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Blurred femtoscopy in two-proton decay

C.A. Bertulani [a,*], M.S. Hussein [b,c], G. Verde [d]

[a] Department of Physics, Texas A&M University, Commerce, TX 75429, USA
[b] Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, Brazil
[c] Max-Planck-Institut für Physik komplexer Systeme, 01187 Dresden, Germany
[d] INFN, Sezione di Catania, Via S. Sofia 64 (Cittadella Universitaria), I-95123 Catania, Italy

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ABSTRACT

We study the effects of final state interactions in two-proton emission by nuclei. Our approach is based on the solution the time-dependent Schrödinger equation. We show that the final relative energy between the protons is substantially influenced by the final state interactions. We also show that alternative correlation functions can be constructed showing large sensitivity to the spin of the diproton system.

Two-proton emission has been observed for numerous excited states in nuclei, populated both in β decay and in nuclear reactions [1–6]. Although these decays are thought to be sequential one-proton emissions proceeding through states in the intermediate nucleus [7,8], there is an intriguing possibility [9] that the diproton (^{2}He) correlation may play an important role in the mechanism of the two-proton emission. This has been nicely demonstrated in the analysis of the two-proton decay of the 21^+ isomeric state in ^{94}Ag, where 19 events were clearly assigned to the simultaneous emission of two correlated protons [4,6]. The traditional idea of diproton radioactivity is due to the pairing effect. Two protons form a quasiparticle (diproton) under the Coulomb barrier and this facilitates penetration. In a more formal description, one has a system with two valence protons in the same shell and coupled to $J^\pi = 0^+$. This question, being still open, continues to motivate studies in this field. In order to assess this information, it is necessary to understand final state interactions between the protons and between each proton and the daughter nucleus.

In nuclear decays the emission of correlated, identical, particles is sensitive to the geometry of the system. Measurements of correlation functions are often performed with charged particle pairs, which interact via the short-range nuclear interaction and the long-range Coulomb interaction and they also interact with the remaining source. As a result, theoretical corrections are needed to subtract the final state interactions (FSI) before one can extract any useful information about the emitting source from the measurements [10–13]. At first sight, the FSI can be regarded as a contamination of "pure" particle correlations. However, it should be noted that the FSI depend on the structure of the emitting source and thus provide information about source dynamics as well.

Two-proton decay in s-wave states can also be used for testing quantum mechanics versus local realism by means of Bell's inequalities [14]. Since the final state of the two protons can be either in a singlet or in a triplet state, their wavefunction is spin entangled. The identification of the spins of the proton in two detectors separated far away would be useful to test the Einstein–Podolski–Rosen (EPR) paradox [15,16]. In fact, these tests should be performed in different and complementary branches of physics to avoid the loopholes encountered in photon experiments. The advantage of using massive fermions to test Bell-type inequalities is that the particles are well localized and the spin state of the pair can be well established by measuring the internal energy of the two-proton system. However, the validity of this method highly depends on our ability to treat FSI. Coincidence measurements of the two proton momenta require knowledge of FSI in order to extract information about their original wavefunction. Here will propose a new method to calculate FSI based on the numerical solution of the time-dependent Schrödinger equation. We hope with that to get a quantitative estimate of the FSI and how they can be used to address the points raised above. Though tests of Bell's inequality using proton–proton spin correlation in low energy scattering con-

* Corresponding author.
E-mail addresses: carlos_bertulani@tamu-commerce.edu (C.A. Bertulani), mhussein@mpipks-dresden.mpg.de (M.S. Hussein), verde@ct.infn.it (G. Verde).

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formed to quantum mechanics (see, e.g., [17]), one would still like to find other means to perform the verification of the complete nature of quantum mechanics.

One should distinguish this work from Hanbury-Brown–Twiss (HBT) studies in high energy nucleus–nucleus collisions. Indeed, in the case of HBT, the whole game is played by FSI. FSI are the means by which one can determine information about the source. In this case FSI is not viewed as a "contamination". The important difference in our case is that we are interested in a case where the protons are not emitted chaotically like in the case of HIC (Heavy-Ion Collisions). We are studying nuclear structure. In HIC (and HBT) protons are assumed to be emitted independently and chaotically from the source (any information about their initial spins is lost and they are assumed to be "evaporated" with a boiling pot). Then, there are no initial state correlations and FSI make the whole physics. In the case of two-proton radioactivity, the emission of the two protons is not chaotic because their correlation function keeps memory of their spin admixture and wave function in the parent nucleus.

