Nuclear Physics A442 (1985) 739-752

RELATIVISTIC COULOMB COLLISIONS AND THE VIRTUAL RADIATION SPECTRUM

C.A. BERTULANI* and G. BAUR

Institut für Kernphysik, Kernforschungsanlage Jülich, D-5170 Jülich, West Germany

Received 15 March 1985

Abstract: We evaluate the Coulomb-excitation cross sections in relativistic heavy-ion collisions by means of the plane-wave Born approximation. The final total cross section is shown to be equal to that obtained by a semiclassical method. As a byproduct the virtual photon spectrum for similar electromagnetic processes is derived. Comparison with other methods is performed.

1. Introduction

Recent experiments [1]) and theoretical works [2-5]) created vivid interest in Coulomb excitation in relativistic heavy-ion collisions. The basic assumption in this kind of reaction is that the nuclei do not penetrate each other. When they penetrate the reaction is overwhelmingly due to the strong interaction so that the cross sections for the two different processes do not interfere. Since the Coulomb scattering for high energies is predominantly forward-peaked, Winther and Alder [2]) used a retarded Coulomb potential for a projectile moving in a straight line, i.e. the so-called Liénard-Wiechart potential, in order to calculate the total Coulomb-excitation cross section of the target nucleus. Corrections due to the finite size of the projectile with respect to the Coulomb excitation of the target, and vice versa, were performed by Jäckle and Pilkuhn [5]) by means of the eikonal approximation. But, as already mentioned by Olson *et al.* [1]), their results are questionable, especially in their limiting form for a point projectile. The relation between the electric-dipole excitation cross section obtained by Winther and Alder and the virtual photon theory of Weizsäcker and Williams was demonstrated by Hoffman and Baur [3]). Later on, it was shown by Goldberg [4]) how one can extend the Weizsäcker-Williams method in order to calculate the virtual photon numbers not only for the E1 but also for all other multipolarities of the radiation.

In sect. 2 we outline the calculation of the transition amplitude for the Coulomb-excitation process in the plane-wave Born approximation (PWBA). We found that some steps of our calculation are equal to those introduced by Winther and Alder [2]). As a matter of fact, it is shown in sect. 3 that, under certain assumptions, the

* On leave of absence from the Federal University of Rio de Janeiro, Brazil, and supported by the Deutscher Akademischer Austauschdienst/CAPES.

excitation cross section integrated over the scattering angle is equal to that obtained by these authors, integrated over impact parameters. This resembles the non-relativistic case, where the equality between the cross sections obtained by the two different approaches is demonstrated [see ref. [8])].

In sect. 4 it is shown that the Coulomb-excitation cross section can be separated in a dynamical factor, which describes the photon-nucleus interaction process, and a kinematical factor, which is related to the virtual photon numbers of the Weizsäcker-Williams method. This later factor is written in an analytical form for all multipolarities and presents a final solution to the problem proposed by Goldberg. The result is also compared to the ultra-relativistic electron scattering calculations of an old paper from Thie *et al.* [9]) on the virtual-photon theory.

2. Transition amplitude in the PWBA

We shall consider the target nucleus as fixed, neglecting its recoil, and we place the origin of our coordinate system in its center of mass. The target will be described by an eigenstate $|IM\rangle$, where I is its angular momentum and M the magnetic quantum number. The projectile will be described by a plane wave $|\boldsymbol{k}\rangle$, where $\boldsymbol{k}$ denotes its wave vector. The transition amplitude in the Born approximation [see e.g. ref. [10])] is given by[†]

$$a_{\mathrm{fi}} = \frac{1}{c}\int \mathrm{d}^3 r\, A_\mu(\boldsymbol{r}) \langle I_\mathrm{f} M_\mathrm{f} | j_\mu(\boldsymbol{r}) | I_\mathrm{i} M_\mathrm{i}\rangle \tag{2.1a}$$

with

$$A_\mu(\boldsymbol{r}) = \frac{1}{c}\int \mathrm{d}^3 r' \frac{\mathrm{e}^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|} \langle \boldsymbol{k}_\mathrm{f} | J_\mu(\boldsymbol{r}') | \boldsymbol{k}_\mathrm{i}\rangle\,, \tag{2.1b}$$

where $j_\mu \equiv (c\rho, \boldsymbol{j})$ is the target four-current and J_μ the projectile one; $\boldsymbol{r}$ ($\boldsymbol{r}'$) denotes the target (projectile) coordinate, and

$$k = \frac{\omega}{c} = \frac{E_\mathrm{i} - E_\mathrm{f}}{\hbar c} \tag{2.2}$$

with E equal to the relative motion energy. The function $A_\mu(\boldsymbol{r})$ represents the four-potential created by the transition current of the projectile. Asssuming that the velocity of the projectile is not appreciably changed during the collision, we can put

$$\langle \boldsymbol{k}_\mathrm{f} | J_\mu(\boldsymbol{r}') | \boldsymbol{k}_\mathrm{i}\rangle = Z_\mathrm{p} e v_\mu\, \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{r}'}\,, \tag{2.3}$$

where $Z_\mathrm{p} e$ is the projectile's charge,

$$\boldsymbol{q} = \boldsymbol{k}_\mathrm{i} - \boldsymbol{k}_\mathrm{f} \tag{2.4}$$

