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Nuclear Physics A 627 (1997) 324–333

NUCLEAR PHYSICS A

Small effects in astrophysical fusion reactions

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Received 1 July 1997; accepted 24 September 1997

Abstract

We study the combined effects of vacuum polarization, relativity, bremsstrahlung, and atomic polarization in nuclear reactions of astrophysical interest. It is shown that these effects do not solve the longstanding differences between the experimental data of astrophysical nuclear reactions at very low energies and the theoretical calculations which aim to include electron screening. © 1997 Elsevier Science B.V.

PACS: 95.30; 26.10; 24.60.L

Keywords: Nuclear astrophysics; Thermonuclear reactions

1. Introduction

Understanding the dynamics of fusion reactions at very low energies is essential to understand the nature of stellar nucleosynthesis. These reactions are measured at laboratory energies and are then extrapolated to thermal energies. This extrapolation is usually done by introducing the astrophysical S-factor,

$$\sigma(E) = \frac{1}{E} S(E) \exp\left[-2\pi\eta(E)\right] , \tag{1.1}$$

where the Sommerfeld parameter, $\eta(E)$, is given by $\eta(E) = Z_1 Z_2 e^2/\hbar v$. Here Z_1, Z_2, and v are the electric charges and the relative velocity of the target and projectile combination, respectively.

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PII S0375-9474(97)00589-7

The term $\exp(-2\pi\eta)$ is introduced to separate the exponential fall-off of the cross section due to the Coulomb interaction from the contributions of the nuclear force. The latter is represented by the astrophysical S-factor, which is expected to have a very weak energy dependence. The form given in Eq. (1.1) assumes that the electric charges on nuclei are "bare". However, neither at very low laboratory energies, nor in stellar environments is this the case. In stars the bare Coulomb interaction between the nuclei is screened by the electrons in the plasma surrounding them. A simple analytic treatment of plasma screening was originally given by Salpeter [1]. In most cases of astrophysical interest Salpeter's treatment still remains to be a sufficient approximation [2]. In the very low energy laboratory experiments the bound electrons in the projectile or the target may also screen the Coulomb potential as the outer turning point gets very large (> 500 fm). As experimental techniques improve one can measure the cross section in increasingly lower energies where the screened Coulomb potential can be significantly less than the bare one. This deviation from the bare Coulomb potential should manifest itself as an increase in the astrophysical S-factor extracted at the lowest energies. This enhancement was indeed experimentally observed for a large number of systems [4–8]. The screening effects of the atomic electrons can be calculated [3] in the adiabatic approximation at the lowest energies and in the sudden approximation at higher energies with a smooth transition in between [9].

In the adiabatic approximation one assumes that the velocities of the electrons in the target are much larger than the relative motion between the projectile and the target nuclei. In this case, the electronic cloud adjusts to the ground state of a "molecule" consisting of two nuclei separated by a time-dependent distance $R(t)$, at each time instant t. Since the closest approach distance between the nuclei is much smaller than typical atomic cloud sizes, the binding energy of the electrons will be given by the ground-state energy of the $Z_P + Z_T$ atom, $B(Z_P + Z_T)$. Energy conservation implies that the relative energy between the nuclei increases by $U_e = B(Z_T) - B(Z_P + Z_T)$. This energy increment increases the fusion probability. In other words, the fusion cross section measured at laboratory energy E represents in fact a fusion cross section at energy $E + U_e$, with U_e being called by the *screening potential*. Using Eq. (1.1), one gets

$$\sigma(E+U_e) \cong \exp\left[\pi\eta(E)\frac{U_e}{E}\right]\sigma(E)\,, \tag{1.2}$$

where one assumes that the factor $S(E)/E$ varies much slower with E, as compared to the energy dependence of $\exp\left[-2\pi\eta(E)\right]$.

