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TWO-NEUTRON REMOVAL CROSS SECTIONS OF ^{11}Li PROJECTILES

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Abstract: We investigate the interplay of the nuclear and Coulomb interaction in the fragmentation of relativistic ^{11}Li projectiles incident on several targets. The ^{11}Li nucleus is assumed to have a cluster-like structure, with a (bound) di-neutron system coupled to a ^{9}Li core in an s-state. The obtained nuclear cross sections show marked differences with the "experimental" data. Taking the calculated nuclear contribution at face value results in a Coulomb dissociation cross section, σ_C, which is well reproduced by recent modified-RPA calculation. The pure cluster model then overestimates σ_C by about 20%.

1. Introduction

The fragmentation of neutron-rich nuclei has led to many unusual speculative ideas about their structure. Perhaps, the most interesting one is due to Hansen and Jonson [1]), who proposed a clusterlike structure for ^{11}Li as composed by a di-neutron system loosely bound to a ^{9}Li-core. This hypothesis has had a general support from several other authors [2-6]). It seems that such cluster structure occurs very often in light neutron-rich nuclei and results from a delicate balance between the neutron-neutron and neutron-core interactions [2]). The Hansen–Jonson model is supported by several facts. Firstly, the separation energy of two neutrons from ^{11}Li is very low [7,8]), $S_{2n} = 250 \pm 80$ keV. Otherwise, the nucleus ^{10}Li does not exist [9]), having a resonant continuum state at 800 ± 250 keV. This means that the neutron–neutron interaction acquires a stronger attractive character in the presence of the ^{9}Li core. Secondly, the experimental measurements of total reaction cross sections [10]) of neutron-rich nuclei incident on several targets at 0.8 GeV/nucleon reveal an r.m.s. radius of 3.14 ± 0.06 fm for ^{11}Li, compared to an r.m.s. radius of 2.41 ± 0.02 fm for ^{9}Li. A large increase of matter radius from ^{12}Be to ^{14}Be, and possibly from ^{15}B to ^{17}B, is also observed. The last two neutrons are responsible for the unusual increase of the matter radius and for the appearance of a "neutron halo" in these nuclei. In the cluster model the existence of such a halo can easily be explained as being due

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to the low binding energy of the di-neutron system. In fact, by assuming a deuteron-like wavefunction for ^{11}Li and adjusting it to reproduce the binding energy of the di-neutron system, an approximate r.m.s. mean distance of the di-neutron to the core of 6 fm is obtained. This would essentially explain the r.m.s. radius of ^{11}Li as roughly given by $\frac{2}{11}R^{(2n)}_{\text{m.s.}}+\frac{9}{11}R^{(9)}_{\text{m.s.}}=\frac{2}{11}\times 6+\frac{9}{11}\times 2.41\simeq 3.1$ fm.

Another support for the cluster-model for ^{11}Li is that the experimentally determined [10]) electromagnetic dissociation cross sections for ^{11}Li can be well described theoretically [1-6]). The momentum distribution of the ^{9}Li fragments are also well fitted within this model, as was shown in ref. [4]). On the other hand, conventional shell model calculations performed by Bertsch and collaborators [11-13]) were able to produce an amount of electric dipole strength in ^{11}Li which is about 20% less than the reported value of the electromagnetic dissociation cross section of 0.9 b. The above result was obtained with a very small value of the binding of the $1P_{3/2}$ level. Similar reults as in ref. [11]) were obtained recently with a hybrid RPA cluster calculation, where the di-neutron is given a distine role [14]). As concluded by Bertsch and Foxwell [11]) it may be essential to take cluster aspects into account. Still remaining differences between model calculations to determine the calculated value of 0.7 b and the reported experimental value of 0.6 b has lead the authors of ref. [13]) to argue if experimental values of the electromagnetic dissociation cross sections [10]) have been correctly extracted from the total cross sections.

In sect. 2 we develop a theoretical calculation of the two-neutron removal cross sections of ^{11}Li projectiles. The basic inputs are the nucleon–nucleon cross sections and the nucleon densities of the projectile and target nuclei. It is considered separately the direct and the stripping contributions to the process. The part of the cross section induced by the Coulomb interactionis given for the E1 and E2 multipolarities; following ref. [3]).

In sect. 3 we present the results of the numerical calculations and the analysis of the results. In sect. 4 we present our conclusions.

