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NUCLEAR PHYSICS A

Nuclear astrophysics with unstable nuclei

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I discuss the challenges in obtaining the nuclear physics input for the Big Bang and stellar evolution. Then I will show how a new generation of theoretical developments and experiments can shed light on the complex nuclear processes that control the evolution of stars and stellar explosions.

1. INTRODUCTION

In stars, the electrons screen the nuclear charges, therefore increasing the fusion probability by reducing the Coulomb repulsion. Fusion cross sections measured in the laboratory have to be corrected by the electron screening when used in a stellar model. This is a purely theoretical problem as one can not reproduce the interior of stars in the laboratory. A screening mechanism occurs in laboratory experiments due to the bound atomic electrons in the nuclear targets. The experimental findings disagree systematically by a factor of two with theory. Dynamical calculations, and other small effects, such as vacuum polarization, atomic and nuclear polarizabilities, relativistic effects, etc., have also been considered [1]. But the discrepancy between experiment and theory remains [1,2].

A possible solution of the laboratory screening problem was sought in ref. [3] by studying the stopping power of slow protons. The obtained stopping power is proportional to v^α, where v is the projectile velocity and $\alpha = 1.35$. Although this result seems to indicate the stopping mechanism as a possible reason for the laboratory screening problem, the theoretical calculations tend to disagree on the power of v at low energy collisions. Ref. [4] found $S \sim v_p^{3.34}$ for protons in the energy range of 4 keV incident on helium targets. Another calculation of the stopping power in atomic He^++He collisions using the two-center molecular orbital basis published in ref. [5]. The agreement with the data of ref. [4] at low energies is excellent. The agreement disappears completely if the nuclear recoil is included. The unexpected "disappearance" of the nuclear recoil was also observed in ref. [6]. This seems to violate a basic principle of nature, as the nuclear recoil is due to Coulomb repulsion between the projectile and the target atoms.

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2. SOLUTIONS WITH RADIOACTIVE BEAMS

2.1. Intermediate energy Coulomb excitation

The importance of relativistic effects in Coulomb excitation of a projectile by a target with charge Z_2, followed by gamma-decay, in nuclear reactions at intermediate energies was studied in details [7–9]. The Coulomb excitation cross section is given by

$$\frac{d\sigma_{i\to f}}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm el} \frac{16\pi^2 Z_2^2 e^2}{\hbar^2} \sum_{\pi\lambda\mu} \frac{B(\pi\lambda, I_i \to I_f)}{(2\lambda+1)^3} \mid S(\pi\lambda,\mu) \mid^2, \tag{1}$$

where $B(\pi\lambda, I_i \to I_f)$ is the reduced transition probability of the projectile nucleus, $\pi\lambda = E1,\ E2,\ M1, \ldots$ is the multipolarity of the excitation, and $\mu = -\lambda, -\lambda+1, \ldots, \lambda$.

Relativistic corrections to the Rutherford formula for $(d\sigma/d\Omega)_{\rm el}$ have been investigated in ref. [10]. For a given impact parameter the scattering angle increases up to 6% when relativistic corrections are included in collisions at 100 MeV/nucleon. Relativistic corrections of the elastic scattering cross section are even more drastic: up to 13% for center-of-mass scattering angles of 0-4 degrees. The orbital integrals $S(\pi\lambda,\mu)$ also change appreciably with relativistic corrections. Inclusion of absorption effects in $S(\pi\lambda,\mu)$ due to the imaginary part of an optical nucleus-nucleus potential were worked out in ref. [8].

A study in ref. [11] has shown that at 10 MeV/nucleon the relativistic corrections are important only at the level of 1%. On the other hand, at 500 MeV/nucleon, the correct treatment of the recoil corrections is relevant on the level of 1%. Thus the non-relativistic treatment of Coulomb excitation can be safely used for energies below 10 MeV/nucleon and the relativistic treatment with a straight-line trajectory is adequate above 500 MeV/nucleon. However, at energies around 100 MeV/nucleon, common to most radioactive beam facilities (MSU, RIKEN, GSI, GANIL), it is very important to use a correct treatment of recoil and relativistic effects, both kinematically and dynamically. At these energies, the corrections can add up to 50%. These effects were also shown in Ref. [9] for the case of excitation of giant resonances in collisions at intermediate energies.

