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Pygmy resonances probed with electron scattering

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Pygmy resonances in light nuclei excited in electron scattering are discussed. These collective modes will be explored in future electron-ion colliders such as ELISe/FAIR (spokesperson: Haik Simon - GSI). Response functions for direct breakup are explored with few-body and hydrodynamical models, including the dependence upon final state interactions.

1. Introduction

Reactions with radioactive beams have attracted great experimental and theoretical interest during the last two decades [1]. Progresses of this scientific endeavor were reported on measurements of nuclear sizes [2], use of secondary radioactive beams to elucidate reactions of astrophysical interest [3,4], fusion reactions with neutron-rich nuclei [5,6], tests of fundamental interactions [7], dependence of the equation of state of nuclear matter upon the asymmetry energy [8], and many other research topics.

New research areas with nuclei far from the stability line will become possible with newly proposed experimental facilities. One of the projects at the future FAIR facility of the GSI laboratory/Germany is the study of electron scattering off unstable nuclei in an electron-ion collider mode [9]. A similar proposal exists for the RIKEN laboratory facility in Japan [10]. By means of elastic electron scattering, these facilities will become the main probe of charge distribution in unstable nuclei [11,12]. Coulomb excitation has been very useful in determining the electromagnetic response in light nuclei [13]. For neutron-rich isotopes [14] the resulting photo-neutron cross sections are characterized by a pronounced concentration of low-lying $E1$ strength. But it is well known that non-perturbative effects leading to distortion of the energy spectrum of the fragments interacting with the target (see, e.g. ref. [15]) is a problem of difficult nature. The nuclear response probed with electrons is free from such effects.

The interpretation of the low-lying $E1$ strength in neutron-rich nuclei engendered a debate: are these "soft dipole modes" just a manifestation of the loosely-bound character of light neutron-rich nuclei, or are they the result of the excitation of a resonance? [16–19]. The electromagnetic response in light nuclei, leading to their dissociation, is related to the nuclear physics needed in several astrophysical sites [3,4,15]. The existence of pygmy resonances have important implications on theoretical predictions of radiative

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neutron capture rates in the r-process nucleosynthesis, and consequently on the calculated elemental abundance distribution in the universe [20].

2. Inelastic scattering in electron-ion colliders

Here, J_i (J_f) is the initial (final) angular momentum of the nucleus, $(E, \mathbf{p})$ and $(E', \mathbf{p}')$ are the initial and final energy and momentum of the electron, and $(q_0, \mathbf{q}) = ((E - E')/\hbar c, (\mathbf{p} - \mathbf{p}')/\hbar)$ is the energy and momentum transfer in the reaction. For low energy excitations, $E, E' \gg \hbar c q_0$, for electron energies $E \simeq 500$ MeV and excitation energies $\Delta E = \hbar c q_0 \simeq 1 - 10$ MeV.

In the plane wave Born approximation (PWBA) and using the Siegert's theorem, one can show that [21]

$$\frac{d\sigma}{d\Omega dE_\gamma} = \sum_L \frac{dN_e^{(EL)}(E, E_\gamma, \theta)}{d\Omega dE_\gamma}\ \sigma_\gamma^{(EL)}(E_\gamma), \tag{1}$$

where $\sigma_\gamma^{(EL)}(E_\gamma) \propto dB(EL)/dE_\gamma$, is the photo-nuclear cross section for the EL-multipolarity, and $E_\gamma = \hbar c q_0$. The response function, $dB(EL)/dE_\gamma$, is given by

$$\frac{dB(EL)}{dE_\gamma} = \frac{\left|\langle J_f \| Y_L(\widehat{\mathbf{r}}) \| J_i\rangle\right|^2}{2J_i + 1} \left[\int_0^\infty dr\ r^{2+L}\ \ \delta\rho_{if}^{(EL)}(r)\right]^2 \rho(E_\gamma), \tag{2}$$

where $\rho(E_\gamma)$ is the density of final states (for nuclear excitations into the continuum) with energy $E_\gamma = E_f - E_i$. The geometric coefficient $\langle J_f \| Y_L(\widehat{\mathbf{r}}) \| J_i\rangle$ and the transition density $\delta\rho_{if}^{(EL)}(r)$ will depend upon the nuclear model adopted.