[†] Here we use the notation $A_\mu = (A_0, \boldsymbol{A})$ and the sum convention $A_\mu B_\mu = A_0 B_0 - \boldsymbol{A}\cdot\boldsymbol{B}$. We also use c.g.s. units with c equal to the velocity of the light and h to the Planck constant.

is the momentum transfer, and

$$v_\mu = (c, \boldsymbol{v})$$

with $\boldsymbol{v}$ equal to the projectile's velocity. Choosing cylindrical coordinates for the projectile space integration and the z-axis in the incident beam direction, we obtain

$$e^{i\boldsymbol{q}\cdot\boldsymbol{r}'} = e^{iq_L z'}\, e^{iq_T\rho'\cos(\psi-\phi')}\,, \tag{2.5}$$

where ψ is the azimuthal scattering angle and q_L (q_T) is the longitudinal (transverse) momentum transfer to the projectile. For relativistic energies the polar scattering angle θ due to the electromagnetic interaction is very small and we can put

$$q_L = k_i - k_f\cos\theta \simeq k_i - k_f \simeq \omega/v\,, \tag{2.6a}$$

$$q_T = k_f\sin\theta \simeq (E/\hbar c)(v/c)\sin\theta\,, \tag{2.6b}$$

where we also assumed that the excitation energy $E_i - E_f = \hbar\omega$ is much smaller than the relative motion energy $E = E_i \simeq E_f$.

Using these approximations we can write

$$A_\mu(\boldsymbol{r}) = Z_p e(v_\mu/c)\int d^3r'\, e^{i(\omega/v)z'}\, e^{iq_T\rho'\cos(\psi-\phi')}\,\frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}\,. \tag{2.7}$$

The z' integration can be performed by defining

$$d^2 = \rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi-\phi')\,.$$

This leads to

$$\int_{-\infty}^{\infty} dz'\, e^{i(\omega/v)z'}\,\frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|} = 2\, e^{i(\omega/v)z} K_0(\omega d/\gamma v)\,, \tag{2.8}$$

where K_0 is the modified Bessel function of zeroth order and γ is the relativistic factor

$$\gamma = (1 - v^2/c^2)^{-1/2}\,. \tag{2.9}$$

Using the Graf addition theorem [see ref. [11]] in order to separate the target and projectile coordinates, the ϕ' integration is easily obtained and $A_\mu(\boldsymbol{r})$ becomes

$$A_\mu(\boldsymbol{r}) = 4\pi Z_p e(v_\mu/c)\, e^{i(\omega/v)z}\sum_{n=-\infty}^{\infty} i^n\, e^{-in\psi}\, e^{in\phi}\{J_n(\omega\rho/\gamma v)\chi_n(R) + \Gamma_n(R,\rho)\}\,, \tag{2.10}$$

where

$$\chi_n(R) = \int_R^\infty J_n(q_T\rho')K_n(\omega\rho'/\gamma v)\rho'\, d\rho'\,, \tag{2.11}$$

$$\Gamma_n(R,\rho) = \{\chi_n(\rho) - \chi_n(R)\}J_n(\omega\rho/\gamma v) + \left\{\int_0^\rho J_n(q_T\rho')J_n(\omega\rho'/\gamma v)\rho'\, d\rho'\right\}K_n(\omega\rho/\gamma v)\,, \tag{2.12}$$

and $J_n(K_n)$ is the common (modified) Bessel function of nth order. In these expressions R is taken as some nuclear radius inside of which the strong interaction is present. The function $\chi_n(R)$ involves the part of the projectile current that is outside a cylindrical hole of radius R. The function $\Gamma_n(R, \rho)$, on the other hand, takes into account the part of the projectile current inside of this hole and includes the projectile-target penetration case.

In contrast to the non-relativistic collisions, below the Coulomb barrier, it is impossible to obtain a pure Coulomb-excitation process in relativistic heavy-ion reactions. It is then necessary to introduce the "cutoff" parameter R which defines a frontier between Coulomb and non-Coulomb processes. In order to limit ourselves within the Coulomb-excitation contribution to the total reaction cross section we are also forced to disregard the function $\Gamma_n(R, \rho)$ in eq. (2.10). But we note that this is an *ad hoc* assumption which, was not contained in the original PWBA expression (2.1). It amounts to punching a cylindrical hole in the plane wave and resembles the semiclassical approach where R is identified as the minimum impact parameter that still leads to a pure Coulomb interaction. Such a cutoff approximation reminds us of the so-called "Butler" cutoff [12]), a critical overview of which is found in ref. [13]).