A reliable extraction of useful nuclear properties, like the electromagnetic response (B(E2)-values, γ-ray angular distribution, etc.) from Coulomb excitation experiments at intermediate energies requires a proper treatment of special relativity [11,12]. Dynamical relativistic effects have often been neglected in the analysis of experiments elsewhere (see, e.g. [13]). The effect is highly non-linear, i.e. a 10% increase in the velocity can lead to a 50% increase (or decrease) of certain physical observables [15,14].

2.2. The Coulomb dissociation method

The differential Coulomb breakup cross section for $a + A \longrightarrow b + c + A$ follows from eq. 1. It can be rewritten as

$$\frac{d\sigma_C^{\pi\lambda}(\omega)}{d\Omega} = F^{\pi\lambda}(\omega;\Omega)\ .\ \sigma^{\pi\lambda}_{\gamma+a\ \to\ b+c}(\omega), \tag{2}$$

where ω is the energy transferred from the relative motion to the breakup, and $\sigma^{\pi\lambda}_{\gamma+a\ \to\ b+c}(\omega)$ is the photo-nuclear cross section for the multipolarity $\pi\lambda$ and photon energy ω. The function $F^{\pi\lambda}$ depends on ω, the relative motion energy, nuclear charges and radii, and the scattering angle $\Omega = (\theta, \phi)$. $F^{\pi\lambda}$ can be reliably calculated [7] for each multipolarity $\pi\lambda$.

Time reversal allows one to deduce the radiative capture cross section $b+c \longrightarrow a+\gamma$ from $\sigma^{\pi\lambda}_{\gamma+a \to b+c}(\omega)$ [16]. It has been tested successfully in a number of reactions of interest for astrophysics. The most celebrated case is the reaction $^7\mathrm{Be}(\mathrm{p},\gamma)^8\mathrm{B}$ [17].

Eq. 2 is based on first-order perturbation theory. It also assumes that the nuclear contribution to the breakup is small, or that it can be separated under certain experimental conditions. The contribution of the nuclear breakup has been examined by several authors (see, e.g. [18]). ^{8}B has a small proton separation energy ($\approx$ 140 keV). For such loosely-bound systems multiple-step, or higher-order effects, are important [19]. These effects are manifest in continuum-continuum transitions. Detailed studies of dynamic contributions to the breakup were explored in refs. [20,21] and in several other publications which followed. The role of higher multipolarities (e.g., E2 contributions [22–24] in the reaction $^7\mathrm{Be}(\mathrm{p},\gamma)^8\mathrm{B}$) and the coupling to high-lying states needs to be investigated carefully. In the later case, a recent work has shown that the influence of giant resonance states is small [25].

2.3. Charge exchange reactions

Charge exchange induced in (p,n) reactions are often used to obtain Gamow-Teller matrix elements, $B(GT)$, which cannot be extracted from beta-decay experiments. This approach relies on the similarity in spin-isospin space of charge-exchange reactions and β-decay operators. As a result of this similarity, the cross section $\sigma(\mathrm{p},\mathrm{n})$ at small momentum transfer q is proportional to $B(GT)$ for strong transitions [26]. *Taddeucci*'s formula reads

$$\frac{d\sigma}{dq}(q=0) = K N_D |J_{\sigma\tau}|^2 B(\alpha), \tag{3}$$

where K is a kinematical factor, N_D is a distortion factor (accounting for initial and final state interactions), $J_{\sigma\tau}$ is the Fourier transform of the effective nucleon-nucleon interaction, and $B(\alpha = F, GT)$ is the reduced transition probability for non-spin-flip, $B(F) = (2J_i+1)^{-1}|\langle f||\sum_k \tau_k^{(\pm)}||i\rangle|^2$, and spin-flip, $B(GT) = (2J_i+1)^{-1}|\langle f||\sum_k \sigma_k\tau_k^{(\pm)}||i\rangle|^2$, transitions.