2. Removal of two neutrons from ^{11}Li

In ref. [10]) it is assumed that the nuclear cross section scales as $\sigma_N = 2\pi(R_P+R_T)\Delta$, which is characteristic of a peripheral process concentrated in a small ring width Δ at the surface of the projectile. By adjusting the parameters of this scaling law for ^{12}C targets, where the Coulomb contribution to the total cross section is negligible, the "experimental" values of σ_N were obtained for other targets, and the Coulomb contribution σ_C to the cross section were inferred by substraction. But since ^{11}Li has a long tail in its matter distribution, such procedure is doubtful. Assuming that the target is a "black dish" the nuclear stripping of the outer nucleons in ^{11}Li should be

$$\sigma_N \sim 2\pi(R_P+R_T)\Delta P(R_T)\,, \tag{2.1}$$

where $P(R_T)$ is the probability that the outer neutrons will be removed from ^{11}Li. Due to the long matter tail, this probability is not independent of R_T. Actually it should be approximately proportional to the area A of overlap between the target and the neutron halo in ^{11}Li. From simple geometrical considerations it is possible to show that $A \propto R_T$. That is, σ_N should increase like R_T^2, which has also as a consequence that "σ_C^{exp}" should be smaller than the values determined by Kobayashi *et al.* [10]), and would come closer to the RPA calculations of Bertsch *et al.* [11]) and Teruya *et al.* [14]) for σ_C. This is indeed a very relevant point since the electromagnetic dissociation of neutron-rich nuclei reveals important aspects of their intrinsic structure.

We analyse the interplay of the nuclear and the Coulomb interaction in the reaction process

$$^{11}\mathrm{Li} + \mathrm{target} \rightarrow {}^{9}\mathrm{Li} + \mathrm{anything} \tag{2.2}$$

at kinetic energies of 800 MeV/nucleon. As shown in ref. [4]) the nuclear Coulomb interference for the process (2.2) should be at most 5% of the total cross section. Then, we may write the cross section as

$$\sigma = \sigma_D^{(N)} + \sigma_S^{(N)} + \sigma_C , \tag{2.3}$$

where $\sigma_D^{(N)}$ is the elastic (diffractive) nuclear breakup of $^{11}\mathrm{Li} \rightarrow {}^{9}\mathrm{Li} + (2n)$ by the target and $\sigma_S^{(N)}$ is the inelastic (stripping) cross section arising when the 2n-system suffers an inelastic collision with the target, while ^{9}Li survives intact. σ_C is the electromagnetic dissociation (Coulomb) cross section for $^{11}\mathrm{Li} \rightarrow {}^{9}\mathrm{Li} + (2n)$.

Nuclear peripheral process in high energy collisions involve the calculation of eikonal phases which are dependent on the nuclear densities at the surface and on the nucleon–nucleon scattering amplitudes. For a projectile an incident on a target A, the cross sections for peripherally induced processes are well described by adjusting the tails of the density functions so as to reproduce the correct values of the eikonal phases. This procedure results in an effective optical potential [15,16]) of the form

$$U_{aA} = \langle t_{NN} \rangle \pi^{3/2} \rho_A(0) \rho_a(0) \frac{a_a^3 a_A^3}{a^3} e^{-r^2/a^2} \tag{2.4}$$

where the nucleon parameters are given by

$$a = \sqrt{a_a^2 + a_A^2} , \qquad a_i^2 = \frac{4R_i t + t^2}{4 \ln 5} ,$$

$$R_i = 1.07 A_i^{1/3}\ \mathrm{fm} , \qquad t = 2.4\ \mathrm{fm} ,$$

$$\rho_i(O) = \frac{3A_i\, e^{R_i^2/a_i^2}}{8\pi R_i^3} [1 + (\pi^2 t^2 / 19.36 R_i^2)]^{-1} . \tag{2.5}$$

The free nucleon-nucleon amplitude $\langle t_{NN}(E)\rangle$ in forward direction ($\Theta = 0°$) can be deduced from the experiment. It can be written as

$$\langle t_{NN}(E)\rangle = -\frac{E}{K}\langle\sigma_{NN}\rangle[\langle\alpha_{NN}\rangle + i]\,,$$

where the angle brackets mean an isospin average of $t_{NN}(E)$ and α_{NN} over the projectile and target nucleons. For 800 MeV/nucleon, one may use [17])

$$\sigma_{pp} = 47.3\text{ mb} \qquad \sigma_{pn} = 37.9\text{ mb}\,,$$

$$\alpha_{pp} = 0.06 \qquad \alpha_{pn} = -0.2\,. \tag{2.6}$$

One observes that at such energy the nucleon-nucleon scattering amplitude is almost totally imaginary, meaning that the optical potential (2.4) is almost completely absorptive.