The exponential factor on the right-hand-side of Eq. (1.2) is the enhancement factor due to the screening by the atomic electrons in the target. For light systems the velocity of the atomic electrons is comparable to the relative motion between the nuclei. Thus, a dynamical calculation has to be done for the effect of atomic screening [9]. However, the screening potential U_e obtained from a dynamical calculation cannot exceed that obtained in the adiabatic approximation because the dynamical calculation includes atomic excitations which reduce the energy transferred from the electronic binding to the relative motion.

Contributions from the nuclear recoil caused by the atomic electrons are expected to further increase the screening effect for asymmetric systems [9,10]. In almost all the cases observed screening effects are found to be equal to or more than the theoretical predictions. Recently, including improved energy loss data for atomic targets has been shown to lead agreement between theory and data [11,12]; however, the situation is still not resolved for molecular and solid targets. Electron screening enhancement was not observed for the heavier symmetric system $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ [13], which is expected to have about 20% enhancement at the energies studied. Recent measurements [14] have not yet clarified the effects of electron screening in this reaction. A mechanism which reduces the screening enhancement for this system (and possibly for other systems with large values of $Z_1 Z_2$ and the reduced mass) seems to be needed.

In this article we show that the contributions from the polarization of the vacuum, relativity, bremsstrahlung, and atomic polarization cannot achieve this task. The motivation for this investigation is that U_e/E, appears multiplied by a large number, $\eta(E)$, in the exponential factor of Eq. (1.2). One needs only a small value of U_e to obtain a sizeable enhancement factor; typically $U_e/E \sim 0.001$. The effects of the vacuum polarizability were previously investigated in Ref. [15] for elastic scattering below the Coulomb barrier and in Ref. [16] for subbarrier fusion reactions using the formalism developed by Uehling [17]. Effects of vacuum polarization in ^{12}C–^{12}C scattering at 4 MeV were subsequently experimentally observed [18]. Other small effects in elastic scattering at low energies have also been studied by several authors [19–21]. They have also been studied in the context of astrophysical reactions in Refs. [22,23]. However, to our knowledge, the other effects have not been studied.

In Section 2 we study the effects of vacuum polarization, relativity, bremsstrahlung, and atomic polarization for the astrophysical reactions listed in Ref. [14], and for which a set of "experimental " values of screening energies ΔU_e are given. These experimental values are chosen so that Eq. (1.1) reproduces the enhancement of the fusion cross sections at very low energies. In Section 3 we present our conclusions.

2. Small effects in thermonuclear reactions

To calculate the fusion cross-section corrections we use for simplicity the (s-wave) WKB penetrability factor

$$P(E) = \exp\left[-\frac{2}{\hbar}\int_{R_n}^{R_C} dr|p(r)|\right], \tag{2.1}$$

where $p(r)$ is the (imaginary) particle momentum inside the repulsive barrier. The corrected fusion cross section is given by

$$\sigma = \sigma_C \cdot R, \tag{2.2}$$

where σ_C is the pure Coulomb repulsion cross section, and $R = P_{C+\alpha}(E)/P(E)$, with $\alpha = \{\text{scr, VPol, rel, Brems, At}\}$, are the corrections due to atomic screening, vacuum polarization, relativity, bremsstrahlung, and atomic polarization, respectively.

The atomic screening effect is calculated using $|p(r)| = \sqrt{2m\,[V_c(r) - E - U_e]}$, where E is the relative energy between the nuclei. The atomic screening potential, assumed to be a constant function of r (valid for $r \ll a_B = 0.529$ Å), is given by U_e.