After these considerations, many steps of our calculations are exactly the same as those performed by Winther and Alder [2]) in the semiclassical approach. Doing a multipole expansion of $A_\mu(\boldsymbol{r})$ we find

$$A_\mu(\boldsymbol{r}) = (4\pi)^{3/2} Z_{\mathrm{p}} e (v_\mu / c) \sum_{lm} i^l \sqrt{2l+1} \left[\frac{(l-m)!}{(l+m)!}\right]^{1/2} (2m-1)!!(c/v\gamma)^m$$
$$\times \mathrm{e}^{-im\psi} \chi_m(R) C_{l-m}^{m+1/2}(c/v) j_l(kr) Y_{lm}^*(\hat{\boldsymbol{r}}) , \tag{2.13}$$

where $C_{l-m}^{m+1/2}$ are the Gegenbauer polynomials, j_l are the spherical Bessel functions, and Y_{lm} are the spherical harmonic functions. Now, inserting this relation into eq. (2.1), using the continuity equation for the nuclear current and the recursion relations of the Gegenbauer polynomials, one can write a_{fi} in terms of the usual multipole matrix elements of nuclear excitation:

$$a_{\mathrm{fi}} = (2\pi Z_{\mathrm{p}} e/\gamma) \sum_{\pi l m} i^m k^l \sqrt{2l+1}\, \mathrm{e}^{-im\psi} \chi_m(R) G_{\pi l m}(c/v) \langle I_{\mathrm{f}} M_{\mathrm{f}} | \mathcal{M}(\pi l, -m) | I_{\mathrm{i}} M_{\mathrm{i}} \rangle , \tag{2.14}$$

where $\pi = \mathrm{E}$ or M, for electric or magnetic excitations, and

$$\mathcal{M}(\mathrm{E}lm) = \frac{(2l+1)!!}{k^{l+1} c(l+1)} \int \boldsymbol{j}(\boldsymbol{r}) \cdot \nabla \times \boldsymbol{L}[j_l(kr) Y_{lm}(\hat{\boldsymbol{r}})]\, \mathrm{d}^3 r , \tag{2.15a}$$

$$\mathcal{M}(\mathrm{M}lm) = -i \frac{(2l+1)!!}{k^l c(l+1)} \int \boldsymbol{j}(\boldsymbol{r}) \cdot \boldsymbol{L}[j_l(kr) Y_{lm}(\hat{\boldsymbol{r}})]\, \mathrm{d}^3 r . \tag{2.15b}$$

The functions $G_{\pi l m}(x)$ are given in terms of the Legendre polynomials $P_{lm}(x)$

calculated for $x > 1$. For $m \geq 0$ they are

$$G_{\mathrm{E}lm}(x) = i^{l+m} \frac{\sqrt{16\pi}}{l(2l+1)!!} \left[\frac{(l-m)!}{(l+m)!}\right]^{1/2} (x^2-1)^{-1/2}$$

$$\times \left[\frac{(l+1)(l+m)}{2l+1} P^m_{l-1}(x) - \frac{l(l-m+1)}{2l+1} P^m_{l+1}(x)\right], \tag{2.16a}$$

$$G_{\mathrm{M}lm}(x) = i^{l+m+1} \frac{\sqrt{16\pi}}{l(2l+1)!!} \left[\frac{(l-m)!}{(l+m)!}\right]^{1/2} (x^2-1)^{-1/2} m P^m_l(x), \tag{2.16b}$$

while for $m < 0$ one can use

$$G_{\mathrm{E}l,-m}(x) = (-1)^m G_{\mathrm{E}lm}(x), \tag{2.17a}$$

$$G_{\mathrm{M}l,-m}(x) = -(-1)^m G_{\mathrm{M}lm}(x). \tag{2.17b}$$

A table of explicit expressions for $G_{\pi lm}(x)$ and for $l \leq 3$ is presented in the appendix of ref. [2]).

3. Cross sections

The differential cross section, for the case in which the orientation of the target is ignored, is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{E}{2\pi\hbar^2 c^2}\right)^2 \frac{1}{2I_\mathrm{i}+1} \sum_{M_\mathrm{i} M_\mathrm{f}} |a_{\mathrm{fi}}|^2. \tag{3.1}$$

From eq. (2.14), the Wigner-Eckart theorem and the orthogonality properties of the Clebsch-Gordan coefficients, one can show that

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{Z_\mathrm{p}\alpha E}{\gamma\hbar c}\right)^2 \sum_{\pi lm} k^{2l} \frac{B(\pi l)}{e^2} |G_{\pi lm}(c/v)|^2 [\chi_m(R)]^2, \tag{3.2}$$

where α is the fine structure constant and $B(\pi l)$ is the reduced transition probability for Coulomb excitation of the target nucleus:

$$B(\pi l) = \frac{1}{2I_\mathrm{i}+1} \sum_{M_\mathrm{i} M_\mathrm{f}} |\langle I_\mathrm{f} M_\mathrm{f} | \mathcal{M}(\pi lm) | I_\mathrm{i} M_\mathrm{i} \rangle|^2. \tag{3.3}$$