The transition matrix element for the elastic (diffractive) breakup in DWBA is

$$T_{fi} = \langle\chi^{(-)}_{k_a}(\boldsymbol{R})\phi^{(-)}_{xb,f}(\boldsymbol{r})|[U_{xA}(\boldsymbol{r}_{xA}) + U_{bA}(\boldsymbol{r}_{bA}) - U_{aA}(\boldsymbol{R}_{aA})]|\chi^{(+)}_{k_a}(\boldsymbol{R})\phi^{(+)}_{xb,i}\rangle\,, \tag{2.7}$$

where ϕ_{xb} is the wave function for the relative motion of x + b clusters (in our case b = di-neutron, a = ^{11}Li and x = ^{9}Li), and $\chi^{(+)}_a$ is the distorted wave for particle a. In the final state, $\chi^{(-)}_a$ represents the distorted wave in the c.m. of x + b. In the way (2.7) is written, the matrix element of U_{aA} is zero because $\langle\phi^{(-)}_{xb,f}|\phi^{(+)}_{xb,i}\rangle = 0$.

We use the c.m. distorted waves

$$\chi^{(+)}_{a,i}(\boldsymbol{R}) = e^{i\boldsymbol{K}_i\cdot\boldsymbol{R}}\exp\left\{-\frac{ik}{2E}\int_{-\infty}^{z} U_{aA}(z', b)\,dz' + i\phi_C(b)\right\}\,, \tag{2.8a}$$

$$\chi^{(-)*}_{a,f} = e^{-i\boldsymbol{K}_f\cdot\boldsymbol{R}}\exp\left\{-\frac{ik}{2E}\int_{z}^{\infty} U_{aA}(z', b)\,dz' + i\phi_C(b)\right\}\,, \tag{2.8b}$$

where

$$\phi_C(b) = \frac{Z_a Z_A \alpha}{v/c}\ln(kb)$$

is the Coulomb phase, and $\alpha \simeq 1/137$.

For the relative motion wave functions $\phi^{(+)}_{xb,i}$ and $\phi^{(-)}_{xb,f}$ we use simple Yukawa and plane-wave functions as in ref. [3]). All coordinates refer to the lab system, with the target as origin. The coordinates $\boldsymbol{r}_{xA}$ and $\boldsymbol{r}_{bA}$ are defined by

$$\boldsymbol{r}_{xA} = \boldsymbol{R} - \frac{m_b}{m_a}\boldsymbol{r}\,, \qquad \boldsymbol{r}_{bA} = \boldsymbol{R} + \frac{m_x}{m_a}\boldsymbol{r}\,.$$

Most of the integrals involved in (2.7) may be calculated analytically and the details of the calculations will be shown elsewhere [18]). The breakup cross section is obtained by standard integrations over the phase space of the fragments [18]). For $R_{^{11}\text{Li}}$, $R_{^{9}\text{Li}}$ and R_{2n} we use, 5.8, 2.41 and 1.6 fm, respectively. These values are compatible with the cluster wave function of ^{11}Li, adjusted to reproduce the binding energy of the di-neutron. The three-body calculations of ref. [6]) have shown that the most probable separation between these neutrons is 3.3 fm.

The "stripping" (inelastic breakup) cross section is given by [19])

$$\sigma_S = \frac{\sqrt{\pi}}{\Lambda} \int d^2 b_x \, |S_x(b_x)|^2 \int d^2 b_{2n} \, |\phi_{^{11}\mathrm{Li}}(|\boldsymbol{b}_x - \boldsymbol{b}_{2n}|)|^2 [1 - |S_{2n}(b_{2n})|^2] \,, \qquad (2.9)$$

