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Excitation and photon decay of giant resonances from high-energy collisions of heavy ions

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Abstract: We develop closed-form expressions for the cross section for the electromagnetic excitation induced by heavy-ion collisions at intermediate and high energies. These expressions directly relate the excitation cross section to the corresponding photonuclear cross section. The effects of strong absorption, relativity and retardation are incorporated from the outset and are shown to be important. We apply our results to several situations of recent interest, including the excitation and photon decay of giant resonances.

1. Introduction

There has been considerable interest in recent years in the study of giant resonances through the excitation induced by heavy-ion collisions [1]). One important reason for such studies is that the Coulomb interaction results in very large cross sections for the excitation of giant resonances at bombarding energies around 100 MeV/nucleon and above. These large cross sections make it feasible to study finer features of the structure of giant resonances that are not easily accessible by other means. For example, since the Coulomb interaction couples to isoscalar and isovector states with the same weight, one gets precise information on the excitation of isovector quadrupole (and higher multipole) resonances which are difficult to find using other probes. As an example of the usefulness of the method, recent studies with pion probes [2-4]) challenged the assertion that the giant quadrupole resonance which one sees at low energies ($E_{\mathrm{GQR}} = 63A^{-1/3}$ MeV) is of purely isoscalar nature, as normally thought. However, Coulomb-excitation studies

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performed by Beene *et al.* [5]) have shown that the ratio of the neutron to proton matrix elements, which is sensitive to the isospin character of the state, is equal to the value of N/Z, as expected for a purely isoscalar excitation.

The large cross section for Coulomb excitation of giant resonances also makes feasible the investigation of photon decays of giant resonances, despite the small branching ratios. This is potentially a very powerful tool to study the structure of giant resonances, especially those of low multipolarity. For example, it has long been recognized that the photon-decay branches of the GDR to low-lying excited states contain valuable information about the coupling of the GDR to collective surface modes, such as rotations and vibrations [6]). Also, weakly excited modes, such as the isovector giant quadrupole resonance, can be extracted from the dominant electric-dipole background using information contained in the angular correlation between the scattered projectile and the decay photon. The pioneering experiments of Beene and collaborators [5]) have demonstrated both the power and the feasibility of this technique.

At high energies, around 1 GeV/nucleon, heavy-ion excitation of giant resonances has been studied for many years [as an example see ref. [7])]. The intriguing possibility of using the Coulomb field of heavy ions to access the excitation of multiphonon states of giant resonances, as suggested by Baur and Bertulani [8]), has also been the subject of intense scrutiny [9,10]). The existence of such states has been tentatively identified in pion-scattering experiments by Mordechai *et al.* [11,12]), and one hopes that the use of the strong electromagnetic field of heavy ions in high-energy collisions can give a deeper insight into the quantitative features of these states.

These examples demonstrate the utility of heavy-ion-induced Coulomb excitation at intermediate and high energies in the study of interesting nuclear properties. However, in all these experiments there exists the difficulty of separating the contribution to the excitation process of the Coulomb interaction from that of the strong interaction. One would like to extract with the highest accuracy the contribution of the electromagnetic interaction to the excitation process, because this contribution is directly linked to the same matrix elements as in photonuclear processes. These photonuclear processes, which are often not accessible by means of direct photoabsorption experiments, give us clear information on the nuclear response function.

It is the aim of this paper to develop a tractable closed-form theory of Coulomb excitation for intermediate- and high-energy collisions based on the eikonal approximation. The effects of strong absorption, relativity and retardation are included from the outset. In sect. 2 we derive expressions that directly relate the excitation cross section to the corresponding photoabsorption cross section. We also show how one obtains from these expressions the semiclassical limit for Coulomb excitation. In sect. 3 we apply our results to the analysis of the excitation of giant resonances. We also extend the formalism to the excitation/photon decay process and propose a new expression that directly relates the cross section for this process to the photon-scattering cross section. Our conclusions are given in sect. 4.

2. Eikonal description of Coulomb excitation

2.1. INELASTIC AMPLITUDES AND VIRTUAL-PHOTON NUMBERS

We consider a situation in which the field of the incident projectile, nucleus 1, excites the target, nucleus 2, with the projectile remaining in its ground state. The direction of the projectile is along the z-axis. We define $\boldsymbol{r}$ to be the separation between the centers of mass of the two nuclei and $\boldsymbol{r}'$ to be the intrinsic coordinate of the target nucleus. Then, in first-order perturbation theory the inelastic-scattering amplitude is given by

$$f(\theta) = \frac{ik}{2\pi\hbar v}\int \mathrm{d}^3r\,\mathrm{d}^3r' \langle \Phi_{\boldsymbol{k}'}^{(-)}(\boldsymbol{r})\phi_{\mathrm{f}}(\boldsymbol{r}')|V_{\mathrm{int}}(\boldsymbol{r},\boldsymbol{r}')|\,\Phi_{\boldsymbol{k}}^{(+)}(\boldsymbol{r})\phi_{\mathrm{i}}(\boldsymbol{r}')\rangle, \tag{1}$$

where $\Phi_{\boldsymbol{k}'}^{(-)}(\boldsymbol{r})$ and $\Phi_{\boldsymbol{k}}^{(+)}(\boldsymbol{r})$ are the incoming and outgoing distorted waves, respectively, for the scattering of the center of mass of the nuclei, and $\phi(\boldsymbol{r}')$ is the intrinsic nuclear wave function of the target nucleus.