The dependence of the differential cross section on the scattering angle is given implicitly by the function $\chi_m(R)$. For forward scattering, $q_\mathrm{T} = 0$, and we obtain

$$\chi_m(R, \theta=0)/R^2 = \delta_{m0} \frac{1}{\xi} K_1(\xi), \tag{3.4}$$

where

$$\xi = \omega R/\gamma v. \tag{3.5}$$

This implies that

$$\frac{d\sigma}{d\Omega}(\theta=0)=(Z_p\alpha)^2\left(\frac{E}{\hbar\omega}\frac{v^2}{c^2}\right)^2\xi^2[K_1(\xi)]^2\sum_l k^{2(l-1)}|G_{El0}(c/v)|^2\frac{B(El)}{e^2}. \qquad (3.6)$$

The quantity m is equal to the angular momentum transfer to the target in the direction of the incident beam,

$$m=M_i-M_f, \qquad (3.7)$$

and eq. (3.4) shows that, for exact forward scattering, it is equal to zero. In this case there is no magnetic excitation of the target. This can be explained in terms of the symmetry properties of the scattering of spin-zero particles. The conservation of parity of the total system forbids the change of internal parity of the target by $(-1)^{l+1}$ in the case $m=0$ [see e.g. ref. [14]), eq. (43)]. Since magnetic excitations are accompanied by this change of parity, the forward scattering amplitude must vanish in this case.

For $\theta\neq 0$ one can analytically solve the integral (2.11) to obtain the dependence of the cross section on the scattering angle. But the exact form of this dependence is of little importance. It suffices to say that it is extremely forward-peaked with a diffraction angle of about

$$\theta_d\simeq\frac{1}{\Lambda} \qquad \text{with } \Lambda=\frac{Ev}{\hbar c^2}R. \qquad (3.8)$$

The parameter Λ is equal to the ratio between the nuclear dimension R and the quantum wavelength of the relative motion energy. For relativistic heavy ion collisions this quantity is much greater than unit and θ_d will be too small. We can also compare the diffraction angle θ_d with the classically expected Coulomb deflection

$$\theta_C\simeq\frac{2Z_pZ_te^2}{ER}. \qquad (3.9)$$

The ratio of these two quantities is

$$\frac{\theta_C}{\theta_d}\simeq\frac{2}{137}Z_pZ_t, \qquad (3.10)$$

showing that only for small projectile and (or) target charge the diffraction effects will be comparable to the Coulomb deflection.

The total cross section is obtained by integrating (3.2) over the scattering angle θ. But, by means of (2.6b) we can transform the angular integration to one involving the momentum transfer q_T:

$$d\Omega=(\hbar c^2/Ev)^2 q_T\, dq_T\, d\psi. \qquad (3.11)$$

Accordingly, the integration in q_T must go from 0 to $Ev/\hbar c^2$. Nevertheless, expressions (2.6b) and (3.8) imply that already for $q_T\simeq 1/R \ll Ev/\hbar c^2$ the differential cross section is negligible. It then makes no difference if we take the integral in q_{T} until infinity. In this case we can use the closure relation of the Bessel functions

$$\int_0^\infty q_{\mathrm{T}} J_m(q_{\mathrm{T}}\rho') J_m(q_{\mathrm{T}}\rho'')\, \mathrm{d}q_{\mathrm{T}} = \frac{1}{\rho'}\delta(\rho'-\rho'') \tag{3.12}$$

in order to obtain the total cross section

$$\sigma(\hbar\omega) = (Z_{\mathrm{p}}\alpha)^2 \sum_{\pi l m} k^{2(l-1)} g_m(\xi) |G_{\pi l m}(c/v)|^2 B(\pi l)/e^2 , \tag{3.13a}$$

where

$$g_m(\xi) = g_{-m}(\xi) = 2\pi(\omega/\gamma v)^2 \int_R^\infty \rho[K_m(\omega\rho/\gamma v)]^2 \, \mathrm{d}\rho$$

$$= \pi\xi^2\{[K_{m+1}(\xi)]^2 - [K_m(\xi)]^2 - (2m/\xi)K_m(\xi)K_{m+1}(\xi)\} . \tag{3.13b}$$

Expression (3.13a) is equal to the Coulomb excitation cross section first derived by Winther and Alder [2]) on the basis of semiclassical calculations. At first it may appear that this equality is due to the introduction of the cutoff parameter R in our calculations. To show that this is not really the case, we prove in the appendix that also in the absence of such a cutoff the PWBA total cross section is equal to the semiclassical one.

The cross section (3.13a) depends on the validity of first-order perturbation theory. Normally, this is a reasonable assumption since the excitation amplitudes (2.14) are very small. Only in extreme cases it will break down. If, for example, one uses eq. (2.14) to calculate the excitation amplitude for the giant dipole resonance in the reaction $^{238}\mathrm{U} + {}^{238}\mathrm{U}$ by taking $\theta = 0$, then one finds that it will approach unit magnitude. In that case, higher-order effects could be taken into account by means of e.g. Glauber approximation or semi-classical coupled-channel calculations.

4. Virtual-photon numbers

According to the virtual photon theory or Weizsäcker-Williams method [6,7]), the excitation of the target nucleus can be described as the absorption of virtual photons whose spectrum is determined by the Fourier time-integral of the electromagnetic interaction [4]). But, alternatively, we can also use the total cross section (3.13a) in order to obtain the virtual-photon spectrum.