where $|S_x(b_x)|^2$ is to be interpreted as the probability that the fragment x(^{9}Li) will survive when hitting the target at an impact parameter b_x. Otherwise, $1 - S_{2n}(b_{2n})^2$ is the probability that the 2n system will suffer an inelastic collision with the target, and $|\phi_{^{11}\mathrm{Li}}(|\boldsymbol{b}_x - \boldsymbol{b}_{2n}|)|^2$ is the probability that the 2n system is found at distance $|\boldsymbol{b}_x - \boldsymbol{b}_{2n}|$ from ^{9}Li. The factor in front of (2.9) comes from the assumption that $\phi_{^{11}\mathrm{Li}}$ can be described by a gaussian wave function, so that

$$|\phi_{^{11}\mathrm{Li}}|^2 = \frac{\Lambda^3}{\pi\sqrt{\pi}} \exp[-\Lambda^2 (z_x - z_{2n})^2] \exp[-\Lambda^2 (\boldsymbol{b}_x - \boldsymbol{b}_{2n})^2] \,. \qquad (2.10)$$

Eq. (2.10) was obtained after an integration over z_x and z_{2n}. The parameter Λ was chosen so that the stripping cross sections obtained by using (2.10) do not differ appreciably from what is obtained by using Yukawa-type wave functions. The proper value of Λ was found to be given by $\Lambda = (11.2 \text{ fm})^{-1}$. This parametrization allows us to write the stripping cross section in an elegant form as

$$\sigma_S = \frac{\pi}{\Lambda^2} \sum_{j=0}^{\infty} [1 - T_j^{(2n)}(\Lambda)] T_j^{x}(\Lambda) \,, \qquad (2.11a)$$

$$T_j^{(i)}(\Lambda) = \frac{2(\Lambda^2)^{j+1}}{j!} \int_0^\infty b_i^{2j+1} \, e^{-\Lambda^2 b_i^2} |S_i(b_i)|^2 \, db_i \qquad (i = x \text{ or } b) \,. \qquad (2.11b)$$

The expression (2.11b) is obtained by means of a series expansion of the Bessel function which results from the integration of (2.9) over the azimuthal angle. The factors $|S_i(b_i)|^2$ are given by

$$|S_i(b_i)|^2 = \exp\left\{ -\frac{k}{E} \int_{-\infty}^{\infty} |\mathrm{Im}\, U_i(b_i, z_i)| \, dz_i \right\} , \qquad (2.12)$$

where U_i are the optical potentials for 2n + target and ^{9}Li + target, parametrized by eq. (2.4).

In addition to the nucleon fragmentation there is an important contribution from Coulomb dissociation, especially for large-Z targets. We can use the formulas obtained in ref. [3]) for the Coulomb dissociation of *cluster nuclei*, which in the limit of very low binding energy, can be written as

$$\sigma_{E1} = \tfrac{4}{3}\pi Z_T^2\alpha^2\left(\frac{c}{v}\right)^2\left[\frac{m_x Z_b - m_b Z_x}{m_a}\right]^2 \frac{1}{\eta^2}\left[\ln\left(\frac{\gamma\hbar v}{\delta\varepsilon R}\right) - \frac{v^2}{2c^2}\right], \tag{2.13a}$$

$$\sigma_{E2} = \tfrac{1}{5}\pi Z_T^2\alpha^2\left(\frac{c}{v}\right)^4\left[\frac{m_x^2}{m_a^2}Z_b + \frac{m_b^2}{m_a^2}Z_x\right]^2 \frac{\varepsilon^2}{\eta^4(\hbar c)^2}$$
$$\times\left[\frac{2}{\gamma^2\xi^2} + \left(2-\frac{v^2}{c^2}\right)^2 \ln\left(\frac{1}{\delta\xi}\right) - \frac{v^4}{2c^4}\right]. \tag{2.13b}$$

The total Coulomb cross section is given quite accurately by (M1 does not contribute significantly)

$$\sigma_C = \sigma_{E1} + \sigma_{E2}\,. \tag{2.13c}$$

In the above equations, $\gamma = (1 - v^2/c^2)^{-1/2}$, $\delta = 0.891\ldots$, $\varepsilon = \hbar^2\eta^2/(2\mu_{bx})$ is the binding energy of the cluster nucleus, and $\xi = \varepsilon b_{min}/(\gamma\hbar v)$. We use $b_{min} = R_{^{11}Li} + R_T$, with $R_T = 1.2A_T^{1/3}$ fm.