One can also define a differential cross section integrated over angles so that

$$\frac{dN_e^{(EL)}(E, E_\gamma)}{dE_\gamma} = 2\pi \int_{E_\gamma/E}^{\theta_m} d\theta\ \sin\theta\ \frac{dN_e^{(EL)}(E, E_\gamma, \theta)}{d\Omega dE_\gamma}, \tag{3}$$

and θ_m is the maximum electron scattering angle, which depends upon the experimental setup. Notice that the lowest limit in the above integral is $\theta_{\min} = E_\gamma/E$, and not zero.

Eqs. 1-3 show that under the conditions of the proposed electron-ion colliders, electron scattering will offer the same information obtained with real photons. The reaction dynamics information is in the virtual photon spectrum, $N_e^{(EL)}(E, E_\gamma, \theta)$, while the nuclear response dynamics information is in eq. 2. $dN_e^{(EL)}/d\Omega dE_\gamma$ is interpreted as the number of equivalent (real) photons incident on the nucleus per unit scattering angle Ω and per unit photon energy E_γ. For larger scattering angles the Coulomb multipole matrix elements are in general larger than the electric (EL) multipoles, and monopole transitions become relevant [22]. Eq. 1 will not be valid under these conditions.

It is found that the spectrum $dN_e^{(EL)}(E, E_\gamma)/dE_\gamma$ increases rapidly with decreasing energies [21]. For $E = 500$ MeV and excitation energies $\Delta E = 1$ MeV, the spectrum yields the ratios $dN_e^{(E2)}/dN_e^{(E1)} \simeq 500$ and $dN_e^{(E3)}/dN_e^{(E2)} \simeq 100$. However, although $dN_e^{(EL)}/dE_\gamma$ increases with the multipolarity L, the nuclear response decreases rapidly with L, and $E1$ excitations tend to dominate the cross sections. For larger electron energies the ratios $N^{(E2)}/N^{(E1)}$ and $N^{(E3)}/N^{(E1)}$ decrease rapidly. A comparison between the $E1$ virtual photon spectrum, $dN_e^{(E1)}/dE_\gamma$, of 1 GeV electrons with the spectrum generated by 1 GeV/nucleon heavy ion projectiles was published in ref. [21]. The virtual spectrum for the electron contains more hard photons, i.e. the spectrum decreases slower with photon energy than the heavy ion photon spectrum. This is because, in both situations, the rate at which the spectrum decreases depends on the ratio of the projectile kinetic energy to its rest mass, E/mc^2, which is much larger for the electron ($m = m_e$) than for the heavy ion (m = nuclear mass).

3. Dissociation of weakly-bound systems

3.1. One-neutron halo

In a two-body model, the single-particle picture has been used previously to study Coulomb excitation of halo nuclei with success [23–28]. The $E1$ transition integral $\mathcal{I}_{l_i l_f} = \int_0^\infty dr\ r^3\ \ \delta\rho_{if}(r)$ becomes

$$\mathcal{I}_{s\to p} \simeq \frac{e_{ff}\hbar^2}{2\mu}\frac{2E_r}{(S_n+E_r)^2}\left[1+\left(\frac{\mu}{2\hbar^2}\right)^{3/2}\frac{\sqrt{S_n}\,(S_n+3E_r)}{-1/a_1+\mu r_1 E_r/\hbar^2}\right], \tag{4}$$

in terms of the effective range expansion of the phase shift, $k^{2l+1}\cot\delta \simeq -1/a_l + r_l k^2/2$.

As shown in previous works [23,24], a peak is manifest in the response function, $dB(EL)/dE \propto |\mathcal{I}_{s\to p}|^2 \propto E_r^{L+1/2}/(S_n+E_r)^{2L+2}$. It appears centered at the energy [24] $E_0^{(EL)} \simeq (L+1/2)S_n/(L+3/2)$ for a generic electric response of multipolarity L. For $E1$ excitations, the peak occurs at $E_0 \simeq 3S_n/5$. The second term inside brackets in eq. 4 is a modification due to final state interactions. It is important in many cases [21].