Integrating (3.13a) for all energy transfers $\varepsilon = \hbar\omega$ and summing over all possible final states of the target, we obtain

$$\sigma_{\mathrm{c}} = \sum_{\mathrm{f}} \int \sigma(\varepsilon)\rho_{\mathrm{f}}(\varepsilon)\, \mathrm{d}\varepsilon , \tag{4.1}$$

where $\rho_{\mathrm{f}}(\varepsilon)$ represents now the density of final states of the target, with energy

$E_f^t = E_i^t + \varepsilon$. Inserting (3.13a) in (4.1), we can rewrite it in the form

$$\sigma_c = \sum_l \int \{n_{El}(\omega)\sigma_l^E(\omega) + n_{Ml}(\omega)\sigma_l^M(\omega)\}\, d\omega/\omega\,, \tag{4.2}$$

where $\sigma_l^{E/M}$ are the photonuclear absorption cross sections for a given multipolarity l:

$$\sigma_l^{E/M}(\omega) = \frac{(2\pi)^3(l+1)}{l[(2l+1)!!]^2}\sum_f \rho_f(\varepsilon)k^{2l-1}B(E/Ml)\,. \tag{4.3}$$

We then obtain the "equivalent photon numbers" $n_{\pi l}(\omega)$ given by

$$n_{\pi l}(\omega) = Z_p^2\alpha\frac{l[(2l+1)!!]^2}{(2\pi)^3(l+1)}\sum_m |G_{\pi lm}(c/v)|^2 g_m(\xi)\,. \tag{4.4}$$

Since all nuclear excitation dynamics is contained in the photon absorption cross section, the virtual-photon number (4.4) is independent of this process. It only depends on the way that the projectile moves. The virtual-photon theory consists of using its kinematics to calculate the intensity of the virtual radiation for a certain frequency interval. From that, one derives the virtual-photon numbers, which for a straight-line-moving projectile must be the same as those of eq. (4.4).

It was shown by Hoffmann and Baur [3]) that, for E1 excitations, the virtual-photon numbers obtained from the total cross section (3.13a) are really equal to that calculated by the Weizsäcker–Williams method. Nevertheless, while it is implicit in the Weizsäcker–Williams method that the virtual-photon numbers are the same for all multipolarities, eq. (4.4) shows that this is not the case. Indeed, a merit of eq. (4.4) is that it gives an analytical expression to calculate the virtual-photon numbers for all different multipolarities and radiation types, in contrast to the method followed by Goldberg [4]), which results in complicated integrals along the projectile trajectory.

By means of a more sophisticated version of the virtual-photon theory, Jäckle and Pilkuhn [5]) derived other expressions for n_{E1} and n_{M1}. In their calculations it was assumed that the projectile had a Yukawa charge distribution with parameter $a = \sqrt{\langle r_p^2\rangle/6}$, where $\sqrt{\langle r_p^2\rangle}$ is the charge mean square radius of the projectile. We can compare their expressions with the eq. (4.4) if we take in their results the projectile as a point particle ($a \to 0$). This leads to [1])

$$n_{E1}^{JP} = Z_p^2\alpha\frac{1}{\pi}\left\{\xi^2[K_0K_2 - K_1^2 - 2K_0(\phi)(K_2 - K_0)] + \frac{\xi^2}{\gamma^2}(K_1^2 - K_0^2) + 4\phi K_0(\phi)K_1(\phi)\right\}, \tag{4.5a}$$

where the K's are the modified Bessel functions as a function of ξ given by (3.5), except for the ones that are explicitly written as functions of

$$\phi = \omega R/v\,. \tag{4.6}$$

In the same way, one obtains

$$n_{\mathrm{M1}}^{\mathrm{JP}}(\omega)=Z_{\mathrm{p}}^{2}\alpha\frac{1}{\pi}\xi^{2}(K_0K_2-K_1^2)\,. \tag{4.5b}$$

On the other hand, using our expression (4.4), together with the definitions (2.16) and (3.13b), we find that

$$n_{\mathrm{E1}}(\omega)=Z_{\mathrm{p}}^{2}\alpha\frac{2}{\pi}\left(\frac{c}{v}\right)^{2}\left[\xi K_0K_1-\frac{v^2\xi^2}{2c^2}(K_1^2-K_0^2)\right], \tag{4.7a}$$

$$n_{\mathrm{M1}}(\omega)=Z_{\mathrm{p}}^{2}\alpha\frac{2}{\pi}[\xi K_0K_1-\tfrac{1}{2}\xi^2(K_1^2-K_0^2)]\,, \tag{4.7b}$$

and also

$$n_{\mathrm{E2}}(\omega)=Z_{\mathrm{p}}^{2}\alpha\frac{2}{\pi}\left(\frac{c}{v}\right)^{4}\left[2\left(1-\frac{v^2}{c^2}\right)K_1^2+\xi\left(2-\frac{v^2}{c^2}\right)^{2}K_0K_1+\frac{\xi^2}{2}\frac{v^4}{c^4}(K_0^2-K_1^2)\right]. \tag{4.7c}$$