We consider first a single proton described at the initial time by a localized wave-packet $\psi_0(\mathbf{r}_1)$. The probability amplitude to find the proton at the detector with momentum $\mathbf{p}_1$ is given by

$$\mathcal{A}(\mathbf{p}_1, \mathbf{r}_1) = \int d\mathbf{r}\, \chi^{(+)}(\mathbf{p}_1, \mathbf{r}) K(\mathbf{r}, \mathbf{r}_1) \psi_0(\mathbf{r}_1), \tag{1}$$

where $\chi^{(+)}(\mathbf{p}_1, \mathbf{r})$ is an asymptotic outgoing Coulomb wave with energy $E = \mathbf{p}_1^2/2m_p$, and $K(\mathbf{r}, \mathbf{r}_1)$ is the propagator which accounts for the time evolution of the particle from the source to the detector.

We now look at the case of two-protons interacting with the residual nucleus and between themselves. We will consider the distortion caused by the Coulomb plus nuclear interaction between each proton i with the nucleus, $V_C(\mathbf{r}_i) + V_N(\mathbf{r}_i)$, and between themselves, $v_C^{12}(\mathbf{r}) + v_N^{12}(\mathbf{r})$, where $\mathbf{r}_i$ is the coordinate of proton i, and $\mathbf{r}$ is their relative coordinate. The proton–nucleus interaction, $V_N(\mathbf{r}_i)$ yields smaller final state interaction effects than the Coulomb counterpart.

We adopt a classical description of the center-of-mass motion for the two-protons and solve the time-dependent Schrödinger equation for the relative motion between them. The Coulomb field that distorts the relative motion of the particles is given by

$$V_C(t) = Ze^2\left(\frac{1}{|\mathbf{r}_1 - \mathbf{R}(t)|} - \frac{1}{|\mathbf{r}_2 - \mathbf{R}(t)|} - \frac{2}{R(t)}\right), \tag{2}$$

where Z is the charge of the daughter nucleus and $\mathbf{r}_1$ and $\mathbf{r}_2$ are the positions of the protons with respect to the center of the nucleus (nuclear recoil is neglected). $V_C(t)$ acts on the relative position $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ through the transformations $\mathbf{r}_1 = \mathbf{R} - \mathbf{r}/2$ and $\mathbf{r}_2 = \mathbf{R} + \mathbf{r}/2$.

One can perform a multipole expansion of this interaction and for r smaller than $R(t)$ one can express the result in terms of a multipole-dependent effective charge, $e_L = e[(-1/2)^L + (1/2)^L]$ where L is the multipole degree. The dipole field ($L = 1$) is only important for particles with different charge-to-mass ratios while the quadrupole field is dominant when these ratios are equal (e.g. for two-proton emission). For the quadrupole interaction, $e_{L=2} = e/2$ and

$$V_C(t) = \frac{Ze^2}{2} \frac{r^2}{R^3(t)} P_2(\cos\theta), \tag{3}$$

where θ is the angle between $\mathbf{R}$ and $\mathbf{r}$ and P_2 is the Legendre polynomial of order 2. We now assume that the protons are produced simultaneously and nearly at rest at position $2a_0$ and time $t = 0$. Their center-of-mass follows a radial trajectory described by

$$R(t) = \frac{a_0}{2}(\cosh w + 1), \qquad t = \frac{a_0}{2v}(\sinh w + w), \tag{4}$$

where the asymptotic velocity is given by $v = \sqrt{E/m_p}$, E is the two-proton decay energy, and $a_0 = e^2/2E$. This assumes that the relative energy between the protons is much smaller than E, which is not a good approximation, as we will show later. It is important to notice that Eqs. (4) only account for the motion of the protons after they emerge from inside the nucleus through the Coulomb barrier and propagate from the closest distance $2a_0$ to infinity. Hence, our calculations neglect what happens during the tunneling process and treat only the external motion. Hence, neglecting the proton–nucleus strong FSI is justified.