3. Results and discussion

As a byproduct of our approach to the nuclear part of the cross section, we can also calculate the total reaction cross section for the reaction ^{11}Li+A by means of the relation

$$\sigma_R^N = 2\pi\int_0^\infty b\,db\,[1 - |S(b)|^2]$$

where $|S(b)|^2$ is given by eq. (2.12), but with the potential $U_{Li,A}$ constructed in the way of the eqs. (2.4)-(2.6). To this reaction cross section one should add the contribution of the Coulomb interaction. The most important channel in this case is the two-neutron emission, where can be obtained within an RPA approach as in refs. [11,14]) or within the cluster model approach, as described above.

The cross section of the nuclear elastic breakup $\sigma_{elast}^{(N)}$, stripping $\sigma_{inel}^{(N)}$, electric dipole σ_{E1}^{C} and electric quadrupole σ_{E2}^{C} are given in table 1 together with the experimental data for the two-neutron removal of ^{11}Li incident on ^{12}C, ^{63}Cu and ^{208}Pb. The $\sigma_{elast}^{(N)}$ and $\sigma_{inel}^{(N)}$ for $\varepsilon = 0.2$ MeV were multiplied by a factor 1.23 in order that their sum with the Coulomb contribution would result in the experimental value for ^{12}C, which is 220 mb. The cross sections were also calculated for several other binding energies, from 0.17 to 0.33 MeV.

The elastic breakup and particularly the total Coulomb cross section decrease appreciably with the binding energy, whereas the stripping cross section, having a geometrical character, does not depend on ε (if one assumes that the ^{11}Li radius is fixed).

In fig. 1 we plot the nuclear contribution to the two-neutron removal cross section as compared to the experimental data. Due to the uncertainty of the binding energy of the di-neutron, the calculated values lie between the two solid curves. One indeed

TABLE 1

The elastic (σ_N^{elast}), inelastic (σ_N^{inel}), nuclear ($\sigma_N = \sigma_N^{elast} + \sigma_N^{inel}$), electric dipole ($\sigma_{E1}$), electric quadrupole ($\sigma_{E2}$), Coulomb ($\sigma_C + \sigma_{E1} + \sigma_{E2}$), nuclear experimental (σ_N^{exp}), and Coulomb experimental (σ_C^{exp}) cross sections for the dissociation of ^{11}Li (0.8 GeV/nucleon) projectiles incident on several targets, as a function of the binding energy of the ^{9}Li + di-neutron system

ε	σ_{elast}^N	σ_{inel}^N	σ_N	σ_{E1}	σ_{E2}	σ_C	σ_N^{exp}	σ_C^{exp}
^{11}Li + ^{12}C								
0.17	79	136	215	9.1	0.5	9.6		
0.2	76	136	212	7.6	0.4	8.0	220	0
0.25	73	136	209	5.9	0.3	6.2	±10	
0.3	70	136	206	4.8	0.2	5.0		
0.33	69	136	205	4.3	0.2	4.5		
^{11}Li + ^{63}Cu								
0.17	187	223	410	203	8	211		
0.2	180	223	403	169	6	175	320	210
0.25	170	223	393	131	5	136	±20	±40
0.3	162	223	385	105	4	109		
0.33	158	223	381	94	3	97		
^{11}Li + ^{208}Pb								
0.17	339	315	654	1565	43	1608		
0.2	324	315	639	1295	33	3128	420	890
0.25	304	315	619	996	24	1020	±30	±100
0.3	289	315	604	803	17	820		
0.33	281	315	596	717	15	732		

observes that the calculated cross sections grow faster than the $A^{1/3}$ law, a result that was also obtained by Bertsch *et al.* [13]) with a different method.

By choosing the binding energy of $\varepsilon = 0.2$ MeV, we find the following parametrization of σ_N with A_T

$$\sigma_N = (aA_T^{1/3} + bA_T^{2/3} + c)\ \text{mb}\,, \tag{3.1a}$$

with

$$a = 98.7\,, \qquad b = 2.284\,, \qquad c = -25.89\,. \tag{3.1b}$$

For large values of A_T, the above equation results in an appreciable deviation from the $A_T^{1/3}$ scaling law [10]).

In contrast to the above results, the nuclear contribution to the total reaction cross section agrees perfectly with the experimental data, as shown in table 2, for five different targets. Data are from ref. [10]). As expected, the nuclear reaction cross section is given by the sum of the geometrical areas of the nuclei. Due to the low binding energy of ^{11}Li it also practically agrees with the definition of ref. [10]) for the "interaction" cross section

$$\sigma_I = \pi(R_A + R_{Li})^2\,,$$

where $R_A = 1.355A^{1/3} - 0.365$ fm, and $R_{Li} = 3.14 \pm 0.06$ fm. Again, we adjust our results so that the data for a berilium target could be reproduced. This amounted to a normalization factor of 1.18.