By means of the recurrence relations for the K-functions, one can immediately show that

$$n_{\mathrm{M1}}^{\mathrm{JP}}(\omega)=n_{\mathrm{M1}}(\omega)\,.$$

But one cannot reduce eq. (4.5a) to eq. (4.7a). Moreover, one cannot understand why the consideration of a charge distribution for the projectile would modify the final results apart from influencing the value of the minimum impact parameter R. The Coulomb potential for a projectile, with a spherical distribution of charge in its rest frame is the same as that for a point particle with equal total charge. A Lorentz transformation to another inertial frame of reference obviously cannot modify this equality. All following results, such as cross sections or virtual-photon numbers, are therefore not changed by introduction of a spherical charge distribution for the projectile.

In fig. 1 we show $n_{\pi l}$ (with $Z_{\mathrm{p}}=$ unity) as a function of $\omega R/c$. We see that the E2 spectrum exceeds that for E1 by a factor 3–10. Nevertheless, over the range of ω where the virtual-photon numbers are large, the quadrupole cross sections of most nuclei are much smaller than the dipole cross sections. The same happens to the M1 transitions as compared to the E1 ones. We also see that there is an appreciable increase of the virtual-photon numbers for high frequencies with the increasing of the projectile energy. This implies that one can directly excite high-lying states by means of relativistic Coulomb collisions. In fig. 2 we see that, only in the extreme relativistic limit ($\gamma\gg 1$), the result of Jäckle and Pilkuhn (JPE1) agrees with the prediction of eq. (4.7a).

For a small-mass projectile one can improve eq. (4.4) based on semiclassical ideas. In order to see how it works we compare the final improved expressions with

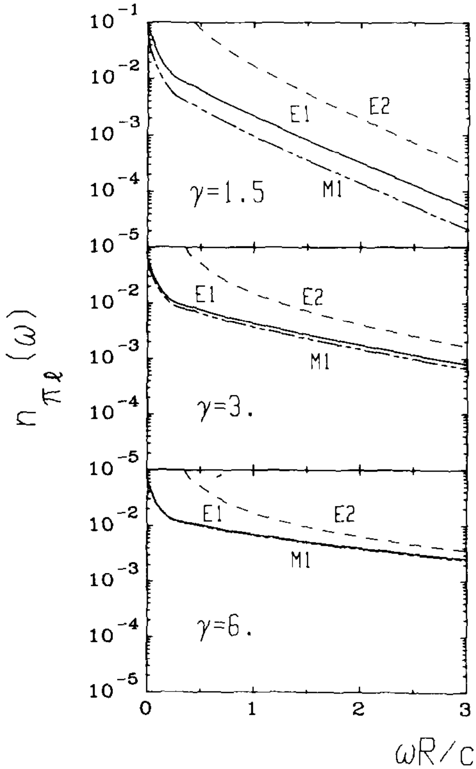

Fig. 1. Virtual-photon number per unit projectile charge, for E1, M1 and E2 radiation, and as a function of the ratio between R and the photon wavelength. γ is the ratio of the projectile energy to its rest energy.

those obtained in the ultra-relativistic electron-nucleus scattering. But, besides the spin interactions, electron scattering is different from Coulomb excitation because the electrons can penetrate the nucleus and continue interacting only electromagnetically with it. Nevertheless, in the long-wavelength limit $q_e R \ll 1$, where q_e is the momentum transfer of the electron, the nuclear volume plays a minor role and the matrix elements contributing to the excitation in the near-forward scattering are just that appearing in the photo-excitation process, with $q_e \simeq k$. To disregard the nuclear volume means to put $R = 0$ in expression (4.4). But in that case it goes to infinity, due to the neglect of the function $\Gamma(R, \rho)$ in eq. (2.10). If we now evoke semiclassical ideas we note that a normal procedure [6]) within the virtual-photon theory is to use the quantum wavelength $\hbar/\gamma M_p v$ of the projectile, instead of the nuclear radius, as the minimum impact parameter when the projectile's mass M_p is small. This assumption is based on the uncertainty principle, which introduces a "smearing out" of the projectile's coordinate in a space interval of about its wavelength. By means of this recipe, we then replace (3.5) by