We can still use Eq. (1) to calculate the probabilities for relative motion of the protons, with the wavefunction for the relative motion given by $\Psi(\mathbf{r}) = K(\mathbf{r}, \mathbf{r}_0)\psi_0(\mathbf{r}_0)$. In the time-dependent description, at time t this wave function can be expanded in spherical harmonics

$$\Psi(\mathbf{r}) = \frac{1}{r}\sum_{lm} u_{lm}(r,t) Y_{lm}(\hat{\mathbf{r}}), \tag{5}$$

and the Schrödinger equation, describing the time evolution of the relative motion between the protons can be solved by the finite difference method, calculating the wavefunction at time $t + \Delta t$ in terms of the wavefunction at time t, according to the algorithm

$$u_{lm}(t+\Delta t) = \left[\frac{1}{i\tau} - \Delta^{(2)} + \frac{\Delta t}{2\hbar\tau}U\right]^{-1} \times \left[\frac{1}{i\tau} + \Delta^{(2)} + \frac{\Delta t}{2\hbar\tau}U + \frac{\Delta t}{\hbar\tau}S_{l'm';lm}\right]u_{lm}(t), \tag{6}$$

where $\tau = \hbar\Delta t/m_p(\Delta r)^2$. The second difference operator is defined as

$$\Delta^{(2)}u_{lm}^{(j)}(t) = u_{lm}^{(j+1)}(t) + u_{lm}^{(j-1)}(t) - 2u_{lm}^{(j)}(t), \tag{7}$$

with $u_{lm}^{(j)}(t) = u_{lm}(r_j, t)$, where r_j is a position in the radial lattice. In Eq. (6), $U = v_C^{12}(r_j) + v_N^{12}(r_j)$ is the Coulomb + nuclear interaction between the two protons as a function of their distance, r_j, and the function $S_{l'm';lm}$ is given by

$$S_{l'm';lm}(r,t) = \sum_{l'm'} \langle Y_{l'm'}|V_C(r,t)|Y_{lm}\rangle u_{l'm'}(r,t). \tag{8}$$

This method of solving the time-dependent equation is the same as used in Ref. [18] for studying reacceleration effects in breakup reactions in nucleus–nucleus collisions at intermediate energies. A grid adequate for our purposes has 5000 spatial mesh points separated by 0.1 fm and 2000 time mesh points separated by 0.5 fm/c.

We use the quantization-axis along the $\mathbf{R}(t)$ center-of-mass radial trajectory. As a consequence, $P_2(\cos\theta) = \sqrt{4\pi/5}Y_{20}(\theta,\phi)$, and one only needs to consider the $m = 0$ component of the spherical harmonics implicitly contained in the potential $V_C(t)$. The initial $l = 0$ state cannot develop a final $l = 1$ component, and only $l = 0$ (s-waves) and $l = 2$ (d-waves) will be present in the final state. Higher l values will be small and need not be considered.

The proton–proton potential is taken as $v_N^{12}(r) + v_C^{12}(r) = e^2/r + v_0(b/r)\exp(-r/b)$. The set of parameters $v_0 = -46.124$ MeV and $b = 1.1809$ fm yields the proton–proton scattering length, $a_p = -7.8196$ fm and the effective range $\rho_0 = 2.790$ fm, in accordance with experimental data. But we choose a higher absolute value of v_0 which allows the presence of a single weakly bound s-wave state. We use this localized wavefunction for the relative motion of the two protons in the initial state: $u_0 \equiv u_{l=0}(r, t = 0)$. This is an artifact of the numerical method chosen as to allow for a localization of the initial wavefunction. The observables associated with the final state will depend on the binding energy, reflecting the dependence on the initial average separation between them.

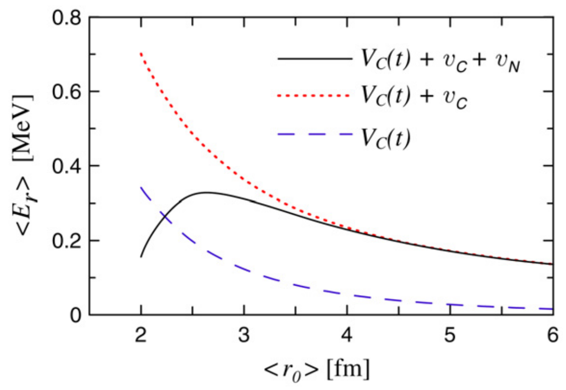

Fig. 1. Average relative motion energy of the two protons as a function of the average initial distance between them. The dashed curve is the final proton–proton relative energy if their interaction is neglected. The dotted curve includes the Coulomb repulsion between them and the solid curve includes their Coulomb and nuclear interaction.