At intermediate energies, $\Delta E/E_{\mathrm{lab}} \ll 1$, and forward angles, $\theta \ll 1$, we can use eikonal wave functions for the distorted waves; i.e.

$$\Phi_{\boldsymbol{k}'}^{(-)*}(\boldsymbol{r})\Phi_{\boldsymbol{k}}^{(+)}(\boldsymbol{r}) = \exp\left\{-i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{i}{\hbar v}\int_{-\infty}^{\infty} U_{\mathrm{N}}^{\mathrm{opt}}(z',b)\,\mathrm{d}z' + i\psi_{\mathrm{C}}(b)\right\}, \tag{2}$$

where $\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$, $U_{\mathrm{N}}^{\mathrm{opt}}$ is the nuclear optical potential, and

$$\psi_{\mathrm{C}}(b) = 2\frac{Z_1Z_2e^2}{\hbar v}\left\{\ln(kb) + \tfrac{1}{2}E_1\left(\frac{b^2}{R_{\mathrm{G}}^2}\right)\right\}, \tag{3}$$

with

$$E_1(x) = \int_x^{\infty}\frac{\mathrm{e}^{-t}}{t}\,\mathrm{d}t. \tag{4}$$

We have defined the impact parameter b by $b = |\boldsymbol{r}\times\hat{\boldsymbol{z}}|$. R_{G} is related to the "gaussian" radii of the nuclei: $R_{\mathrm{G}}^2 = [R_{\mathrm{G}}^{(1)}]^2 + [R_{\mathrm{G}}^{(2)}]^2$. For a heavy nucleus, $R_{\mathrm{G}}^{(i)} = \sqrt{2a_iR_i}$, where a_i is the diffusivity and R_i is the radius of the corresponding Fermi density distribution. For a light nucleus, $R_{\mathrm{G}}^{(i)}$ is equal to the size parameter of the gaussian matter density.

The first term in eq. (3) is the contribution to the Coulomb phase of a point-like charge distribution. It reproduces the elastic Coulomb amplitude when introduced into the eikonal expression for the elastic-scattering amplitude (see eq. (34)). The second term in eq. (3) is a correction due to the extended gaussian charge distribution [13]). It eliminates the divergence of the Coulomb phase at $b = 0$, so that

$$\psi_{\mathrm{C}}(0) = 2\frac{Z_1Z_2e^2}{\hbar v}\,[\ln(kR_{\mathrm{G}}) - C], \tag{5}$$

where $C = 0.577...$ is the Euler constant.

The interaction potential, assumed to be purely Coulomb, is given by

$$V_{\mathrm{int}}(\boldsymbol{r},\boldsymbol{r}') = \frac{v^{\mu}}{c^2}j_{\mu}(\boldsymbol{r}')\frac{\mathrm{e}^{i\kappa|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}, \tag{6}$$

where $v^\mu = (c, \boldsymbol{v})$, with $\boldsymbol{v}$ equal to the projectile velocity, $\kappa = \omega/c$, and $j_\mu(\boldsymbol{r}')$ is the charge four-current for the intrinsic excitation of nucleus 2 by an energy of $\hbar\omega$. Inserting eqs. (2) and (6) in eq. (1) and following the same steps as in ref. [14]), one finds

$$f(\theta) = i\frac{Z_1 ek}{\gamma\hbar v}\sum_{\pi\ell m} i^m \left(\frac{\omega}{c}\right)^\ell \sqrt{2\ell+1}\,\mathrm{e}^{-im\phi} \times \Omega_m(q) G_{\pi\ell m}\left(\frac{c}{v}\right)\langle I_\mathrm{f} M_\mathrm{f}|\mathcal{M}(\pi\ell, -m)|I_\mathrm{i} M_\mathrm{i}\rangle\,, \tag{7}$$

where $\pi\ell m$ denotes the multipolarity, $G_{\pi\ell m}$ are the Winther–Alder relativistic functions [15]), and $\langle I_\mathrm{f} M_\mathrm{f}|\mathcal{M}(\pi\ell, -m)|I_\mathrm{i} M_\mathrm{i}\rangle$ is the matrix element for the electromagnetic transition of multipolarity $\pi\ell m$ from $|I_\mathrm{i} M_\mathrm{i}\rangle$ to $|I_\mathrm{f} M_\mathrm{f}\rangle$, with $E_\mathrm{f} - E_\mathrm{i} = \hbar\omega$. The function $\Omega_m(q)$ is given by

$$\Omega_m(q) = \int_0^\infty \mathrm{d}b\; b J_m(qb) K_m\left(\frac{\omega b}{\gamma v}\right)\exp\{i\chi(b)\}\,, \tag{8}$$

where $q = 2k\sin(\frac{1}{2}\theta)$ is the momentum transfer, θ and ϕ are the polar and azimuthal scattering angles, respectively, and

$$\chi(b) = -\frac{1}{\hbar v}\int_0^\infty U_\mathrm{N}^\mathrm{opt}(z', b)\,\mathrm{d}z' + \psi_\mathrm{C}(b). \tag{9}$$