$$\xi_e = \hbar\omega / \gamma^2 m_e v^2 , \tag{4.8}$$

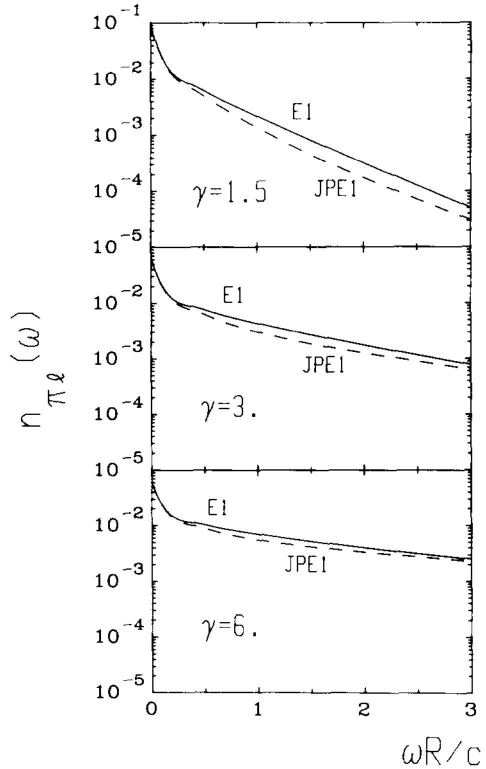

Fig. 2. Comparison of the virtual-photon number per unit charge with the Jäckle and Pilkuhn result (JPE1) for the electric-dipole radiation (see text).

where m_e is the electron rest mass. This quantity is generally much less than one, so that the expressions (4.7) become

$$n_{E1}^{\text{electron}} = (2/\pi)\alpha[\ln(1/\xi_e) - \delta] = n_{M1}^{\text{electron}}, \tag{4.9a}$$

$$n_{E2}^{\text{electron}} = (2/\pi)\alpha[1/\gamma^2\xi_e^2 + \ln(1/\xi_e) - \delta], \tag{4.9b}$$

where $\delta = 0.384\ldots$.

Aside from an irrelevant factor, these are just the results that one derives from the virtual-photon numbers for electron scattering, first obtained by Thie *et al.* [9]), in the ultra-relativistic limit with small energy loss, where one can put $E_i \simeq E_f = \gamma m_e c^2 \gg m_e c^2$.

Besides of a more transparent description of the excitation process, the virtual photon numbers can be used to obtain the cross sections for relativistic Coulomb fragmentation in heavy-ion collisions by using the experimentally measured photon-absorption cross sections, instead of the theoretical ones, in the eq. (4.2). This was indeed the procedure followed by Olson *et al.* [1]) but with the bad assumption that the virtual-photon numbers should be equal to $n_{E1}(\omega)$ for all multipolarities.

5. Conclusions

The study of relativistic Coulomb collisions by means of the PWBA was useful in obtaining new insight into this subject. One interesting feature in this approximation is the absence of magnetic excitations of the target nucleus in the case of forward scattering of the projectile. The cross section integrated over angles is shown to be equal to the semiclassical one integrated over impact parameters. In order to obtain this result, we had to introduce some assumptions, like the consideration of small momentum transfers to the projectile and a cylindrical hole cutoff. Nevertheless, these assumptions are expected to be suitable in the high-energy region.

By factorizing the cross section integrated over the excitation energy, we reached an expression for the number of equivalent photons, related to the virtual-photon theory, for different multipolarities and frequencies of the electromagnetic radiation. A comparison with the results derived by other methods was useful to clarify some points in this matter.

We are grateful to Professor J. Speth for the hospitality given to one of the authors (C.A.B.).

Appendix

We shall prove that the PWBA and semiclassical total cross sections are the same for the electromagnetic interaction of relativistic spin-zero particles with an atomic or a nuclear system. The proof is similar to the one given by ref. [8]) in the non-relativistic case. Using the integral representation

$$\frac{\mathrm{e}^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}=\frac{1}{2\pi^2}\int\frac{\mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{p^2-k^2}\,\mathrm{d}^3p\,, \tag{A.1}$$

eq. (2.1) can be written as

$$a_{\mathrm{fi}}=\frac{Z_{\mathrm{p}}e}{2\pi^2}\int\!\!\int \mathrm{d}^3p\,\mathrm{d}^3r'\,\frac{\mathrm{e}^{i(\boldsymbol{q}-\boldsymbol{p})\cdot\boldsymbol{r}'}}{p^2-k^2}F(\boldsymbol{p})\,, \tag{A.2}$$

where

$$\boldsymbol{q}=(q_x,q_y,q_z)\equiv(\boldsymbol{q}_{\mathrm{T}},\omega/v)\,, \tag{A.3}$$

$$F(\boldsymbol{p})=\int \mathrm{d}^3r\left\langle I_{\mathrm{f}}M_{\mathrm{f}}\left|\frac{v_\mu}{c^2}j_\mu(\boldsymbol{r})\,\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{r}}\right|I_{\mathrm{f}}M_{\mathrm{f}}\right\rangle. \tag{A.4}$$

Integrating over $\boldsymbol{r}'$ we obtain

$$a_{\mathrm{fi}}=4\pi Z_{\mathrm{p}}e\frac{F(\boldsymbol{q})}{q^2-k^2}\,. \tag{A.5}$$

According to the relations (3.1) and (3.11) the total cross section is

$$\sigma_{\mathrm{PWBA}} = 8\pi\left(\frac{Z_{\mathrm{P}}e}{\hbar v}\right)^2 \frac{1}{2I_{\mathrm{i}}+1} \sum_{M_{\mathrm{i}}M_{\mathrm{f}}} \int_0^{q_{\mathrm{T}}^{\max}} \frac{|F(\boldsymbol{q})|^2}{(q^2-k^2)^2} q_{\mathrm{T}}\,\mathrm{d}q_{\mathrm{T}}\,. \tag{A.6}$$