The average initial separation, r_0, and the binding energy, B, are approximately related by $r_0 = \hbar(4Bm_p)^{-1/2}$.

As time evolves the initial state will acquire components in the continuum due to the action of the interaction $V_C(t)$. The continuum component propagates as a wavepacket which moves away from the source with a final asymptotic momentum $\mathbf{p}$. The continuum wavefunction is obtained by removing the bound-state part from the solution of Eq. (6)

$$\Psi_c(t) = \mathcal{N}\big[\Psi(t) - \langle\Psi(t)|\Psi_0\rangle\Psi_0\big],$$

where $\mathcal{N}$ normalizes the continuum wavefunction, Ψ_c, to unity.

The probability amplitude to find the protons with a final relative momentum $\mathbf{p}$ is given by

$$\begin{aligned}\mathcal{A}(\mathbf{p}) &= \big\langle \chi^{(+)}(\mathbf{p},\mathbf{r})\big|\Psi(\mathbf{r}, t \longrightarrow \infty)\big\rangle \\ &= \int u_{l=0}(r, t\to\infty) H_0(pr)\,dr \\ &\quad + \int u_{l=2}(r, t\to\infty) H_2(pr)\,dr, \qquad (9)\end{aligned}$$

where

$$H_l(pr) = \exp\left[i\left(pr + \frac{l\pi}{2} - \eta\ln(2pr) + \sigma_l\right)\right] \qquad (10)$$

is the asymptotic Coulomb wavefunction for angular momentum l, with $p = \hbar k$, $\eta = e^2/\hbar v_r$, $v_r = \sqrt{4E_r/m_p}$ is the asymptotic relative velocity, and $E_r = \hbar^2 k^2/m_p$ the relative energy.

We consider two-proton decay from ^{45}Fe with a decay energy of 1.1 MeV. In Fig. 1 we show the results for the average relative motion energy $\langle E_r\rangle = \langle\mathcal{A}|p^2/m_p|\mathcal{A}\rangle/\langle\mathcal{A}|\mathcal{A}\rangle$ as a function of the average initial distance between the two protons. The dashed curve is the final proton relative energy if their mutual interaction is neglected. The dotted curve includes the Coulomb repulsion between the protons and the solid curve includes both their Coulomb and nuclear interaction.

The physical reasons for the results shown in Fig. 1 are transparent: two strongly interacting particles emitted from the volume of $r_0 \approx 2$ fm, when their "own mean radius", reflected by their attractive strong potential v_N, is about $b \approx 1$ fm, should sufficiently "feel" each other through mutual attraction. On the other hand, when the emission zone is much larger than the particle mean radius, for instance if the radius is $r_0 \approx 6$ fm, the contribution of the short-range attraction is negligible and only the long-range Coulomb repulsion acts, as is seen in Fig. 1. It is also clear that the final relative energy increases when the distance r_0 decreases. But as r_0 increases the contribution of the "tidal" Coulomb interaction of the diproton with the daughter nucleus decreases much faster than the contribution of their own mutual Coulomb repulsion.

It is also important to notice that all the contributions to the FSI energies are not small compared to the decay energy E. This is contrary to our initial assumption used to justify our dynamical model. Hence, the results point to the important conclusion that final-state interactions are very important in determining the relation between the proton energies and the spatial distribution of the protons in the decay process. The FSI contributions due to the Coulomb tidal interaction depend on the square of the charge of the daughter nucleus while the contribution of the strong force between the protons is approximately independent of the nuclear mass. Thus, for lighter nuclei (e.g. ^{18}Ne), the dashed curve in Fig. 1 becomes negligible.