For intermediate energies the nuclear optical potential $U_\mathrm{N}^\mathrm{opt}$ is obtained from fits to the available elastic-scattering data. For relativistic energies one constructs a "potential" which gives the expected transparency for a given impact parameter [16]) in terms of the nucleon–nucleon scattering t-matrix. Rasmussen *et al.* [17]) have shown that a good parameterization is given by

$$U_\mathrm{N}(r) = \pi^{3/2}\langle t_\mathrm{NN}(E)\rangle\rho_1(0)\rho_2(0)\left[\frac{R_\mathrm{G}^{(1)}R_\mathrm{G}^{(2)}}{R_\mathrm{G}}\right]^3 \exp\left\{-\frac{r^2}{R_\mathrm{G}^2}\right\}, \tag{10}$$

where t_NN is the nucleon–nucleon t-matrix, which can be obtained from nucleon–nucleon scattering at high energies [18]), and $\rho_i(0) = \frac{1}{2}\rho_0\exp(R_i/2a)$, with $\rho_0 = 0.17$ fm^{-3}, $a = 0.65$ and $R_i = 1.2\,A_i^{-1/3}$.

Using the Wigner–Eckart theorem, one can calculate the inelastic differential cross section from eq. (7), using techniques similar to those of ref. [14]). One obtains

$$\frac{\mathrm{d}^2\sigma_\mathrm{C}}{\mathrm{d}\Omega\,\mathrm{d}E_\gamma}(E_\gamma) = \frac{1}{E_\gamma}\sum_{\pi\ell}\frac{\mathrm{d}n_{\pi\ell}}{\mathrm{d}\Omega}\sigma_\gamma^{\pi\ell}(E_\gamma)\,, \tag{11}$$

where $\sigma_\gamma^{\pi\ell}(E_\gamma)$ is the photonuclear cross section for the absorption of a real photon with energy E_γ by nucleus 2, and $\mathrm{d}n_{\pi\ell}/\mathrm{d}\Omega$ is the "equivalent", or virtual, photon number, which is given by

$$\frac{\mathrm{d}n_{\pi\ell}}{\mathrm{d}\Omega} = Z_1^2\alpha\left(\frac{\omega k}{\gamma v}\right)^2 \frac{\ell\,[(2\ell+1)!!]^2}{(2\pi)^3(\ell+1)}\sum_m |G_{\pi\ell m}|^2|\Omega_m(q)|^2\,, \tag{12}$$

where $\alpha = e^2/\hbar c$.

The total cross section for Coulomb excitation can be obtained from eqs. (11) and (12), using the approximation $\mathrm{d}\Omega \simeq 2\pi q\,\mathrm{d}q/k^2$, valid for small scattering angles and small energy losses. Using the closure relation for the Bessel functions, we obtain

$$\frac{\mathrm{d}\sigma_C}{\mathrm{d}E_\gamma}(E_\gamma) = \frac{1}{E_\gamma}\sum_{\pi\ell} n_{\pi\ell}(E_\gamma)\,\sigma_\gamma^{\pi\ell}(E_\gamma)\,, \tag{13}$$

where

$$n_{\pi\ell}(\omega) = Z_1^2\alpha\,\frac{\ell\,[(2\ell+1)!!]^2}{(2\pi)^3(\ell+1)}\sum_m |G_{\pi\ell m}|^2 g_m(\omega) \tag{14}$$

and

$$g_m(\omega) = 2\pi\left(\frac{\omega}{\gamma v}\right)^2\int \mathrm{d}b\; bK_m^2\left(\frac{\omega b}{\gamma v}\right)\exp\{-2\chi_{\mathrm{I}}(b)\}\,, \tag{15}$$

where $\chi_{\mathrm{I}}(b)$ is the imaginary part of $\chi(b)$, which is obtained from eq. (9) and the imaginary part of the optical potential.

Eqs. (11)–(15) are the main results of this work. They express the Coulomb-excitation cross section directly in terms of the photoabsorption cross section. We emphasize that the relationship between the Coulomb-excitation and photoabsorption cross sections is not an approximation. Rather it is an exact result which emerges from the fact that the excitation occurs in a divergence-free field ($\nabla\cdot\boldsymbol{E} = 0$), so that the Coulomb-excitation and photoabsorption processes involve precisely the same transverse matrix elements.

Before proceeding further, it is worthwhile to mention that the present calculations differ from those of refs. [14,15]) by the proper inclusion of absorption. To reproduce the angular distributions of the cross sections, it is essential to include the nuclear transparency. In the limit of a black-disk approximation, the above formulas reproduce the results presented in ref. [14]). One also observes that the Coulomb phase in the distorted waves, which is necessary for the quantitative reproduction of the experimental angular distributions, is not important for the total cross section in high-energy collisions. This fact explains why semiclassical and quantum methods give the same result for the total cross section for Coulomb excitation at relativistic energies [14]). At intermediate energies, however, it is just this important phase which reproduces the semiclassical limit for the scattering of large-Z ions, as we shall see next. Using the semiclassical terminology, for $E_{\mathrm{lab}} \approx 100$ MeV/nucleon or less, the recoil in the Coulomb trajectory is relevant. At the distance of closest approach, when the Coulomb field is most effective at inducing the excitation, the ions are displaced farther from each other due to the Coulomb recoil. Winther and Alder [15]) have shown that one may account for this effect approximately by using the effective impact parameter $b_{\mathrm{eff}} = b + \pi Z_1 Z_2 e^2/4E_{\mathrm{lab}}$ in the semiclassical calculations. This recoil approximation can also be used in eq. (15), replacing b by b_{eff} in the Bessel function and the nuclear phase, in order to obtain the total cross section. Since the modified Bessel function is a rapidly decreasing function of its argument, this modification leads to sizeable modifications of the total cross section at intermediate-energy collisions.