2.1. Vacuum polarization

Vacuum polarization *increases* the electromagnetic potential between two like charges. Like the Coulomb potential itself, the increase due to vacuum polarization is also proportional to the product of the charges [17]. Vacuum polarization contribution increases almost exponentially as the two charges get closer. The Coulomb interaction is smaller for asymmetric systems than for symmetric systems of comparable size. On the other hand, the nuclear force tends to extend farther out for asymmetric systems because of the extra neutrons. Consequently, for asymmetric systems the very tail of the nuclear force can turn the relatively weak Coulomb potential around to form a barrier at a considerable distance from the nuclear touching radius. For symmetric systems, however, the location of the barrier is further inside where the vacuum polarization contribution is stronger. We show that the resulting increase in vacuum polarization is nevertheless not sufficiently large to make an appreciable contribution to the extracted astrophysical S-factor. For light symmetric systems with small values of $Z_1 Z_2$ this effect should be negligible. Indeed, for the pp reaction the vacuum polarization contribution was shown to be very small [22]. Similarly the measured S-factor for the $d(d,p)^3\text{H}$ reaction [8] agrees well with theoretical calculations of atomic screening [24]. On the other hand one may expect that already for the $^3\text{He}(^3\text{He},2p)^4\text{He}$ reaction the increase in the potential due to the vacuum polarization could be large enough to counter the decrease due to electron screening. We show that this is not the case.

The vacuum polarization potential is according to Uehling [17] given by

$$V_{\text{Pol}}(r) = \frac{Z_1 Z_2 e^2}{r}\frac{2\alpha}{3\pi} I\left(\frac{2r}{\lambda_e}\right), \tag{2.3}$$

where $\alpha = 1/137$ is the fine structure constant, and $\lambda_e = 386$ fm is the Compton wavelength of the electron. The function $I(x)$ is given by

$$I(x) = \int_1^\infty \mathrm{e}^{-xt}\left(1 + \frac{1}{2t^2}\right)\frac{\sqrt{t^2-1}}{t^2}\,dt. \tag{2.4}$$

As shown by Pauli and Rose [25] this integral can be rewritten as

$$I(x) = \alpha(x) K_0(x) + \beta(x) K_1(x) + \gamma(x)\int_x^\infty K_0(t)\,dt, \tag{2.5}$$

where

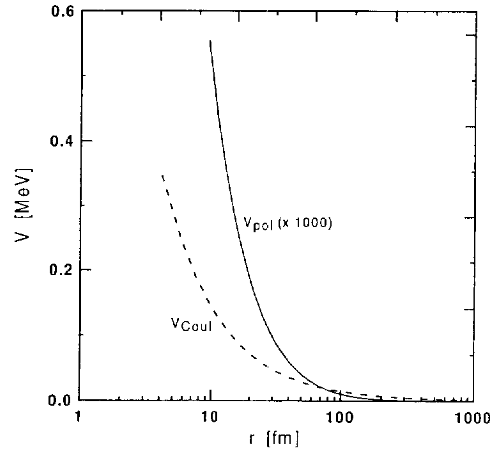

Fig. 1. Comparison between the Coulomb potential and the vacuum polarization potential as a function of the nuclear separation distance for $Z_1 Z_2 = 1$. The vacuum polarization potential has been multiplied by a factor 1000 in order to be visible in the same plot.

$$\alpha(x) = 1 + \tfrac{1}{12}x^2\,, \qquad \beta(x) = -\tfrac{5}{6}x(1 + \tfrac{1}{10}x^2)\,,$$
$$\gamma(x) = \tfrac{3}{4}x(1 + \tfrac{1}{9}x^2)\,, \qquad \text{with } x = 2r/\lambda_e\,. \tag{2.6}$$

In Ref. [15] it was shown that the modified Bessel functions K_0 and K_1 as well the integral over K_0 can be expanded in a very useful series in Chebyshev polynomials which converge rapidly and for practical purposes only a few terms ($\approx$ 5–10) is needed, allowing a very fast and accurate computation of the Uehling potential.

In Fig. 1 we plot the Coulomb potential and the vacuum polarization potential for $Z_1 Z_2 = 1$. Both the Coulomb potential and the screening potential scale with the product $Z_1 Z_2$. However, the vacuum polarization potential has a stronger dependence on the nuclear separation distance.