Taddeucci's formula, valid for one-step processes, was proven to work rather well for (p,n) reactions (with a few exceptions). For heavy ion reactions the formula might not work so well. In ref. [27] it was shown that multistep processes involving the physical exchange of a proton and a neutron can still play an important role up to bombarding energies of 100 MeV/nucleon. Refs. [28,29] use the isospin terms of the effective interaction to show that deviations from Taddeucci's formula are common under many circumstances. As shown in ref. [30], for important GT transitions whose strength are a small fraction of the sum rule the direct relationship between $\sigma(\mathrm{p},\mathrm{n})$ and $B(GT)$ values fails to exist. Similar discrepancies have been observed [31] for reactions on some odd-A nuclei including ^{13}C, ^{15}N, ^{35}Cl, and ^{39}K and for charge-exchange induced by heavy ions [29,32]. It is still an open question if Taddeucci's formula is valid in general.

2.4. Knock-out reactions

Single-nucleon knockout reactions with heavy ions, at intermediate energies and in inverse kinematics, have become a specific and quantitative tool for studying single-particle occupancies and correlation effects in the nuclear shell model. It was shown in ref. [33] that the longitudinal component of the momentum (taken along the beam or z direction) gives the most precise information on the intrinsic properties of the halo and that it is insensitive to details of the collision and the size of the target. In contrast, the transverse momentum distributions are significantly broadened by diffractive effects and by Coulomb scattering. This is confirmed in ref. [34], which extends the theory to include a proper momentum dependence of the differential cross section.

3. RECONCILING NUCLEAR STRUCTURE WITH NUCLEAR REACTIONS

Very often one solves a set of coupled integro-differential equations of the form

$$\sum_{\alpha'} \int d^3r' \left[H^{AB}_{\alpha\alpha'}(\mathbf{r},\mathbf{r}') - E N^{AB}_{\alpha\alpha'}(\mathbf{r},\mathbf{r}') \right] g_{\alpha'}(\mathbf{r}') = 0, \quad (4)$$

where $H^{AB}_{\alpha\alpha'}(\mathbf{r},\mathbf{r}') = \langle \Psi_A(\alpha,\mathbf{r})|H|\Psi_B(\alpha',\mathbf{r}')\rangle$ and $N^{AB}_{\alpha\alpha'}(\mathbf{r},\mathbf{r}') = \langle \Psi_A(\alpha,\mathbf{r})|\Psi_B(\alpha',\mathbf{r}')\rangle$. In these equations H is the Hamiltonian for the system of two nuclei (A and B) with energy E, $\Psi_{A,B}$ is the wavefunction of nucleus A (and B), and $g_\alpha(\mathbf{r})$ is a function to be found by numerical solution of eq. 4, which describes the relative motion of A and B in channel α. Full antisymmetrization between nucleons of A and B are implicit. Modern nuclear shell-model calculations, including the No-Core-Shell-Model (NCSM) are able to provide the wavefunctions $\Psi_{A,B}$ for light nuclei.

Overlap integrals of the type $I_{Aa}(r) = \langle \Psi_{A-a}|\Psi_A\rangle$ for bound states have been calculated in ref. [35] within the NCSM. This is one of the inputs necessary to calculate S-factors for radiative capture, $S_\alpha \sim |\langle g_\alpha|\mathcal{O}_{EM}|I_{Aa}\rangle|^2$, where $\mathcal{O}_{EM}$ is a corresponding electromagnetic operator. The left-hand side of this equation is to be obtained by solving eq. 4. For some cases, in particular for the p+^{7}Be reaction, the distortion caused by the microscopic structure of the cluster does not seem to be crucial to obtain the wavefunction in the continuum. The wavefunction is often obtained by means of a potential model. The NCSM overlap integrals, I_{Aa}, can also be corrected to reproduce the right asymptotics [36], given by $I_{Aa}(r) \propto W_{-\eta,l+1/2}(2k_0 r)$, where η is the Sommerfeld parameter, l the angular momentum, $k_0 = \sqrt{2\mu E_0}/\hbar$ with μ the reduced mass and E_0 the separation energy.

A step in the direction of reconciling structure and reactions for the practical purpose of obtaining astrophysical S-factors, along the lines described in the previous paragraph, was obtained in ref. [36]. The wavefunctions obtained in this way were shown to reproduce very well the momentum distributions in knockout reactions of the type ^{8}B+$A \longrightarrow$ ^{7}Be+X obtained in experiments at MSU and GSI facilities. The astrophysical S-factor for the reaction ^{7}Be(p, γ)^{8}B was also calculated and excellent agreement was found with the experimental data of direct and indirect measurements [36].