Finally, we point out that for very light heavy-ion partners, the distortion of the scattering wave functions caused by the nuclear field is not important. This distortion is manifested in the diffraction peaks of the angular distributions, characteristic of strong absorption processes. If $Z_1 Z_2 \alpha \gg 1$, one can neglect the diffraction peaks in the inelastic-scattering cross sections and a purely Coulomb-excitation process emerges. One can gain insight into the excitation mechanism by looking at how the semiclassical limit of the excitation amplitudes emerges from the general result (12). We do this next.

2.2. SEMICLASSICAL LIMIT OF THE EXCITATION AMPLITUDES

If we assume that Coulomb scattering is dominant and neglect the nuclear phase in eq. (9), we get

$$\Omega_m(q) \simeq \int_0^\infty \mathrm{d}b\, b J_m(qb) K_m\left(\frac{\omega b}{\gamma v}\right) \exp\{i\psi_C(b)\}\,. \tag{16}$$

This integral can be done analytically by rewriting it as (an unimportant factor $k^{12\eta}$ is omitted)

$$\Omega_m(q) = \int_0^\infty \mathrm{d}b\, b^{1+i2\eta} J_m(qb) K_m\left(\frac{\omega b}{\gamma v}\right), \tag{17}$$

where we used the simple form $\psi_C(b) = 2\eta \ln(kb)$, with $\eta = Z_1 Z_2 e^2/\hbar v$. Using standard techniques found in ref. [19]), we find

$$\begin{aligned}\Omega_m(q) =& 2^{2i\eta}\frac{1}{m!}\Gamma(1+m+i\eta)\Gamma(1+i\eta)\\ &\times \Lambda^m \left(\frac{\gamma v}{\omega}\right)^{2+2i\eta} F\left(1+m+i\eta; 1+i\eta; 1+m; -\Lambda^2\right),\end{aligned} \tag{18}$$

where

$$\Lambda = \frac{q\gamma v}{\omega}, \tag{19}$$

and F is the hypergeometric function [19]).

The connection with the semiclassical results maybe obtained by using the low momentum transfer limit

$$\begin{aligned}J_m(qb) &\simeq \sqrt{\frac{2}{\pi q b}}\cos\left(qb - \tfrac{1}{2}\pi m - \tfrac{1}{4}\pi\right)\\ &= \frac{1}{\sqrt{2\pi q b}}\left\{\mathrm{e}^{iqb}\mathrm{e}^{-i\pi(m+1/2)/2} + \mathrm{e}^{-iqb}\mathrm{e}^{i\pi(m+1/2)/2}\right\},\end{aligned} \tag{20}$$

and using the stationary-phase method, i.e.

$$\int G(x)\mathrm{e}^{i\phi(x)}\,\mathrm{d}x \simeq \left(\frac{2\pi i}{\phi''(x_0)}\right)^{1/2} G(x_0)\mathrm{e}^{i\phi(x_0)}, \tag{21}$$

where

$$\frac{\mathrm{d}\phi}{\mathrm{d}x}(x_0) = 0, \qquad \phi''(x_0) = \frac{\mathrm{d}^2\phi}{\mathrm{d}x^2}(x_0). \tag{22}$$

This result is valid for a slowly varying function $G(x)$.

Only the second term in brackets of eq. (20) will have a positive ($b = b_0 > 0$) stationary point, and

$$\Omega_m(q) \simeq \frac{1}{\sqrt{2\pi q}} \left(\frac{2\pi i}{\phi''(b_0)}\right)^{1/2} \sqrt{b_0} K_m \left(\frac{\omega b_0}{\gamma v}\right) \exp\left\{i\phi(b_0) + \tfrac{1}{2}i\pi(m + \tfrac{1}{2})\right\}, \tag{23}$$

where

$$\phi(b) = -qb + 2\eta \ln(kb). \tag{24}$$

The condition $\phi'(b_0) = 0$ implies

$$b_0 = \frac{2\eta}{q} = \frac{a_0}{\sin(\frac{1}{2}\theta)}, \tag{25}$$

where $a_0 = Z_1 Z_2 e^2/\mu v^2$ is half the distance of closest approach in a classical head-on collision.

We observe that the relation (25) is the same [with $\mathrm{cotg}(\frac{1}{2}\theta) \sim \sin^{-1}(\frac{1}{2}\theta)$] as that between impact parameter and deflection angle of a particle following a classical Rutherford trajectory. Also,

$$\phi''(b_0) = -\frac{2\eta}{b_0^2} = -\frac{q^2}{2\eta}, \tag{26}$$

which implies that in the semiclassical limit

$$\begin{aligned} |\Omega_m(q)|^2_{\text{s.c.}} &= \frac{4\eta^2}{q^4} K_m^2 \left(\frac{2\omega\eta}{\gamma v q}\right) \\ &= \frac{1}{k^2}\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ruth}} K_m^2 \left(\frac{\omega a_0}{\gamma v \sin(\frac{1}{2}\theta)}\right). \end{aligned} \tag{27}$$

Using the above results, eq. (12) becomes

$$\frac{dn_{\pi\ell}}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Ruth}} Z_1^2 \alpha \left(\frac{\omega}{\gamma v}\right)^2 \frac{\ell\,[(2\ell+1)!!]^2}{(2\pi)^3(\ell+1)} \sum_m |G_{\pi\ell m}|^2 K_m^2 \left(\frac{\omega a_0}{\gamma v \sin(\frac{1}{2}\theta)}\right). \tag{28}$$