The limits of the integral are the nuclear radius, R_n, where the nuclear fusion reaction occurs, and the classical turning point in the Coulomb potential, $R_C = Z_1 Z_2 e^2/E'$, where $E' = E + U_e$. At very low energies the inferior limit R_n is not important when vacuum polarization is neglected (the exponential factor in Eq. (1.2) can be obtained with $V_{\rm Pol} = 0$, and $R_n \to 0$, in Eq. (2.1)). However, since the vacuum polarization potential has a strong dependence on the nuclear separation distance, being much stronger at shorter distances, its effect is very much dependent on the choice of this parameter. For all reactions with the deuteron we use the "deuteron radius", $R_n = 4.3$ fm, corresponding to an average distance value associated with matrix elements involving the deuteron. For the other reactions we use R_n values given in the third column of Table 1. In the 4th row we show the ratio between the penetrability factor through the Coulomb barrier and the penetrability factor including atomic screening, $P_{C+\rm Scr}(E)/P_C(E)$. In the 5th row we show the effect of vacuum polarization, $P_{C+\rm VPol}(E)/P_C(E)$. The energy E chosen is the lowest experimental energy for each reaction. The atomic screening corrections U_e were calculated in the adiabatic approximation, given by the differences

Table 1
Lowest experimental energies, E_{min}, energy corrections [24] due to the screening by the atomic electrons, U_e, nuclear radii, and correction factors for the nuclear reaction: (a) due to atomic screening, $1 - R_{Scr}$; (b) vacuum polarization, $1 - R_{VPol}$; (c) relativity, $1 - R_{rel}$; (d) bremsstrahlung, $1 - R_{Br}$; (e) atomic polarization, $1 - R_{At}$

Reaction	E_{min} [keV]	U_e [eV]	R_n [fm]	$1 - R_{Scr}$	$1 - R_{VPol}$ $[\times 10^{-2}]$	$1 - R_{rel}$ $[\times 10^{-3}]$	$1 - R_{Br}$ $[\times 10^{-3}]$	$1 - R_{At}^{dip}$ $[\times 10^{-5}]$	$1 - R_{At}^{mon}$ $[\times 10^{-1}]$
$D(d,p)T$	1.62	20	4.3	0.164	−0.95	0.17	0.54	1.01	0.246
$^3He(d,p)^4He$	5.88	119	4.3	0.331	−1.60	0.47	1.12	0.39	0.314
$D(^3He,p)^4He$	5.38	113	4.3	0.364	−1.58	0.47	1.00	2.11	0.357
$^3He(^3He,2p)^4He$	25	292	3.0	0.196	−3.14	1.75	0.58	0.35	0.321
$^6Li(p,\alpha)^3He$	10.74	186	3.0	0.258	−1.82	1.07	1.36	0.30	0.360
$^7Li(p,\alpha)^4He$	12.70	186	4.3	0.198	−1.88	1.04	1.28	0.17	0.284
$^6Li(d,\alpha)^4He$	14.31	186	3.0	0.218	−2.32	0.72	0.71	0.15	0.313
$H(^6Li,\alpha)^3He$	10.94	186	3.0	0.250	−1.82	1.07	1.23	2.55	0.350
$H(^7Li,\alpha)^4He$	12.97	186	4.3	0.191	−1.88	1.04	1.17	1.42	0.275
$D(^6Li,\alpha)^4He$	15.89	186	3.3	0.184	−2.34	0.71	0.35	0.91	0.262
$^{10}B(p,\alpha)^7Be$	18.70	346	3.3	0.376	−2.38	2.03	1.45	0.57	0.758
$^{11}B(p,\alpha)^8Be$	16.70	346	2.0	0.462	−2.36	2.00	2.13	0.85	0.906

in electron binding energies between the separated atoms and the compound atom [9]. We see that the effect of vacuum polarization is small, but non-negligible for some reactions. Moreover, it increases the discrepancy between the value of the screening potential required to explain the experimental data and the theoretical calculations of this potential as illustrated in Table 1.

