_N(r)$, is evaluated using the "$t\rho\rho$" approximation[6] with due care to medium effects in the calculation of the nucleon-nucleon t-matrix. Since the inelastic transitions considered are peripheral, the amplitudes are sensitive to the surface region only. Therefore, as in Ref. 3, we employ Gaussian forms for the densities $\rho = \rho_{\rm G}\, e^{-r^2/\alpha^2}$ that are adjusted to reproduce the tail region of the realistic densities. This fit imply the relations $\alpha_i = \sqrt{2aR_i}$ and $\rho_{{\rm G},\,i} = (\rho_0/2)e^{R_i/2a}$ with $i = 1, 2; \rho_0 = 0.17$ fm^{-3}; $a = 0.65$ fm and $R_i = 1.2\, A_i^{1/3}$ fm. The resulting U_N becomes Gaussian[3] and the corresponding nuclear phase comes out to be

$$\chi_N(b) = -\frac{\pi^2}{\hbar v}\langle t_{NN}\rangle \frac{\alpha_1^3\alpha_2^3}{\alpha^2}\, \rho_{{\rm G},1}\, \rho_{{\rm G},2}\, e^{-b^2/\alpha^2}\,, \tag{4}$$

where $\alpha = \sqrt{\alpha_1^2 + \alpha_2^2}$. The Gaussian approximation for U_N is essential for a simple derivation of the DTD relations. Reinforcing our previous statements, this is not a bad approximation for heavier systems since what matters for the inelastic scattering is a good description of the surface region of the nuclei, which can be fitted by a Gaussian. For more details on the validity of this approximation see, e.g., Ref. 7.

An analytical formula for the Coulomb phase, which reproduces the Coulomb elastic amplitude when inserted in Eq. (2) is given by

$$\chi_C(b) = \frac{2Z_1Z_2e^2}{\hbar v} \ln(kb) . \tag{5}$$

This formula will be used in our numerical calculations. For the derivation of the DTD relations it is however more useful to work with the definition given by Eq. (3).

In Fig. 1 we show the elastic scattering angular distribution for the system $^{17}O + ^{208}Pb$ at $E_{Lab} = 84$ MeV/nucleon obtained with $\langle\sigma_{NN}\rangle = 60$ mb and $t_{Re}/t_{Im} = 1$, as in Ref. 3. The data[8] are very well reproduced with our calculation. Further we have assessed that within the angular range of interest $0° \leq \theta < 6°$ the cross-section is completely near-side[9] dominated and represents a nice case of Coulomb rainbow scattering with the rainbow angle being $\theta_r = 3.3°$, as can be seen clearly in Fig. 2 which shows the analogy of the deflection function, which we appropriately call here, following Ref. 10, the momentum transfer function. The rainbow momentum transfer, $q_r(b)$, which corresponds to θ_r is 1.86 fm^{-1}.

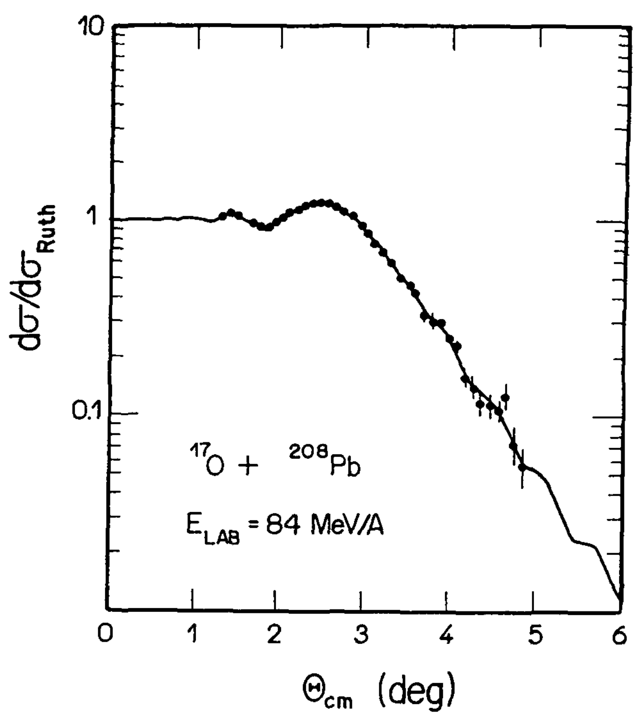

Fig. 1. Elastic cross-section data for the system $^{17}O + ^{208}Pb$ at 84 MeV/nucleon. Data are from Ref. 7. The solid curve is our theoretical prediction.