REFERENCES

1. A.B. Balantekin, C.A. Bertulani, M.S. Hussein, Nucl. Phys. A627 (1997) 324.
2. C. Rolfs, Prog. Part. Nucl. Phys. 46 (2001) 23.
3. C.A. Bertulani and D.T. de Paula, Phys. Rev. C62 (2000) 045802.
4. R. Golser and D. Semrad, Phys. Rev. Lett. 14 (1991) 1831.
5. C.A. Bertulani, Phys. Let. B 585 (2004) 35.

6. A. Formicola et al., Eur. Phys. J. A8 (2000) 443.
7. C.A.Bertulani and G.Baur, Phys. Reports 163 (1988) 299.
8. C.A. Bertulani and A.M. Nathan, Nucl. Phys. A 554 (1993) 158.
9. A.N.F. Aleixo and C.A. Bertulani, Nucl. Phys. A 505 (1989) 448.
10. C.E.Aguiar, A.N.F.Aleixo and C.A.Bertulani, Phys. Rev. C 42 (1990) 2180.
11. C.A. Bertulani, A. Stuchbery, T. Mertzimekis and A. Davies, Phys. Rev. C 68 (2003) 044609.
12. C.A. Bertulani, C.M. Campbell, and T. Glasmacher, Comput. Phys. Commun. 152 (2003) 317.
13. T. Glasmacher, Nucl. Phys. A 693 (2001) 90.
14. C.A.Bertulani, Phys. Rev. Lett. 94 (2005) 072701.
15. C.A. Bertulani, Proceedings of Workshop on "Reaction Mechanisms for Rare Isotope Beams" Michigan State University March 9-12,2005, arXiv.org:nucl-th/0505015.
16. G. Baur, C. Bertulani and H. Rebel, Nucl. Phys. A459 (1986) 188.
17. T. Motobayashi, et al., Phys. Rev. Lett. 73 (1994) 2680.
18. C.A. Bertulani and M. Gai, Nucl. Phys. A636 (1998) 227.
19. C.A.Bertulani and L.F.Canto, Nucl. Phys. A 539 (1992) 163
20. G. Baur, C.A. Bertulani and D.M. Kalassa, Nucl. Phys. A550 (1992) 527.
21. G.F. Bertsch and C.A. Bertulani, Nucl. Phys. A556 (1993) 136; Phys. Rev. C49 (1994) 2834; H. Esbensen, G.F. Bertsch and C.A. Bertulani, Nucl. Phys. A581 (1995) 107.
22. C.A. Bertulani, Phys. Rev. C49 (1994) 2688; Nucl. Phys. A587 (1995) 318; Z. Phys. A356 (1996) 293.
23. M. Gai and C.A. Bertulani, Phys. Rev. C52 (1995) 1706; Nucl. Phys. A636 (1998) 227.
24. H. Esbensen and G. Bertsch, Phys. Rev. Lett. A359 (1995) 13; Nucl. Phys. A600 (1996) 37.
25. C.A. Bertulani, Phys. Lett. B3 (2002) 205.
26. T.N. Tadeucci et al., Nucl. Phys. A469 (1987) 125.
27. H. Lenske, H.H. Wolter and H.G. Bohlen, C.A. Bertulani, Phys. Rev. Lett. 62 (1989) 1457.
28. C.A. Bertulani, Nucl. Phys. A 554 (1993) 493.
29. C.A. Bertulani and P. Lotti, Phys. Lett. B 402 (1997) 237; C.A. Bertulani and D. Dolci, Nucl. Phys. A674 (2000) 527.
30. S.M. Austin, N. Anantaraman and W.G. Love, Phys. Rev. Lett. 73 (1994) 30.
31. J.W. Watson et al., Phys. Rev. Lett. 55 (1985) 1369.
32. M. Steiner, et al., Phys. Rev. Lett. 76 (1996) 26.
33. C. A. Bertulani and K. W. McVoy, Phys. Rev. C 46 (1992) 2638.
34. C. A. Bertulani and P. G. Hansen, Phys. Rev. C 70 (2004) 034609.
35. P. Navratil, Phys. Rev. C. 70 (2004) 054324.
36. P. Navratil, C.A. Bertulani, and E. Caurier, Phys. Lett. B 634 (2006) 191; Phys. Rev. C 73 (2006) 065801.