2.2. Relativistic effects

A classical Hamiltonian may be written which contains relativistic effects to first order in $1/mc^2$. This Darwin Hamiltonian takes the following form in the center-of-mass system:

$$E = \frac{p^2}{2m_0} + \frac{Z_P Z_T e^2}{r} - \frac{p^4}{8c^2}\left(\frac{1}{m_P^3} + \frac{1}{m_T^3}\right) + \frac{Z_P Z_T e^2}{2m_P m_T c^2}\left(\frac{p^2 + p_r^2}{r}\right), \tag{2.7}$$

where $m_0 = m_P m_T/(m_P + m_T)$ is the reduced mass, and p_r is the radial component of the relative momentum. In a head-on collision,

$$E = \frac{p^2}{2m_0} + \frac{Z_P Z_T e^2}{r} - \frac{p^4}{8c^2}\left(\frac{1}{m_P^3} + \frac{1}{m_T^3}\right) + \frac{Z_P Z_T e^2}{m_P m_T c^2}\frac{p^2}{r}. \tag{2.8}$$

The solution of this equation yields, for $R_n \leqslant r \leqslant R_C$,

$$|p| = (2\beta)^{-1/2}\left[-(\alpha+\gamma) + \sqrt{(\alpha+\gamma)^2 + 4\beta(V_C - E)}\right]^{1/2}, \tag{2.9}$$

$$\alpha = 1/2m_0, \qquad \beta = \left(1/m_P^3 + 1/m_T^3\right)/8c^2, \qquad \gamma = Z_P Z_T e^2/\left(m_P m_T c^2 r\right).$$

The correction due to relativity is given in the 6th column of Table 1. Although the correction in the momentum $|p|$ is of the order of 10^{-6}, the penetrability is enhanced by an amount of order of 10^{-3} as compared to the penetrability with only the Coulomb interaction.

2.3. Bremsstrahlung

The energy emitted by bremsstrahlung per frequency interval $d\omega$ and solid angle element $d\Omega$ is

$$dE_{\mathrm{Br}}(\omega) = d\omega d\Omega \frac{\omega^2}{c^3} \left| \int_{-\infty}^{\infty} \frac{dt}{2\pi} \mathrm{e}^{i\omega t} \sum_j q_j \mathrm{e}^{-ik\widehat{n}\cdot\boldsymbol{r}_j(t)} \left[\boldsymbol{v}_j(t) \times \widehat{\boldsymbol{n}}\right] \right|^2 , \tag{2.10}$$

where $k = \omega/c$ and the direction of observation $\widehat{\boldsymbol{n}} = \boldsymbol{k}/k$. The sum goes over the charges q_j, positions $\boldsymbol{r}_j$, and velocity $\boldsymbol{v}_j$ of the moving particles. In the long-wavelength approximation

$$\mathrm{e}^{-ik\widehat{n}\cdot\boldsymbol{r}_j(t)} = 1 - (ik)\widehat{\boldsymbol{n}} \cdot \boldsymbol{r}_j + \ldots , \tag{2.11}$$

$$dE_{\mathrm{Br}}(\omega) = d\omega d\Omega A_R \frac{\omega^2}{c^3} \left| \boldsymbol{d}(\omega) \times \widehat{\boldsymbol{n}} - \frac{ik}{2} \boldsymbol{Q}(\omega) \times \widehat{\boldsymbol{n}} + \ldots \right|^2 . \tag{2.12}$$

where, in the center-of-mass system, with relative positions and velocities given by $\boldsymbol{r}(t)$ and $\boldsymbol{v}(t)$, respectively, and

$$\boldsymbol{d}(t) = f_1 e^2 \boldsymbol{v}(t) , \qquad \boldsymbol{Q}(t) = f_2 e^2 \left[\widehat{\boldsymbol{n}} \cdot \boldsymbol{r}(t)\right] \boldsymbol{v}(t) ,$$
$$f_\lambda = A_R^{\lambda-1} \left(\frac{Z_P}{A_P^\lambda} - \frac{Z_T}{A_T^\lambda} \right) , \qquad A_R = \frac{A_P A_T}{A_P + A_T} . \tag{2.13}$$

In a head-on collision

$$r = a_0(\cosh\xi + 1) , \qquad t = \omega_0(\sinh\xi + \xi) ,$$
$$a_0 = \frac{Z_P Z_T e^2}{2E} , \qquad \omega_0 = \frac{a_o}{v} . \tag{2.14}$$