If strong absorption is not relevant, the above formula can be used to calculate the equivalent photon numbers. The stationary value given by eq. (25) means that the important values of b which contribute to $\Omega_m(q)$ are those close to the classical impact parameter. Dropping the index 0 from eq. (25), we can also rewrite eq. (28) as

$$\frac{1}{2\pi b}\frac{dn_{\pi\ell}}{db} = Z_1^2 \alpha \left(\frac{\omega}{\gamma v}\right)^2 \frac{\ell\,[(2\ell+1)!!]^2}{(2\pi)^3(\ell+1)} \sum_m |G_{\pi\ell m}|^2 K_m^2 \left(\frac{\omega b}{\gamma v}\right). \tag{29}$$

which is equal to the semi-classical expression given in ref. [20]), eq. (A.2).

For very forward scattering angles, such that $\Lambda \ll 1$, a further approximation can be made by setting the hypergeometric function in eq. (18) equal to unity [19]), and we obtain

$$\Omega_m(q) = 2^{2i\eta}\frac{1}{m!}\Gamma(1 + m + i\eta)\Gamma(1 + i\eta)\Lambda^m \left(\frac{\gamma v}{\omega}\right)^{2+2i\eta}. \tag{30}$$

The main value of m in this case will be $m = 0$, for which one gets

$$\Omega_0(q) \simeq 2^{2i\eta}\Gamma(1+i\eta)\Gamma(1+i\eta)\left(\frac{\gamma v}{\omega}\right)^{2+2i\eta}$$
$$= -\eta^2 2^{2i\eta}\Gamma(i\eta)\Gamma(i\eta)\left(\frac{\gamma v}{\omega}\right)^{2+2i\eta}, \tag{31}$$

and

$$|\Omega_0(q)|^2 = \eta^4\left(\frac{\gamma v}{\omega}\right)^4 \frac{\pi^2}{\eta^2\sinh^2(\pi\eta)}, \tag{32}$$

which, for $\eta \gg 1$, results in

$$|\Omega_0(q)|^2 = 4\pi^2\eta^2\left(\frac{\gamma v}{\omega}\right)^4 \mathrm{e}^{-2\pi\eta}. \tag{33}$$

This result shows that in the absence of strong absorption and for $\eta \gg 1$, Coulomb excitation is strongly suppressed at $\theta = 0$. This also follows from semiclassical arguments, since $\theta \to 0$ means large impact parameters, $b \gg 1$, for which the action of the Coulomb field is weak.

3. Applications and discussions

3.1. SINGLES SPECTRA IN COULOMB EXCITATION OF GDR

In this section, we apply our formalism to the analysis of the data of ref. [1]), in which a projectile of ^{17}O with an energy of $E_{\mathrm{lab}} = 84$ MeV/nucleon excites the target nucleus ^{208}Pb to the GDR. We first seek parameters of the optical potential which fits the elastic-scattering data. We use the eikonal approximation for the elastic amplitude in the form given by

$$f_{\mathrm{el}}(\theta) = ik\int J_0(qb)\{1-\exp[i\chi(b)]\}\, b\,\mathrm{d}b, \tag{34}$$

where J_0 is the Bessel function of zeroth-order and the phase $\chi(b)$ is given by eq. (9). In fig. 1 we compare our calculated elastic-scattering angular distribution to the data from ref. [21]). The calculation utilized eq. (34), with $\chi(b)$ obtained from an optical potential of a standard Woods–Saxon form with parameters

$$\begin{aligned} V_0 &= 50\,\mathrm{MeV}, \quad W_0 = 58\,\mathrm{MeV}, \\ R_V &= R_W = 8.5\,\mathrm{fm} \\ a_V &= a_W = 0.85\,\mathrm{fm}. \end{aligned} \tag{35}$$

The data are evidently very well reproduced by the eikonal approximation.

In order to calculate the inelastic cross section for the excitation of the GDR, we use a lorentzian parameterization for the photoabsorption cross section of ^{208}Pb [ref. [22])], assumed to be all E1, with $E_{\mathrm{GDR}} = 13.5$ MeV and $\Gamma = 4.0$ MeV. Inserting this form into eq. (13) and doing the calculations implicit in eq. (12) for $\mathrm{d}n_{E1}/\mathrm{d}\Omega$, we calculate the angular distribution and compare it with the data in fig. 2. The agreement with the

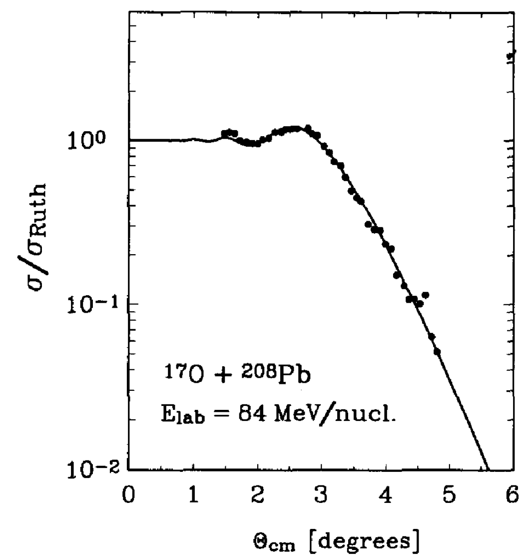

Fig. 1. Ratio to the Rutherford cross section of the elastic cross section for the $^{17}O+^{208}Pb$ reaction at 84 MeV/nucleon, as a function of the center-of-mass scattering angle. Data are from ref. [21]).