Further aspects of the spatial configuration of the protons can be obtained in light of the usual discussion in terms of two-particle interferometry, or correlation functions. The relation between the center-of-mass coordinates and the laboratory are given by

$$\mathbf{R} = \frac{(\mathbf{r}_1 + \mathbf{r}_2)}{2}, \qquad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2,$$
$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2, \qquad \mathbf{p} = \frac{(\mathbf{p}_1 - \mathbf{p}_2)}{2}.$$

The probability amplitude to find one of the protons with momentum $\mathbf{p}_1$ is given by $\mathcal{A}_1(\mathbf{p}_1,\mathbf{r}_1) = \mathcal{A}(\mathbf{p}_1 - \mathbf{P}/2, \mathbf{r}_1)$. The center of mass momentum, $\mathbf{P}$, is set by the decaying energy and the assumption that it follows an outgoing radial motion, i.e. $\mathbf{P} = \hat{\mathbf{R}}\sqrt{E/m_p}$, where $\hat{\mathbf{R}}$ is the unit vector along the radial direction.

Next we show that one can disentangle the contributions of singlet and triplet spin final states of the two-proton system by measuring momentum correlations. This will prove to be a useful method since a direct measurement of the spin orientations of each proton is by far more complicated. The application of the method is very general as it only relies on measured quantities, independent of the models for the treatment of FSI.

The protons are identical particles and their detection requires the consideration of their quantum statistical properties. If proton 1 is detected with momentum $\mathbf{p}_1$ and proton 2 is detected with momentum $\mathbf{p}_2$, the probability amplitude for this is given by product $\mathcal{A}_1(\mathbf{p}_1,\mathbf{r}_1)\mathcal{A}_2(\mathbf{p}_2,\mathbf{r}_2)$. Because of the indistinguishability of the particles, the probability amplitude must be symmetric with respect to the interchange of two particles if they are in a spin-singlet state ($S = 0$), and antisymmetric if they are in a spin-triplet state ($S = 1$). The normalized probability amplitude becomes

$$\begin{aligned}\Lambda^{(\pm)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\sqrt{2}}\big[&\mathcal{A}_1(\mathbf{p}_1,\mathbf{r}_1)\mathcal{A}_2(\mathbf{p}_2,\mathbf{r}_2) \\ &\pm \mathcal{A}_1(\mathbf{p}_2,\mathbf{r}_1)\mathcal{A}_2(\mathbf{p}_1,\mathbf{r}_2)\big],\end{aligned}$$

where the plus or minus sign is the spin-singlet and spin-triplet state, respectively.

The two-particle momentum distribution $P(\mathbf{p}_1,\mathbf{p}_2)$ is the probability to measure a nucleon having momentum $\mathbf{p}_1$ in coincidence with the measurement of the other nucleon having momentum $\mathbf{p}_2$. It is defined as

$$\begin{aligned}P(\mathbf{p}_1,\mathbf{p}_2) = \int d^3r_1 d^3r_2 \big|\big|\Lambda^{(+)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r}_1,\mathbf{r}_2)\big|^2 \\ \pm \mathcal{M}\Lambda^{(-)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r}_1,\mathbf{r}_2)^2\big|^2, \qquad (11)\end{aligned}$$

where $\mathcal{M}$ is the mixing parameter, determining the relative contribution of the triplet state. The correlation function $C(\mathbf{p}_1,\mathbf{p}_2)$ is

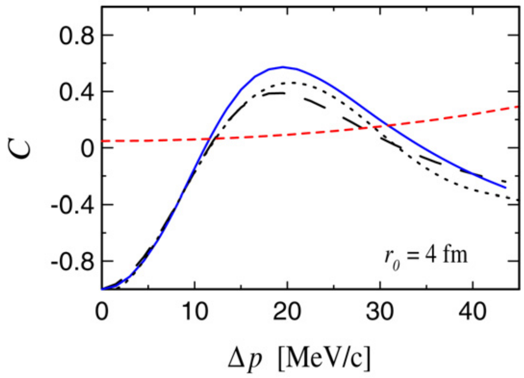

Fig. 2. Correlation function, Eq. (15), for the ^{45}Fe decay with $E = 1.1$ MeV, as a function of the relative momentum of the protons. The short-dashed curve is for the singlet state and all others are for the triplet state. The dotted curve does not include the proton–proton interaction, the long-dashed curve includes the Coulomb interaction between the protons and the solid curve includes both nuclear and Coulomb interaction between the protons.