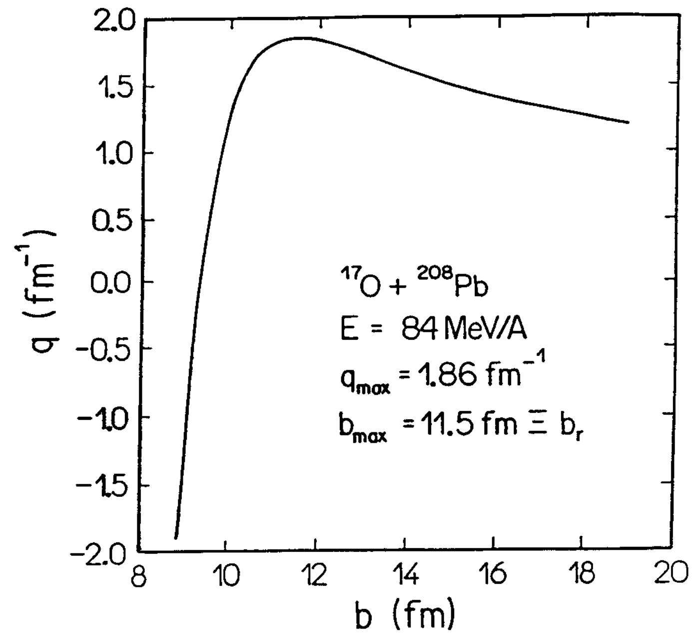

Fig. 2. The momentum transfer function $q(b)$ vs. b. See text for details.

When analyzing the inelastic amplitude, it is clear that the Coulomb-rainbow scattering effect is also present through the phase, which is the same as the one that appears in the elastic one. For $q < 1.86$ fm^{-1} two stationary phase contributions dominate $f_{\rm el}$ and $f_{\rm inel}$. These are the nuclear (inner branch) and the Coulomb (outer branch) and the cross-sections clearly exhibits the known Coulomb-nuclear interference. On the dark-side of the rainbow there is only one complex stationary phase contribution. This is a mixture of Coulomb + nuclear scattering. Therefore, no interference arises in $f_{\rm el}$ or $f_{\rm inel}$.

It is in this region that one would expect a simple linear relationship between the amplitudes to hold. Since the inelastic nuclear amplitude involves an integrand containing the derivative of the potential (Tassie Model[11]), which can be related to the derivative of the eikonal phase with respect to b [see Eq. (3)], a simple integration by parts of Eq. (1) indicate that $f_{\rm inel}(q)$ should be proportional to $q\, f_{\rm el}(q)$ in this angular region. Similar arguments can be applied to the inelastic Coulomb amplitude. Further, since the inelastic amplitude can be written in the form[3] (after integration over the azimuthal angle ϕ defined by $d^2b = b\, db\, d\phi$),

$$f^{\lambda\mu}_{N,C} = \frac{C^{N,C}_{\lambda\mu}\, k}{\hbar v} \int_0^\infty db\, b J_\mu(qb) e^{i\chi(b)} \int_{-\infty}^\infty dz\, e^{i\omega z/v}\, r^\lambda U_{N,C(\lambda)}\left(\sqrt{b^2+z^2}\right) P_{\lambda\mu}(\theta)\,, \tag{6}$$

the q dependence arises entirely from the Bessel function. In the equations above, we used $\mathbf{q}\cdot\mathbf{z} = (\mathbf{k}' - \mathbf{k})\cdot\mathbf{z} \simeq \omega z/v$, valid for $\Delta k \ll k$. $\hbar\omega$ is the excitation energy and v is the projectile velocity. For collisions at intermediate energies, and excitation energies about $\hbar\omega$ = 10–20 MeV, $\omega z/v \ll 1$ and the exponential $\exp(i\omega/v)$ can be dropped off in the integral in (6). In Eq. (6) the factor $r^\lambda U_N$ results from the use of the Tassie model and a Gaussian nuclear potential. For the Coulomb interaction we make a multipole expansion and $r^\lambda U_{C(\lambda)} \equiv 1/r^{\lambda+1}$. The constant $C_{\lambda\mu}$ includes the matrix element for the transition $|0\rangle \rightarrow |\lambda\mu\rangle$, which in the Tassie model (nuclear interaction) is proportional to a deformation parameter β_N. In the Coulomb case this constant includes a matrix element of the form $\langle\lambda\mu|r^\lambda Y_{\lambda\mu}|0\rangle$ which is also related[3] to a deformation parameter β_C.

The integral over z can be related to derivatives, with respect to b, of $\chi(b)$, Eq. (2), if one uses the Gaussian approximation for the tails of the densities. Since the b's that contribute are large (roughly, the sum of the two radii of the two nuclei), and taking q to be near the rainbow value, one may use the asymptotic form of $J_\mu(qb)$, namely $J_\mu(qb) \approx \sqrt{2/\pi qb}\,\cos(qb-\mu\pi/2-\pi/4)$. Therefore, one would expect that the λ and μ dependence to be almost irrelevant to the q-dependence. Accordingly, the inelastic amplitudes on the dark-side should have similar q-dependence, irrespective of the multipolarity.