$\boldsymbol{d}(\omega)$, $\boldsymbol{Q}(\omega)$ are the Fourier transforms of $\boldsymbol{d}(t)$ and $\boldsymbol{Q}(t)$, respectively, which can be calculated analytically. The final result, after an integration over Ω, is

$$\frac{dE_{\mathrm{Br}}(\omega)}{d\omega} = \frac{4}{\pi} \hbar\omega_0 \alpha \left(\frac{v}{c}\right)^2 A_R^2 \left[f_1^2 h_1 + f_2^2 \left(\frac{v}{c}\right)^2 h_2 \right] , \tag{2.15}$$

where

$$h_1 = \frac{2z}{3} \mathrm{e}^{-\pi z} \left[K'_{iz}(z)\right]^2 , \qquad h_2 = \frac{2z}{15} \mathrm{e}^{-\pi z} \left[K_{iz}(z)\right]^2 , \qquad z = \omega/\omega_0 ,$$
$$K_{iz}(y) = \frac{1}{2} \int_0^\infty \mathrm{e}^{-y\cosh\xi} \cos(z\xi)\, d\xi , \qquad K'_{iz}(y) = \frac{dK_{iz}(y)}{dy} . \tag{2.16}$$

The above results give the bremsstrahlung due to the incoming branch of the trajectory only, since we are interested in the energy loss until the fusion occurs. The result for the full trajectory, including the outgoing branch is obtained by replacing the lower limit of the integral in Eq. (2.16) by $-\infty$.

The frequencies $z \ll 1$ ($\omega \ll \omega_0$) dominate the spectrum and we can replace the functions $K_{iz}(z)$ and $K'_{iz}(z)$ by their approximate values for low z. The integration of Eq. (2.15) over ω is straightforward, and we get

$$E_{\mathrm{Br}} = \frac{4}{3\pi^2}\hbar\omega_0\alpha\left(\frac{v}{c}\right)^2 A_R^2\left\{f_1^2 + \frac{2}{5\pi^2}f_2^2\left(\frac{v}{c}\right)^2\left[\left(\frac{3}{2} - \ln(2\pi)\right)^2 + \zeta(2,2)\right]\right\}, \tag{2.17}$$

where $\zeta(2,2) = 0.64493\ldots$ is the Riemann's Zeta function for a particular value of its argument.

The results of energy loss by bremsstrahlung are given in the 7th row of Table 1, where $R_{\mathrm{Br}} = P_{C+\mathrm{Br}}/P_C$ is calculated by using Eq. (1.2), with U_e replaced by $(-E_{\mathrm{Br}})$. It is larger for the systems with a large effective dipole charge f_1, since the quadrupole radiation is smaller by a factor $(v/c)^2$. However, even for the systems with a larger f_1, the bremsstrahlung correction is of order of 10^{-3}.

2.4. Atomic polarizability

The virtual excitations of the atomic electrons in the target yields in second-order perturbation theory a polarization potential given by

$$V_{\mathrm{At}} = -\sum_{n\neq 0}\frac{|\langle 0|V_C(\boldsymbol{r},\boldsymbol{R})|n\rangle|^2}{E_n - E_0}, \tag{2.18}$$

where

$$V_C(\boldsymbol{r},\boldsymbol{R}) = \sum_i \frac{Z_P e^2}{|\boldsymbol{R} - \boldsymbol{r}_i|} = Z_P e^2\begin{Bmatrix} 1/R & \text{if } r_i < R \\ 1/r_i & \text{if } r_i \geqslant R\end{Bmatrix} \quad \text{(monopole approx.)}, \tag{2.19}$$

$$= \sqrt{\frac{4\pi}{3}}Z_P e^2 Y_{10}(\hat{\boldsymbol{r}})\begin{Bmatrix} r_i/R^2 & \text{if } r_i < R \\ R/r_i^2 & \text{if } r_i \geqslant R\end{Bmatrix} \quad \text{(dipole approx.)}. \tag{2.20}$$

The first equation is valid in the monopole approximation and the second equation is valid in the dipole approximation, in a head-on collision. r_i are the positions of the atomic electrons, and R is the distance between the atomic nuclei.