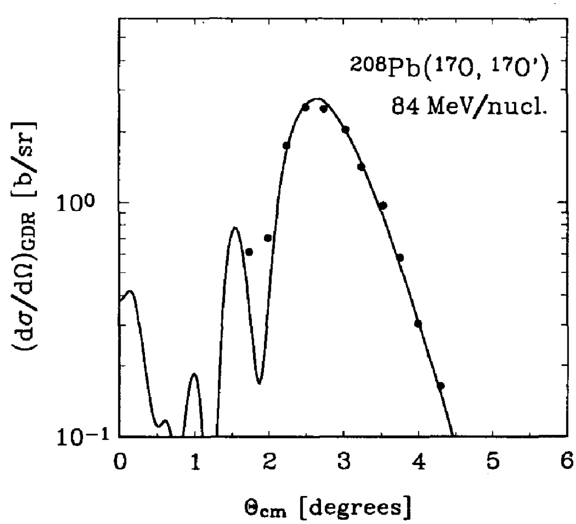

Fig. 2. Differential cross section for the excitation of the isovector giant dipole resonance in ^{208}Pb by means of ^{17}O projectiles at 84 MeV/nucleon, as a function of the center-of-mass scattering angle. Data are from ref. [21]).

data is excellent, provided we adjust the overall normalization to a value corresponding to 93% of the energy-weighted sum rule (EWSR) in the energy interval 7 – 18.9 MeV. Taking into account the ±10% uncertainty in the absolute cross sections quoted in ref. [21]), this is consistent with photoabsorption cross section in that energy range, for which approximately 110% of the EWSR is exhausted.

To unravel the effects of relativistic corrections, we repeat the previous calculations unplugging the factor $\gamma = (1 - v^2/c^2)^{-1/2}$ which appears in the expressions (14) and (15) and using the non-relativistic limit of the functions G_{E1m}, as described in ref. [15]). These modifications eliminate the relativistic corrections on the interaction potential. The result of this calculation is shown in fig. 3 (dotted curve). For comparison, we also show the result of a full calculation, keeping the relativistic corrections (dashed curve). We observe that the two results have approximately the same pattern, except that the non-relativistic result is slightly smaller than the relativistic one. This fact may explain the discrepancy between the fit of ref. [21]) and ours as due to relativistic corrections not properly accounted for in the ECIS code [23]). In fact, if we repeat the calculation for the excitation of IVGDR using the non-relativistic limit of eqs. (14) and (15), we find that the best fit to the data is obtained by exhausting 113% of the EWSR. This value is very close to the 110% obtained by Barrette *et al.* [21]).

In fig. 3 we also show the result of a semiclassical calculation (solid curve) for the IVGDR excitation in lead, using eq. (28) for the virtual-photon numbers. One observes that the semiclassical curve is not able to fit the experimental data. This is mainly be-

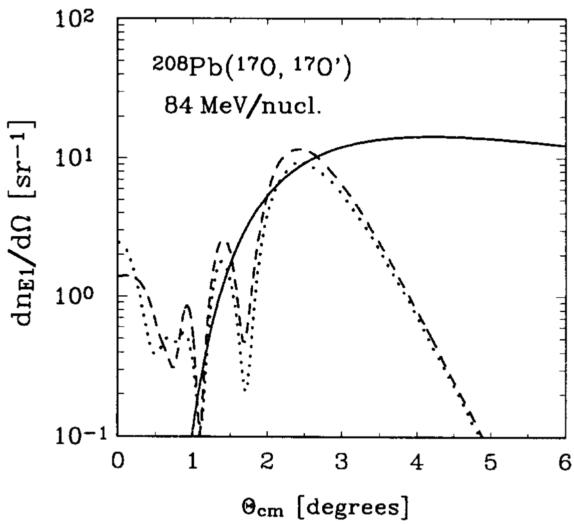

Fig. 3. Virtual-photon numbers for the electric-dipole multipolarity generated by 84 MeV/nucleon ^{17}O projectiles incident on ^{208}Pb, as a function of the center-of-mass scattering angle. The solid curve is a semiclassical calculation. The dashed and dotted curves are eikonal calculations with and without relativistic corrections, respectively.

cause diffraction effects and strong absorption are not included. But the semiclassical calculation displays the region of relevance for Coulomb excitation. At small angles the scattering is dominated by large impact parameters, for which the Coulomb field is weak. Therefore the Coulomb excitation is small and the semiclassical approximation fails. It also fails in describing the large-angle data (dark side of the rainbow angle), since absorption is not treated properly. One sees that there is a "window" in the inelastic-scattering data near $\theta = 2° - 3°$ in which the semiclassical and full calculations give approximately the same cross section.

As discussed above, the semiclassical result works for large-Z nuclei and for relativistic energies where the approximation of eq. (16) is justified. However, angular distributions are not useful at relativistic energies since the scattering is concentrated at extremely forward angles. The quantity of interest in this case is the total inelastic cross section. If we use a sharp-cutoff model for the strong absorption, so that $\chi_{\rm I}(b) = \infty$ for $b < b_{\rm min}$ and 0 otherwise, then eqs. (14) and (15) yield the same result as an integration of the semiclassical expression, eq. (29), from $b_{\rm min}$ to ∞. In fact, this result has been obtained earlier in ref. [14]).