The above can be made more quantitative by analyzing, within the stationary phase approximation, the amplitude $f_{N,C}^{\lambda\mu}$, Eq. (6). Calling the z-integral $F_{N,C}^{\lambda\mu}(b)$, and considering only the incoming component of $J_\mu(qb)$, namely

$$\frac{1}{\sqrt{2\pi qb}}\exp\left[-iqb+i\frac{\mu\pi}{2}+i\frac{\pi}{4}\right]$$

(i.e., taking only the near-side contribution into account, which is quite valid in the angle region of interest), we have

$$f_{N,C}^{\lambda\mu}(q) = C_{\lambda\mu}^{N,C}\int db\, b^{1/2}\, F_{N,C}^{\lambda\mu}(b)\,\exp\left[i\chi(b)-iqb\right], \tag{7}$$

where we include other factors in the constant $C_{\lambda\mu}^{N,C}$. Note that both f_N and f_C are evaluated with the full eikonal phase (Coulomb + nuclear). Since $b^{1/2}\,F_{N,C}^{\lambda\mu}(b)$ varies much slower with b than $\exp[i\chi(b)-iqb]$ we may apply the stationary point approximation to (7) and obtain, in the shadow side of the Coulomb rainbow

$$f_{N,C}^{\lambda\mu}(q) \simeq C_{\lambda\mu}^{C,N}\,F_{N,C}^{\lambda\mu}[\bar{b}(q)]\,f_{\rm el}^{\rm Near}(q), \tag{8}$$

where $\bar{b}(q)$ is the stationary phase point. The evaluation of the quantity $F_{N,C}^{\lambda\mu}[\bar{b}(q)]$ is lengthy but straightforward. The important point that we emphasize here is that both $F_N^{\lambda\mu}[\bar{b}(q)]$ and $F_C^{\lambda\mu}[\bar{b}(q)]$ can be reduced to linear combinations of $\chi_N[\bar{b}(q)]$ and $\chi_C[\bar{b}(q)]$ and their higher derivatives with respect to $\bar{b}$. For the giant dipole case $(\lambda=1,\mu=1,0,-1)$ we find, e.g.,

$$F_N^{1,1}\left[\bar{b}(q)\right] \propto \frac{d\chi_N}{db} \propto q_N \quad \text{and} \quad F_C^{1,1}\left[\bar{b}(q)\right] \propto \frac{d\chi_C}{db} \propto q_C\,. \tag{9}$$

Identical relations can be obtained for $F_N^{1,-1}$ and $F_C^{1,-1}$. As far as $F_{N,C}^{1,0}$ is concerned, one can show that both $\chi[\bar{b}(q)]$ and $d\chi[\bar{b}(q)]/db$ are involved here. However the derivative term is dominant. Note that q and χ are related by the stationary phase condition

$$q = \frac{d\chi_N(b)}{db} + \frac{d\chi_C(b)}{db} \equiv q_N + q_C \,. \tag{10}$$

The full amplitude $f_{\text{inel}}^{11} = f_N^{11} + f_C^{11}$, using (8) and (9) is proportional to $f_{\text{el}}^{\text{Near}}(q)$ on the dark-side of the rainbow as already stated. The dependence on q is of the form

$$f_{\text{inel}}^{11}(q) = \left[C_1^N\, q_N + C_1^C\, q_C\right] f_{\text{el}}(q)\,. \tag{11}$$

For the angular regions where either the Coulomb or the nuclear scattering dominates, the above equation is then approximated by a numerical factor times $q f_{\text{el}}(q)$.

A more accurate derivation of our DTD relations can be done following the arguments presented above, with the use of the detailed form of $P_{\lambda\mu}(\theta)$ in terms of b and z. With the help of the Tassie model[11] and the uniform stationary phase approximation[12] we have obtained the DTD relations for the excitation of the isovector giant dipole (IVGDR) and isoscalar giant quadrupole (ISGQR) resonances. In the dark region we write the data-to-data relations

$$\sigma_\lambda(q) = A_\lambda\, q^2\, \sigma_{\text{el}}(q)\,, \tag{12}$$

where $\lambda = 1^-, 2^+$ and A_λ is a normalization factor.