In the semiclassical calculations the excitation amplitude is given by

$$T_{\mathrm{fi}} = \frac{1}{i h} \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{i\omega t} \langle I_{\mathrm{f}}M_{\mathrm{f}}| \frac{v_\mu}{c^2} j_\mu(\boldsymbol{r}) \phi(\boldsymbol{r}, t) | I_{\mathrm{i}}M_{\mathrm{i}}\rangle\,, \tag{A.7}$$

where

$$\phi(\boldsymbol{r}, t) = \frac{Z_{\mathrm{p}}e\gamma}{[(x-b_x)^2+(y-b_y)^2+\gamma^2(z-vt)^2]^{1/2}} \tag{A.8}$$

is the Liénard–Wiechart potential for a charged particle moving in a straight line with an impact parameter $b=\sqrt{b_x^2+b_y^2}$. By means of the Bethe integral we can write

$$\phi(\boldsymbol{r}, t) = \frac{Z_{\mathrm{p}}e\gamma}{2\pi^2} \int \mathrm{d}^3p \frac{\mathrm{e}^{i\boldsymbol{p}\cdot[\boldsymbol{R}-\boldsymbol{R}'(t)]}}{p^2}\,, \tag{A.9}$$

where

$$\boldsymbol{R} = (x, y, \gamma z)\,, \qquad \boldsymbol{R}' = (b_x, b_y, \gamma v t)\,. \tag{A.10}$$

The integral in t yields

$$\frac{2\pi}{\gamma v}\delta(p_z - \omega/\gamma v)\,,$$

and therefore

$$T_{\mathrm{fi}} = \frac{Z_{\mathrm{P}}e}{\pi i \hbar v} \int \mathrm{d}^2 p_{\mathrm{T}} \frac{F(\boldsymbol{p}')}{p_{\mathrm{T}}^2 + (\omega/\gamma v)^2} \mathrm{e}^{i\boldsymbol{p}_{\mathrm{T}}\cdot\boldsymbol{b}}\,, \tag{A.11}$$

where

$$\boldsymbol{p}' = (\boldsymbol{p}_{\mathrm{T}}, \omega/v)\,. \tag{A.12}$$

The total cross section is obtained by integrating the above squared expression over all possible impact parameters:

$$\begin{aligned}\sigma_{\mathrm{s.c.}} &= \frac{1}{I_{\mathrm{i}}+1} \sum_{M_{\mathrm{i}}M_{\mathrm{f}}} \int |T_{\mathrm{fi}}|^2\,\mathrm{d}^2b \\ &= 8\pi\left(\frac{Z_{\mathrm{P}}e}{\hbar v}\right)^2 \frac{1}{2I_{\mathrm{i}}+1} \sum_{M_{\mathrm{i}}M_{\mathrm{f}}} \int_0^{\infty} \frac{|F(\boldsymbol{p}')|^2}{[p_{\mathrm{T}}^2+(\omega/\gamma v)^2]^2} p_{\mathrm{T}}\,\mathrm{d}p_{\mathrm{T}}\,.\end{aligned} \tag{A.13}$$

Then, since

$$p_{\mathrm{T}}^2 + (\omega/\gamma v)^2 = p'^2 - k^2\,, \tag{A.14}$$

the equality between the semiclassical and the PWBA cross section is guaranteed if we are allowed to replace q_T^{max} by infinity in (A.6), which is generally the case as soon as the form factor $F(\boldsymbol{q})$ is a rapidly decreasing function of q_T.

References

1) D.L. Olson *et al.*, Phys. Rev. **C24** (1981) 1529
2) A. Winther and K. Alder, Nucl. Phys. **A319** (1979) 518
3) B. Hoffmann and G. Baur, Phys. Rev. **C30** (1984) 247
4) A. Goldberg, Nucl. Phys. **A420** (1984) 636
5) R. Jäckle and H. Pilkuhn, Nucl. Phys. **A247** (1975) 521
6) J.D. Jackson, Classical electrodynamics (Wiley, New York, 1975) p. 719
7) C.F. Weizsäcker, Z. Phys. **88** (1934) 612;
E.J. Williams, Phys. Rev. **45** (1934) 729
8) H.A. Bethe and R.W. Jackiw, Intermediate quantum mechanics (Benjamin, New York, 1968) p.326
9) J.A. Thie, C.J. Mullin and E. Guth, Phys. Rev. **87** (1952) 962
10) J.M. Eisenberg and W. Greiner, Excitation mechanisms of the nucleus (North-Holland, Amsterdam, 1970) p. 227
11) Handbook of mathematical functions, ed. M. Abramowitz and I. A. Stegun (National Bureau of Standards, Washington, DC, 1964) p. 363
12) S.T. Butler, Phys. Rev. **80** (1950) 1095
13) G.R. Satchler, Rev. Mod. Phys. **50** (1978) 1
14) M. Jacob and G.C. Wick, Ann. of Phys. **7** (1959) 404