3.2. EXCITATION AND PHOTON DECAY OF THE GDR

We now consider the excitation of the target nucleus to the giant dipole resonance and the subsequent photon decay of that excited nucleus, leaving the target in the ground state. Experimentally, one detects the inelastically scattered projectile in coincidence with the decay photon and demands that the energy lost by the projectile is equal to the energy of the detected photon. To the extent that the excitation mechanism is dominated by Coulomb excitation, with the exchange of a single virtual photon, this reaction is very similar to the photon-scattering reaction, except that in the present case the incident photon is virtual rather than real. In this section, we investigate whether the connection between these two reactions can be formalized.

We first review the excitation mechanism. The physical situation is that of a heavy ion of energy E incident on a target. The projectile loses an energy ΔE while scattering through an angle θ. We have shown that, under the conditions $\Delta E/E \ll 1$, the cross section for excitation of the target nucleus partitions into the following expression (we assume that the contribution of the E1 multipolarity is dominant):

$$\frac{\mathrm{d}^2\sigma_{\rm C}}{\mathrm{d}\Omega\,\mathrm{d}E_\gamma}(E_\gamma) = \frac{1}{E_\gamma}\frac{\mathrm{d}n_\gamma}{\mathrm{d}\Omega}(E_\gamma)\,\sigma_\gamma(E_\gamma)\,, \tag{36}$$

where $\sigma_\gamma(E_\gamma)$ is the photonuclear cross section for the absorption of a real photon with energy $E_\gamma = \Delta E$ by the target nucleus, and the remaining terms on the right-hand side are collectively the number of virtual photons per unit energy with energy E_γ. This latter quantity depends on the kinematics of the scattered heavy ion and on the optical potential but is otherwise independent of the target degrees of freedom. This partitioning allows one to relate the excitation cross section to the photoabsorption cross section.

Now, the usual way to write the cross section $d^2\sigma_{C\gamma}/d\Omega\, dE_\gamma$ for the excitation of the target followed by photon decay to the ground state is simply to multiply the above expression by a branching ratio R_γ, which represents the probability that the nucleus excited to an energy E_γ will emit a photon leaving it in the ground state [5]):

$$\frac{d^2\sigma_{C\gamma}}{d\Omega\, dE_\gamma}(E_\gamma) = \frac{1}{E_\gamma}\frac{dn_\gamma}{d\Omega}(E_\gamma)\sigma_\gamma(E_\gamma)R_\gamma(E_\gamma)\,. \tag{37}$$

Instead, we propose the following expression, in complete analogy with eq. (36):

$$\frac{d^2\sigma_{C\gamma}}{d\Omega\, dE_\gamma}(E_\gamma) = \frac{1}{E_\gamma}\frac{dn_\gamma}{d\Omega}(E_\gamma)\,\sigma_{\gamma\gamma}(E_\gamma)\,, \tag{38}$$

where $\sigma_{\gamma\gamma}(E_\gamma)$ is the cross section for the elastic scattering of photons with energy E_γ. Formally, these expressions are equivalent in that they simply define the quantity R_γ. However, if one treats R_γ literally as a branching ratio, then these expressions are equivalent only if it were true that the photon-scattering cross section is just the product of the photoabsorption cross section and the branching ratio. In fact, it is well-known from the theory of photon scattering that the relationship between the photoabsorption cross section and the photon-scattering cross section is more complicated [25]). In particular, it is not correct to think of photon scattering as a two-step process consisting of absorption, in which the target nucleus is excited to an intermediate state of energy E_γ, followed by emission, in which the emitted photon has the same energy E_γ. Since the intermediate state is not observable, one must sum over all possible intermediate states and not just

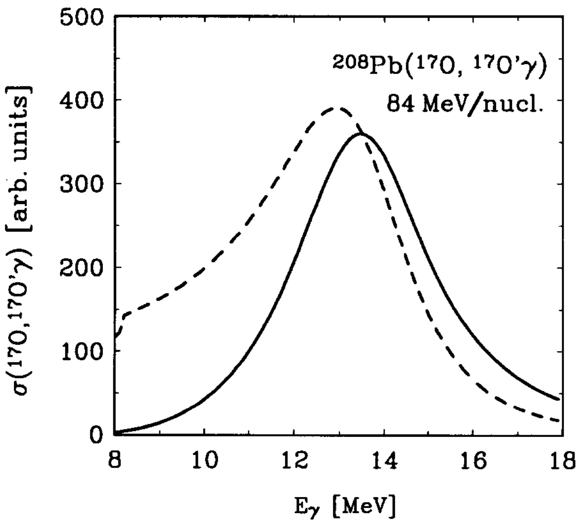

Fig. 4. Calculated cross section for the excitation followed by γ-decay of ^{208}Pb induced by ^{17}O projectiles at 84 MeV/nucleon. The photoabsorption cross section was parameterized by a simple lorentzian representing the GDR, and the statistical component of the photon decay was neglected. The solid curve uses the formalism described in the text (eq. (38)) while the dashed curve uses a constant branching ratio for photon decay (eq. (37)).

those allowed by conservation of energy. Now, if the energy E_γ happens to coincide with a narrow level, then that level will completely dominate in the sum over intermediate states. In that case, it is proper to regard the scattering as a two-step process in the manner described above, and the two expressions for the cross section will be equal. However, for E_γ in the nuclear continuum region (e.g. in the region of the GDR), this will not be the case, as demonstrated in the following discussion.