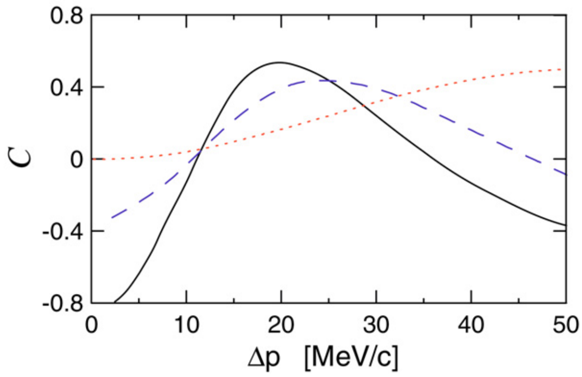

Fig. 3. Correlation function, $C(\Delta p)$, for $r_0 = 4$ fm and for different admixtures of singlet and triplet states. The dotted, dashed and solid lines correspond to $\mathcal{M} = 0.1$, 0.5 and 0.9, respectively. $\mathcal{M}$ is the absolute contribution of the triplet state.

defined as the ratio of the probability for the coincidence of $\mathbf{p}_1$ and $\mathbf{p}_2$ relative to the probability of observing $\mathbf{p}_1$ and $\mathbf{p}_2$ separately,

$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{P(\mathbf{p}_1, \mathbf{p}_2)}{P_1(\mathbf{p}_1)P_2(\mathbf{p}_2)}.$$

Let us assume for the moment the that protons suddenly emerge from the nucleus and that their intrinsic wavefunction is in a pure entangled state. If their wavefunction is approximated by $\exp(i\mathbf{p}_1 \cdot \mathbf{r}_1)\exp(i\mathbf{p}_2 \cdot \mathbf{r}_2) \pm \exp(i\mathbf{p}_2 \cdot \mathbf{r}_1)\exp(i\mathbf{p}_1 \cdot \mathbf{r}_2)$ it will lead to destructive or constructive interferences. If we also assume a Gaussian source of size $r_0 \equiv \sqrt{\langle r^2 \rangle}$, the correlation function without final state interactions would be given by

$$C(\mathbf{p}_1, \mathbf{p}_2) \equiv C(q) = f(\Delta p)\left[1 \pm \exp\left(-\Delta p^2 r_0^2/\hbar^2\right)\right], \tag{12}$$

where $\Delta p = p = |\mathbf{p}_1 - \mathbf{p}_2|/2$. All other features of the reaction mechanism are included in the function $f(\Delta p)$. One can approximately account for the Coulomb interaction between the protons by using a Gamow function for $f(\Delta p)$:

$$f(\Delta p) = \frac{2\pi\eta}{\exp(2\pi\eta) - 1}, \tag{13}$$

but no such simple estimate exists for the effect of the nuclear interaction.

According to Eq. (12), for $\Delta p r_0/\hbar \ll 1$ one should be able to see a destructive interference for triplet final states and constructive interference for singlet final states. It is thus appropriate to redefine the correlation function in terms of the relative momentum between the protons, so that the correlated ($\mathcal{C}$) and uncorrelated ($\mathcal{U}$) measurements of protons 1 and 2 are defined by

$$\mathcal{C}(\Delta p) = \int P(\mathbf{p}_1; \mathbf{p}_1 + 2\Delta\mathbf{p})\, d\mathbf{p}_1\, d\Omega_p,$$

$$\mathcal{U}(\Delta p) = \frac{1}{N}\int P(\mathbf{p}_1)P(\mathbf{p}_1 + 2\Delta\mathbf{p})\, d\mathbf{p}_1\, d\Omega_p. \tag{14}$$

The integration in Ω_p is over all orientations of $\Delta\mathbf{p}$. $P(\mathbf{p}) = \int P(\mathbf{p}; \mathbf{p}')\, d\mathbf{p}'$ is the probability to measure the momentum $\mathbf{p}$ for one of the protons, irrespective of what the momentum of the other proton is. N is the total number of particles measured, i.e., $N = \int P(\mathbf{p})\, d\mathbf{p}$.