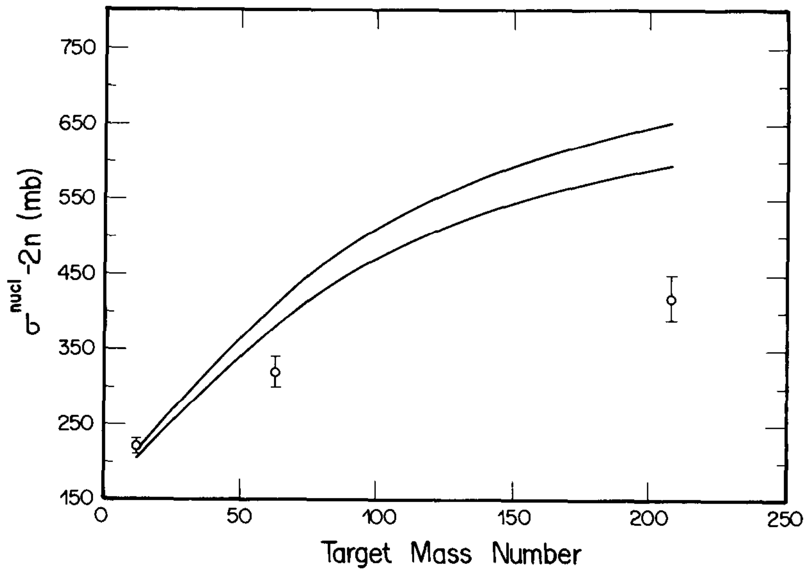

Fig. 1. Two-neutron removal cross sections of ^{11}Li (0.8 GeV/nucleon) projectiles due to the nuclear interaction with the targets, as a function of the target number. Due to the uncertainty of the binding energy of ^{11}Li, the theoretical results lie between the two solid curves. The experimental data of ref. [10]) are also shown.

The electromagnetic dissociation experimental cross sections obtained in ref. [10]) are within the limits of the theoretical pure cluster results, as shown in fig. 2. We observe that the scale is logarithmic and that the Coulomb cross section dependent on the binding energy of the di-neutron+^{9}Li. This dependence is approximately proportional to the inverse of ε (see eq. (13a)). The lower solid curve in fig. 2 corresponds to $\varepsilon = 0.33$ MeV, while the upper curve corresponds to $\varepsilon = 0.17$ MeV. If the nuclear contribution to the process actually scales as in eq. (3.1), the experimental values of the Coulomb contribution (fig. 2) should be smaller. In this case, the cluster model would not reproduce the experimental data on Coulomb dissociation, being larger by 20–30%, especially for high-Z targets. However, the RPA results of refs. [11,14]) would then fall within the "experimental" results.

TABLE 2

Reaction cross sections (in barns) ^{11}Li+target

	Target				
	Be	C	Al	Cu	Pb
σ_{exp}	0.98 ± 0.02	1.04 ± 0.02	1.41 ± 0.04	2.10 ± 0.06	3.66 ± 0.08
σ_{theory}	0.98	1.02	1.36	2.00	3.48

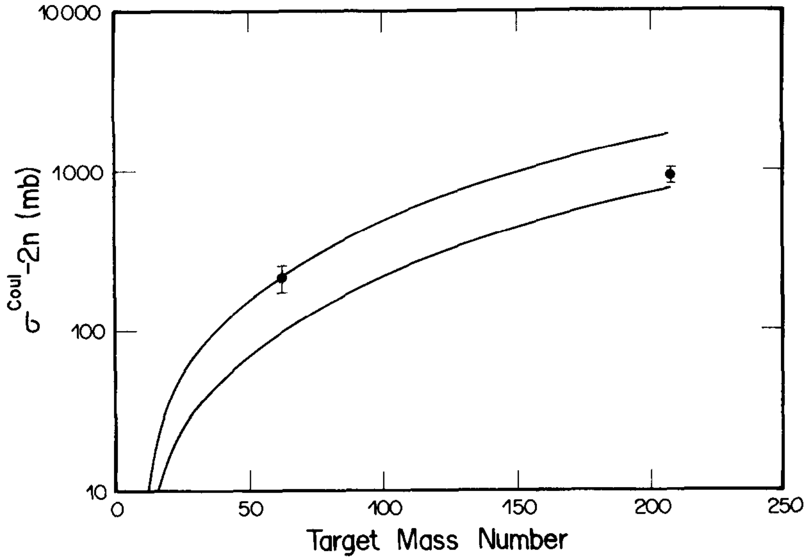

Fig. 2. Same as fig. 1, but for the electromagnetic dissociation of ^{11}Li.