We again consider the inelastic scattering of ^{17}O projectiles of energy $E_{\rm lab}$ = 84 MeV/nucleon from a ^{208}Pb nucleus at an angle of 2.5°. We use eq. (12) to calculate the E1 virtual-photon number and we use a lorentzian parameterization of the GDR of ^{208}Pb. We calculate R_γ and $\sigma_{\gamma\gamma}$ according to the prescriptions of ref. [5]) and ref. [25]), respectively; in both cases we neglect the statistical contribution to the photon decay. The results are compared in fig. 4, where it is very evident that they make very different predictions for the cross section, especially in the wings of the GDR.

We next use our expression to compare directly with the data of ref. [5]). For this pur-

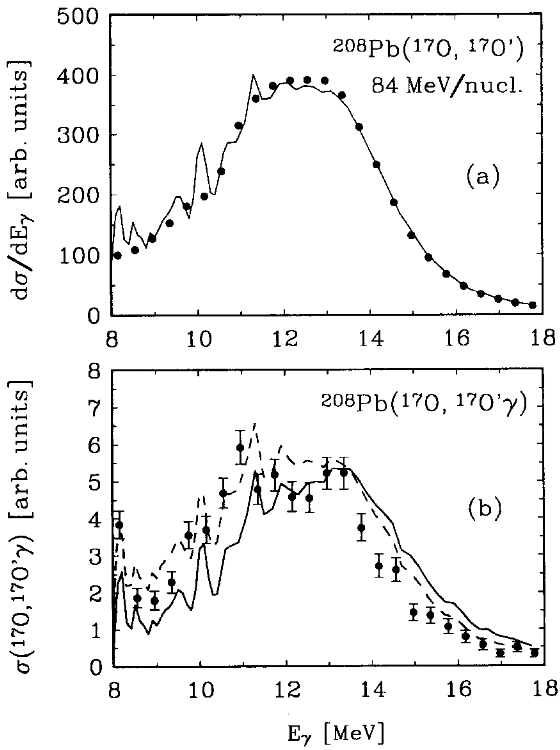

Fig. 5. (top panel) Differential cross section for the excitation of ^{208}Pb by ^{17}O projectiles at 84 MeV/nucleon, as a function of the excitation energy. Data are from ref. [5]). (bottom panel) Cross section for excitation followed by γ-decay of ^{208}Pb in the reaction mentioned in the previous figures. The solid (dashed) line includes (excludes) the Thomson scattering amplitude. Data are from ref. [5]).

pose, we again calculate $\sigma_{\gamma\gamma}$ using the formalism of ref. [25]), which relates $\sigma_{\gamma\gamma}$ to the total photoabsorption. For the latter, we use the numerically defined data set of ref. [22]) rather than a lorentzian parameterization. The effect of the underlying compound nuclear levels (i.e. the statistical contribution to the photon scattering) is also included. The calculation is compared to the data in fig. 5. The top panel shows the cross section for the excitation of the GDR without the detection of the decay photon. The agreement with the data is excellent, giving us confidence that our calculation of the virtual-photon number as a function of E_γ is correct. The bottom panel shows the cross section for the excitation–decay process as a function of E_γ. Although the qualitative trend of the data are well described, the calculation systematically overpredicts the cross section on the high-energy side of the GDR (solid curve). If the Thomson amplitude is not included in $\sigma_{\gamma\gamma}$, the calculation is in significantly better agreement with the data (dashed curve). This last point is puzzling and must be left as an open question.

As a final point, we remark that the relationship of eq. (38) is not new. For relativistic projectiles, this relationship can be obtained classically (for the E1 multipolarity only) via the so-called Weizsäcker–Williams [26]) method of virtual quanta or quantum mechanically via the Primakoff formalism [27]). A formal derivation for the kinematic regime of the Coulomb-excitation/photon-decay experiments of ref. [5]) is still lacking. Nevertheless, we suggest that future experiments be analyzed using the expression eq. (38).

4. Conclusions

We have shown that the cross sections for excitation of giant resonances in intermediate- and relativistic-energy collisions can be well explained in terms of a simple formalism, based on the eikonal approximation. This formalism includes the effects of strong absorption, retardation and relativity from the very beginning. These effects are shown to be of relevance for the determination of the excitation strengths needed to reproduce the experimental data. The resulting cross section can be factorized into a sum over multipoles of products of a density of virtual photons and the photoabsorption cross section for that multipole. A similar factorization had previously been done only for relativistic collisions [14]). The advantage of such a factorization is clear, since the photonuclear processes can often give us important and unambiguous information on the nuclear response function. We have given some examples related to recent experiments [1,5,21]) that demonstrate both the power of the reaction and the utility of our technique in interpreting the data. At intermediate energies, the present technique is a distinct improvement over semiclassical calculations. The latter are not well suited because the effects of diffraction, which are manifest in the experimental data, are not treated properly in that approach.

For the future, it still remains to obtain a rigorous derivation relating the Coulomb-excitation/photon-decay cross section to the photon scattering. Using the formalism developed here, we have obtained good qualitative agreement with the data of ref. [5]), but only by neglecting the Thomson contribution to the scattering cross section. This is not at present understood and further theoretical work along these lines is highly desirable.

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