3.2. Two-neutron halo

Many weakly-bound nuclei, like ^{6}He or ^{11}Li, require a three–body treatment. In one particular model, the bound–state wavefunction in the center of mass system is written as an expansion over hyperspherical harmonics, see e.g. [29]. For weakly-bound systems having no bound subsystems the hyperradial functions behave asymptotically as [30] $\Phi_a(\rho) \longrightarrow \exp(-\eta\rho)$ as $\rho \longrightarrow \infty$, where the two-nucleon separation energy is related to η by $S_{2n} = \hbar^2\eta^2/(2m_N)$. (see also [32]). The $E1$ transition matrix element is obtained by a sandwich of the E1 operator between $\Phi_a(\rho)/\rho^{5/2}$ and scattering wavefunctions. Following ref. [31], but using distorted scattering states,

$$\mathcal{I}(E1) = \int dxdy \frac{\Phi_a(\rho)}{\rho^{5/2}}\ y^2 x u_p(y)\, u_q(x), \tag{5}$$

where $u_p(y) = j_1(py)\cos\delta_{nc} - n_1(py)\cos\delta_{nc}$ is the core-neutron asymptotic continuum wavefunction, assumed to be a p-wave, and $u_q(x) = j_0(qx)\cos\delta_{nn} - n_0(qx)\cos\delta_{nn}$ is the neutron-neutron asymptotic continuum wavefunction, assumed to be an s-wave.

The $E1$ strength function is proportional to the square of eq. 5 integrated over all momentum variables. As pointed out in ref. [31], the $E1$ three-body response function of ^{11}Li can still be described by an expression similar to the two-body case, but with different factors. Explicitly, $dB(E1)/dE_r \propto E_r^3/\left(S_{2n}^{eff}+E_r\right)^{11/2}$. Instead of S_{2n}, one has to use an effective $S_{2n}^{eff} = aS_{2n}$, with $a \simeq 1.5$. With this approximation, the peak of the strength function in the three-body case is situated at about three times higher energy than for the two-body case. In the three-body model, the maximum is thus predicted at $E_0^{(E1)} \simeq 1.8S_{2n}$, which fits the experimental peak position for the ^{11}Li $E1$ strength function very well [31]. It is thus apparent that the effect of three-body configurations is to widen and to shift the strength function $dB\,(E1)\,/dE$ to higher energies.

3.3. The hydrodynamical model

As with giant dipole resonances (GDR) in stable nuclei, one believes that pygmy resonances at energies close to threshold are present in halo, or neutron-rich, nuclei. This was proposed by Suzuki et al. [33] using the hydrodynamical model for collective vibrations. We will use the method of Myers et al. [36], considering collective vibrations in nuclei as an admixture of Goldhaber-Teller (GT) and Steinwedel-Jensen (SJ) modes. For light nuclei Goldhaber-Teller modes dominate.

For spherically symmetric densities, the transition density, $\delta\rho_p\,(\mathbf{r}) = \delta\rho_p\,(r)\,Y_{10}\,(\widehat{\mathbf{r}})$, can be calculated assuming a combination of SJ and GT distributions [21],

$$\delta\rho_p\,(r) = \sqrt{\frac{4\pi}{3}}R\left\{Z_{eff}^{(1)}\alpha_1\frac{d}{dr} + Z_{eff}^{(2)}\alpha_2\frac{K}{R}j_1\,(kr)\right\}\rho_0(r), \tag{6}$$

where R is the mean nuclear radius, and α_i is the percent displacement of the center of mass of neutron and proton fluids in the GT ($i = 1$) and SJ ($i = 2$) modes. For light, weakly-bound nuclei, it is appropriate to assume that the neutrons inside the core (A_c, Z_c) vibrate in phase with the protons. The neutrons and protons in the core are tightly bound. Calling the excess nucleons by $(A_e, Z_e) = (A - A_c, Z - Z_c)$, the effective charge for the GT mode is $Z_{eff}^{(1)} = (Z_cA_e - A_cZ_e)\,/A$. In eq. 6, $j_1(kr)$ is the spherical Bessel function of first order, α_2 the percent displacement of the center of mass of the neutron and proton fluids in the SJ mode, $kR = a = 2.081$, and $K = 2a/j_0\,(a) = 9.93$.