Using hydrogenic wavefunctions and considering only the atomic polarization arising from the transitions from the ground state, $\Phi_{nlm} \equiv \Phi_{100}$, and the s-state, $\Phi_{nlm} \equiv \Phi_{200}$, we get

$$\langle \Phi_{100}|V_C^{\mathrm{mon}}(\boldsymbol{r},\boldsymbol{R})|\Phi_{200}\rangle = \frac{4\sqrt{2}}{27}\frac{Z_T Z_P e^2}{a_0}f(\chi), \tag{2.21}$$

where $\chi = 3Z_T R/2a_0$ and

$$f(\chi) = (1+\chi)\exp(-\chi)\,. \tag{2.22}$$

For the dipole excitations, considering transitions from the ground state, $\Phi_{nlm} \equiv \Phi_{100}$, and the p-state, $\Phi_{nlm} \equiv \Phi_{210}$, we get

$$\left\langle \Phi_{100} \left| V_C^{\mathrm{dip}}(\boldsymbol{r},\boldsymbol{R}) \right| \Phi_{210} \right\rangle = \frac{8}{9\sqrt{2}} \frac{Z_T Z_P e^2}{a_0} g(\chi)\,, \tag{2.23}$$

where

$$g(\chi) = \frac{1}{\chi^2}\left[8 - \left(8 + 8\chi + 4\chi^2 + \chi^3\right)\exp(-\chi)\right]\,. \tag{2.24}$$

For an estimate of the effect of atomic polarizability, we will assume that the contribution of virtual excitations of the s- and p-orbit are the most relevant. For a hydrogen atom $E_2 - E_1 = 10.2$ eV. Using this value in Eqs. (2.21)–(2.24), also for heavier atoms, we find the corrections due to atomic polarizability presented in the two last rows of Table 1. We see that atomic polarizability is more important for monopole excitations. The reasoning here is the same as in the use of the adiabatic approximation for the effect of electron screening: the monopole field of the combined $(Z_P + Z_T)$ atom is stronger than the monopole field of the Z_P atom. Thus, the contribution of the monopole term dominates over other multipolarities. However, its effect on the fusion cross section is still small. This agrees with the hypotheses used in the dynamical calculations [9] of atomic screening that effects due to atomic excitations, and particularly for virtual excitations, are small and can be neglected.

2.5. *Conclusions*

In conclusion, we have shown that the vacuum polarization, relativistic corrections, bremsstrahlung, and atomic polarization contributions to the astrophysical S-factor never exceed a few percent, but may be significant in extrapolating the measured S-factor to lower energies. Although these contributions are not comparable to that of sub-threshold resonances and electron screening, they represent some of the many factors that may contribute to the weak energy dependence of the S-factor. Nuclear polarization effects, were not included, since they are much smaller than effects due to atomic polarization, for light targets.

Vacuum polarization effects are the most important from all small contributions, and sensitive to the inner turning point of the potential barrier, hence to the diffuseness of the nuclear potential employed. Although the energy needed to create a virtual e^+e^- pair is much larger than atomic excitation energies, the magnitude of its effect is greatly compensated by its large matrix element (due to the large overlap of the electron and positron wavefunctions), contrary to the atomic polarization cases. For the same reason, nuclear polarization and excitation should be neglected.

We have shown in this work that none of the corrections beyond the effect of atomic screening can explain the missing enhancement of the fusion cross sections in atomic target experiments. As suggested in Ref. [11], this effect might well be due to a wrong assumption on the dependence of the stopping power on the beam energy.

Acknowledgements

We thank K. Langanke for useful comments. This work was supported in part by MCT/FINEP/CNPQ(PRONEX) under contract No. 41.96.0886.00, in part by the U.S. National Science Foundation Grant No. PHY-9605140, in part by the FAPESP under contract number 96/1381-0, and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

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