Owing to the use of the Glauber theory and the strong Coulomb field felt at small angles, we anticipate that the above relations are qualitative, as far as the absolute values of the cross-sections are concerned. Therefore, we use the normalization factors A_λ as adjustable factors. The momentum transfer q, as defined in Eq. (10) has an imaginary part. As shown by Amado *et al.*[5] this introduces a slight shift in the argument of σ_{el} in Eq. (12). We therefore, guided by the discussion at the beginning of this work, write the inelastic cross-section as

$$\sigma_\lambda(q) = A_\lambda\, q^2\, \sigma_{\text{el}}(q + \Delta_\lambda)\,, \tag{13}$$

where A_λ and Δ_λ are then found by adjusting the data.

In Figs. 3 and 4 we show our data-to-data relation calculation for $\sigma_{1^-}(q)$ (Fig. 3) and $\sigma_{2^+}(q)$ (Fig. 4) using the elastic scattering cross-section shown in Fig. 1. We found $A_1 = 2 \times 10^{-5}$ fm^2, $\Delta_1 = 0.19$ fm^{-1} and $A_2 = 3.9 \times 10^{-6}$ fm^2, $\Delta_2 = -0.66$ fm^{-1}. The shifts in the argument of σ_{el} correspond to angular shifts of $\Delta\theta_1 = 0.7^\circ$ and $\Delta\theta_2 = -1.2^\circ$.

In conclusion we have established in this work, that DTD relations can be derived for heavy-ion scattering at intermediate energies. These relations, though do not predict the absolute values of the inelastic cross-sections supply a very simple mean to obtain their shapes quite well, in the Coulomb rainbow region. Further analysis of these relations and the question of how to obtain the normalization factors in terms of the nuclear excitation parameters is in progress and will be reported elsewhere.

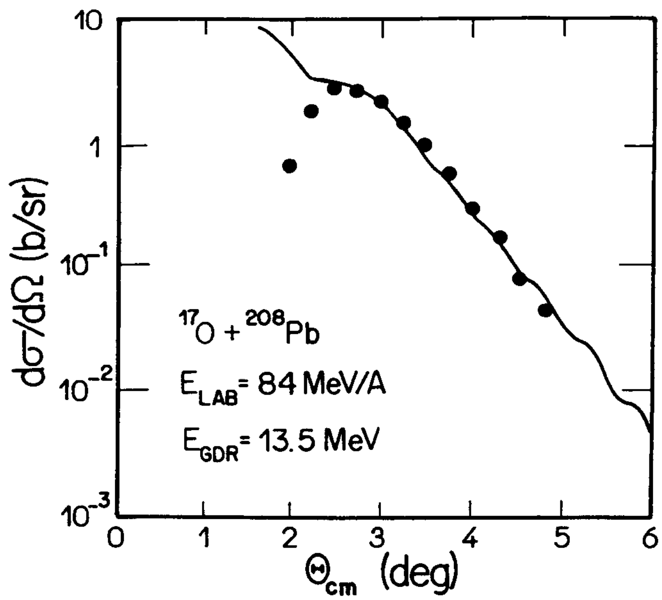

Fig. 3. Cross-section for the excitation of isovector giant dipole resonance. Data are from Ref. 7. The solid curve was obtained from the data-to-data relation given by Eqs. (7), with parameters $A_1 = 2 \times 10^{-5}$ fm^2 and $\Delta_1 = 0.19$ fm^{-1}.

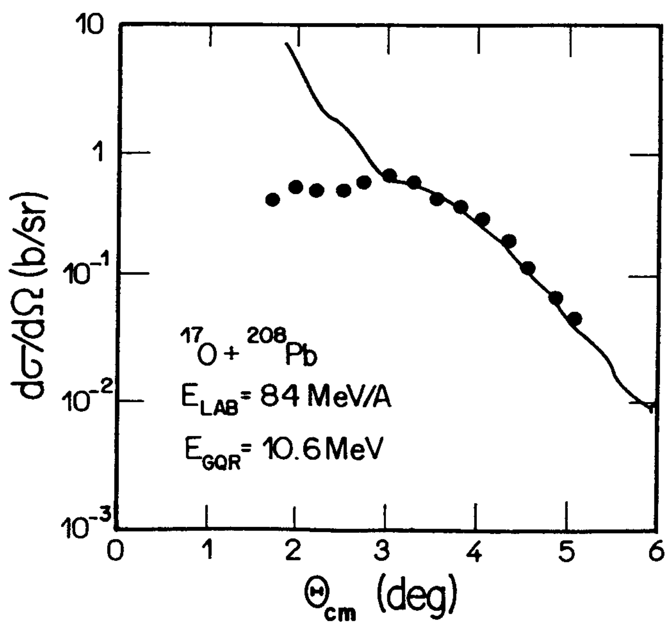

Fig. 4. The same as in Fig. 3, but for the isoscalar giant quadrupole resonance. We used here $A_2 = 3.9 \times 10^{-6}$ fm^2 and $\Delta_2 = -0.66$ fm^{-1}.

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