The hydrodynamical model can be further explored to obtain the energy and excitation strength of the collective excitations. This can be achieved by finding the eigenvalues of the Hamiltonian $\mathcal{H} = \frac{1}{2}\dot{\alpha}\mathcal{T}\dot{\alpha} + \frac{1}{2}\alpha\mathcal{V}\alpha + \dot{\alpha}\mathcal{F}\dot{\alpha}$, where $\alpha = (\alpha_1, \alpha_2)$ is now a vector containing GT and SJ contributions to the collective motion. $\mathcal{T}$ and $\mathcal{V}$ are the kinetic and potential energies 2×2 matrices [36]. The kinetic term can be calculated from GT and SJ velocity fields. The potential term is related to the stiffness parameters of the liquid-drop model and adjusted to a best fit to the nuclear masses. The stiffness of the system is due to the change in symmetry energy of the system as it goes out of the equilibrium position. The last term in the Hamiltonian is the Rayleigh dissipation term, which is related to the Fermi velocity of the nucleons [36] and yields the width of the eigenstate.

As shown by Myers et al. [36], the liquid drop model predicts an equal admixture of SJ+GT oscillation modes for large nuclei. The contribution of the SJ oscillation mode decreases with decreasing mass number, i.e. $\alpha \longrightarrow (\alpha_1,\ 0)$ as $A \longrightarrow 0$. This is even more probable in the case of halo nuclei, where a special type of GT mode (oscillations of the core against the halo nucleons) is likely to be dominant. For this special collective motion an approach different than those used in refs. [36] and [33] has to be considered.

It is easy to make changes in the original Goldhaber and Teller [34] formula to obtain the energy of the collective vibrations, yielding

$$E_{PR} = \left(\frac{3\varphi\hbar^2}{2aRm_NA_r}\right)^{1/2}, \tag{7}$$

where $A_r = A_c\,(A - A_c)\,/A$ and a is the length within which the interaction between a neutron and a nucleus changes from a zero-value outside the nucleus to a high value inside, i.e. a is the width of the nuclear surface. φ is the energy needed to extract one neutron from the proton environment. Goldhaber and Teller [34] argued that in a heavy stable nucleus φ is not the binding energy of the nucleus, but the part of the potential energy due neutron-proton asymmetry. In the case of weakly-bound nuclei this picture changes and it is more reasonable to associate φ to the separation energy of the valence neutrons, S. I will use $\varphi = \beta S$, with a parameter β which is expected to be of order of one. Since for halo nuclei the product aR is proportional to S^{-1}, we obtain the proportionality $E_{PR} \propto S$. Using eq. 7 for ^{11}Li , with $a = 1$ fm, $R = 3$ fm and $\varphi = S_{2n} = 0.3$ MeV, we get $E_{PR} = 1.3$ MeV. Considering that the pygmy resonance will most probably decay by particle emission, one gets $E_r \simeq 1$ MeV for the kinetic energy of the fragments. This is about a factor 2 larger than what is obtained in a numerical calculation [21]. But it is within the right ballpark. The hydrodynamical model is very unlikely to be an accurate model for light, loosely-bound, nuclei and is significant only in that a reasonable magnitude of the resonance energy is found.

Both the direct dissociation model and the hydrodynamical model yield a bump in the response function with position proportional to S, the valence nucleon(s) separation energy. In the direct dissociation model the width of the response function depends on the separation energy and on the nature of the model, i.e. a two- or a three-body model. In the case of the pygmy resonance, this question is completely open. The hydrodynamical model predicts [36] for the width of the collective mode $\Gamma = \hbar\overline{\mathrm{v}}/R$, where $\overline{\mathrm{v}}$ is the average velocity of the nucleons inside the nucleus. This relation can be derived by assuming that the collective vibration is damped by the incoherent collisions of the nucleons with the walls of the nuclear potential well during the vibration cycles. This approach mimics that used in the kinetic theory of gases for calculating the energy transfer of a moving piston to gas molecules in a container. Using $\overline{\mathrm{v}} = 3\mathrm{v}_F/4$, where $\mathrm{v}_F = \sqrt{2E_F/m_N}$ is the Fermi velocity, with $E_F = 35$ MeV and $R = 6$ fm, one gets $\Gamma \simeq 6$ MeV. This is the typical energy width a giant dipole resonance state in a heavy nucleus. But in the case of neutron-rich light nuclei $\overline{\mathrm{v}}$ is not well defined. But the piston model is not able to describe the width of the response function properly. Microscopic models, such as those based on random phase approximation (RPA) calculations, are necessary to tackle this problem. The halo nucleons have to be treated in an special way to get the response at the right energy position, and with approximately the right width. Right now, the problem remains if the experimentally observed peak in dB/dE is due to a direct transition to the continuum, weighted by the phase space of the fragments, or if it proceeds sequentially via a soft dipole collective state.

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