The new correlation function is defined as

$$C(\Delta p) = \frac{\mathcal{C}(\Delta p)}{\mathcal{U}(\Delta p)} - 1. \tag{15}$$

In Fig. 2 we show the correlation function for ^{45}Fe decay with $E = 1.1$ MeV, as a function of the relative momentum of the protons. The short-dashed curve is for the singlet state and all others for the triplet state. The dotted curve does not include the proton–proton interaction, the long-dashed curve includes the Coulomb interaction between them and the solid curve includes both nuclear and Coulomb interaction between the protons.

One sees that the properties of the correlation functions in the singlet and triplet states are completely different. When Δp is small the correlation function is negative only for the triplet state. It is -1 at $\Delta p = 0$ for the triplet state, whereas it is close to zero for the singlet state. While for the former case the correlation function crosses zero at two points, it does not have a null point for the singlet case. It is also worthwhile mentioning that the effect of the Coulomb interaction between the protons can be switched off and the resulting correlation function $\mathcal{C}(\Delta p)$ multiplied by the Gamow factor in Eq. (13) yields a result (not shown in Fig. 2) slightly different than the long-dashed curve. In fact, the Gamow factor of Eq. (13) tends to underestimate the Coulomb final state interaction between the protons.

The method described above is directly applicable to determine the spin mixing of final states in low-energy two-proton nuclear decay for $0^+ \to 0^+$ transitions. In this case the final spin wave function of the pair equals that of the initial wave function. In particular, when singlet states are identified, spin-spin coincidence experiments will generate dichotomic outcomes for each single measurement.

Fig. 3 shows the correlation function, $C(\Delta p)$, for $r_0 = 4$ fm and for different admixtures of singlet and triplet states. The dotted, dashed and solid lines correspond to $\mathcal{M} = 0.1$, 0.5 and 0.9, respectively. $\mathcal{M}$ is the absolute contribution of the triplet state. One clearly sees that different admixtures lead to very different dependence on Δp.

Summarizing the above, we can say that the strong and Coulomb final state interactions cannot be neglected when the volume of spatial separation of the two-proton wavefunction is measured by $r_0 < 6$ fm. When $r_0 < 4$ fm the strong final state interaction is noticeable in the relative motion spectrum of the two-protons (see Fig. 1) and its presence is reflected in the strong reduction of the relative energy. The tidal Coulomb force due to the charge of the daughter nucleus tends to increase considerably the relative motion of the two protons. The same applies for the Coulomb repulsion between the protons due to their own charge. These results point to the importance of considering FSI in the experimental analysis of two-proton decay experiments. The effect of final state interactions are also visible in correlation functions which are considerably modified as the initial separation of the two-protons are probed. This is shown in Fig. 4, where the correlation function, $C(\Delta p)$, is plotted for two-proton triplet state decay of ^{45}Fe with $r_0 = 4$ fm (solid curve) and $r_0 = 10$ fm (solid curve).

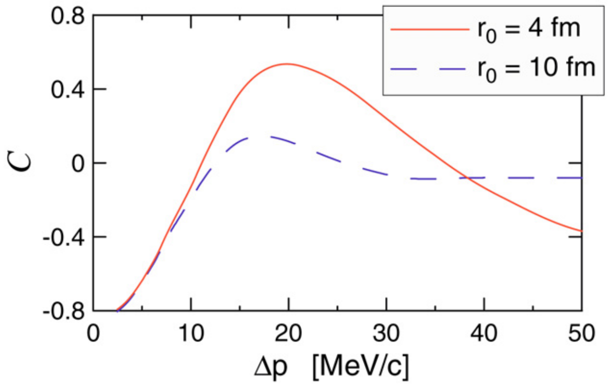

Fig. 4. Correlation function, $C(\Delta p)$, for two-proton triplet state decay of ^{45}Fe with $r_0 = 4$ fm (solid curve) and $r_0 = 10$ fm (solid curve).

We have shown that correlation functions, when defined appropriately, can clearly resolve the statistic nature of the diproton spin state in nuclear decay. This paves another route to study important problems of basic quantum mechanics interest, such as the Einstein–Podolski–Rosen paradox [15] and Schrödinger cat states. Two-proton radioactivity may supply yet another test of the EPR dilemma, namely whether the spin state function provides a complete description of quantum mechanics as we know it. These studies would be complementary to others performed in quantum optics and atomic physics, as well as in some instances of nuclear physics [17,19].

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