The merit of the cluster model is that it gives the necessary amount of the electromagnetic dipole strength at low energies, so that the Coulomb dissociation cross section of ^{11}Li comes out appreciable. The matrix elements for the photo-disintegration of ^{11}Li within the cluster model were firstly calculated in ref. [3]). From their results we obtain for the electric dipole strength distribution

$$\frac{\mathrm{d}B\,(\mathrm{E1};\uparrow)}{\mathrm{d}(\hbar\omega)}=\frac{3\hbar^2e^2}{\pi^2\mu_{\mathrm{bx}}}\left[\frac{Z_{\mathrm{x}}m_{\mathrm{b}}-Z_{\mathrm{b}}m_{\mathrm{x}}}{m_{\mathrm{a}}}\right]^2\frac{\sqrt{\varepsilon}(\hbar\omega-\varepsilon)^{3/2}}{(\hbar\omega)^4}\,, \tag{3.2}$$

where $\mu_{\mathrm{bx}}(\varepsilon)$ is the reduced mass (binding energy) of the cluster system. The dipole strength function for ^{11}Li, assuming $\varepsilon=0.2$ MeV, has a peak at $\hbar\omega=0.32$ MeV. In spite of the fact that the cluster model as described here is very simplified, the above results indicate that in order to obtain the necessary amount of electric dipole strength of ^{11}Li at low energies, it is necessary either to make unconventional changes in the mean field as done in ref. [11]), or include cluster aspects in the shell model calculations, as was done in refs. [5,6]), and in the RPA calculation as was done in ref. [14]).

From (3.2) we obtain that the total dipole strength in the cluster model, integrated over energy, is given by

$$B(\mathrm{E1})=\frac{3\hbar^2e^2}{16\pi\mu_{\mathrm{bx}}\varepsilon}\left[\frac{Z_{\mathrm{x}}m_{\mathrm{b}}-Z_{\mathrm{b}}m_{\mathrm{x}}}{m_{\mathrm{a}}}\right]^2 \tag{3.3}$$

for ^{11}Li, using $\varepsilon=0.2$ MeV, we obtain $B(\mathrm{E1})/e^2=2.25\ \mathrm{fm}^2$ in the cluster model, which is about 80% of the cluster sum rule for dipole excitations [20]) and 7% of the total nuclear dipole sum rule. This means that in order to reproduce the experimental data on the Coulomb dissociation of ^{11}Li, an appreciable amount of the strength of the dipole response in ^{11}Li should be located at the ^{9}Li + 2n channel. The Coulomb cross section is given by

$$\sigma_C = \int n(\omega)\sigma_\gamma(\omega)\frac{d\omega}{\omega},$$

where $\sigma_\gamma(\omega)$ is the photonuclear cross section and $n(\omega)$ is a smooth function of ω (approximately a logarithm of ω). Therefore, the key information about the nuclear structure is contained in $\int \sigma_\gamma(\omega)\, d\omega/\omega$ which is directly proportional to the (non-energy weighted) integrated B(E1) values [21,22].

4. Conclusions

In conclusion, our results in this paper indicate as do those of Bertsch *et al.* [13] that the Coulomb dissociation cross section of ^{11}Li is smaller than reported in ref. [10] and close to the recent modified RPA calculations of refs. [11,14]. The pure cluster model, therefore overestimate σ_C by as much as 20%.

The Coulomb dissociation of neutron-rich nuclei is an extremely useful tool to investigate their structure. If one could perform these measurements at Brookhaven (14.5 GeV/nucleon) and at CERN (E_{lab} = 200 GeV/nucleon) for example, one would obtain a Coulomb dissociation cross section of about two and three times as large as that measured by Kobayashi *et al.* [10]. The nuclear contribution would be not so relevant, and the investigation about the nuclear structure aspects of neutron-rich nuclei would be more free of bias.

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